Exponential Smoothing Forecasts

PREREQUISITE TOOLS

None.

USAGE

PURPOSE

Exponential smoothing provides short-term forecasts of variables by extrapolating from past data.

USES

Exponential smoothing is used to:

1) Forecast demand for services or goods.
2) Obtain economic forecasts.
3) Forecast any variable where past behavior is expected to continue.
4) Provide forecasts at regular intervals.
5) Trace an underlying trend or pattern for a variable when random fluctuations in the data obscure that trend.

KEY DEFINITIONS

1) The *smoothed value* is an estimate of the average value of the variable being forecast. It is calculated each period by the equation:

$$S_t = S_{t-1} + a(x_t - S_{t-1})$$

where

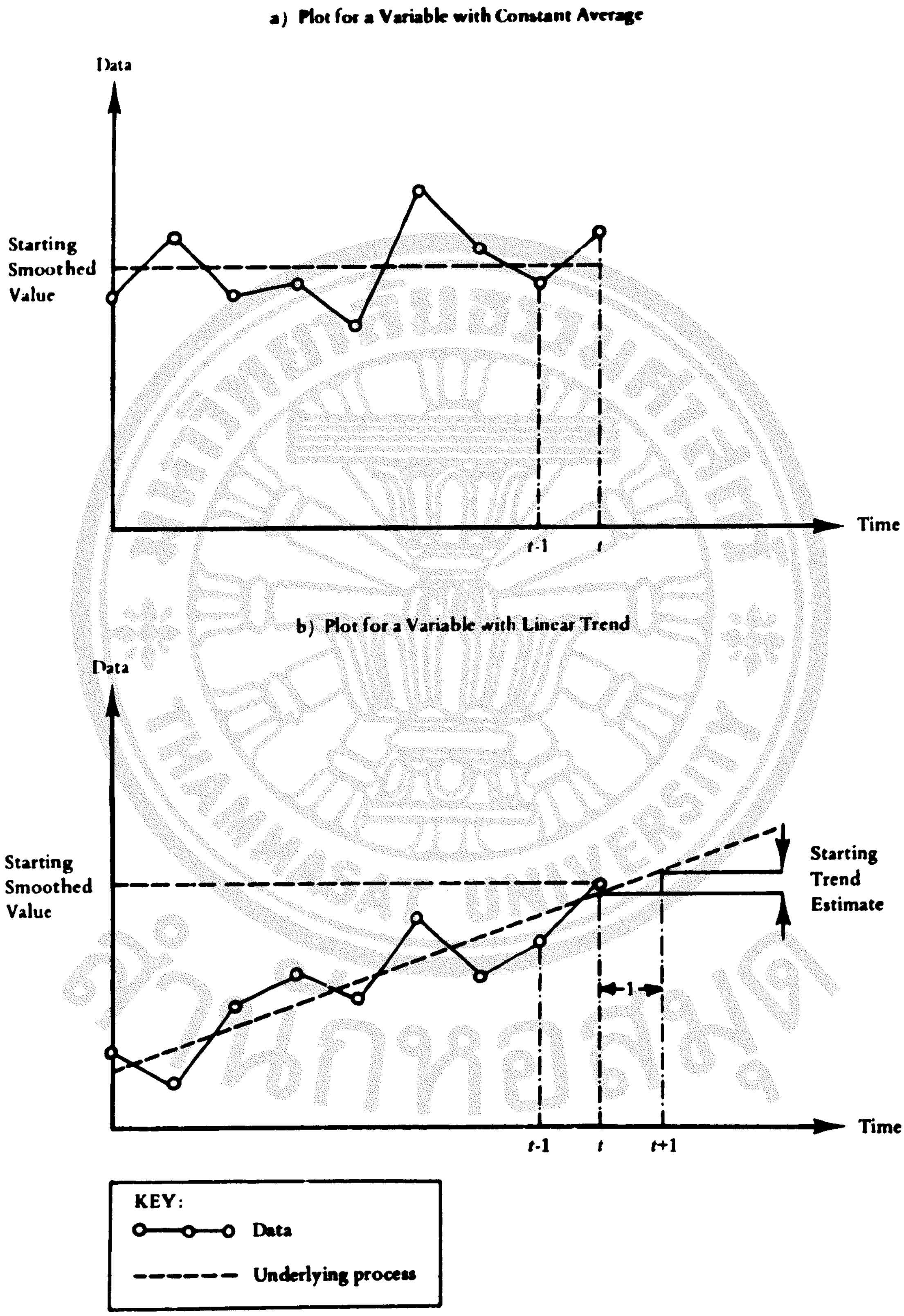
S_t = the new smoothed value
S_{t-1} = the old smoothed value
a = a smoothing constant
x_t = new datum

2) The *period* of a time series is the time interval between successive observations of the underlying process. This interval may be a day, a week, a month, or one or more years. For example, the period would be a week if the datum is the weekly total of immunizations performed.

SHORT DESCRIPTION

A *smoothed value* of the average of the data is the basis for forecasting by exponential smoothing. This value is calculated for each *period* using the data for that period and the smoothed value from the previous period. The new smoothed value becomes the forecast for the next period if the average value for the variable is expected to remain constant (see figure 1a). However, variables with a steadily increasing or decreasing average (a trend) can also be forecast by obtaining a smoothed value for the average and a smoothed estimate for the trend component. The forecast for the next period is the sum of the two estimates (see figure 1b).

FIGURE 1
Plots of Past Data



ADVANTAGES

1) Exponential smoothing is easy to understand and use since it relies on intuition and simple mathematics.

2) When changes occur in the behavior of the variable being forecast, exponential smoothing can continue to be used since gross errors in forecasting smooth out after a few periods.

3) Large quantities of past data need not be retained (see Regression Forecasting, RGF, page 160).

LIMITATIONS

1) Exponential smoothing is not a causal model; it only extrapolates from past data. Since past behavior only partly explains the future, exponential smoothing may not always be sufficient.

2) A more complex smoothing model is necessary to accurately forecast cyclic variations in data (see Montgomery, 1968).

REQUIRED RESOURCES

LEVEL OF EFFORT

Exponential smoothing involves the substitution of numerical values into simple formulae. The effort required is minimal.

SKILL LEVEL

Basic arithmetic skills are needed to use exponential smoothing. Some experience in choosing the *smoothing constant* is necessary to obtain good forecasts.

TIME REQUIRED

Once the smoothing constant has been selected, the forecast calculations are straightforward and require little time. It may be desirable to keep track of the errors in each forecast and take corrective action should they become too large (see Brown, 1965). This requires a minimum of extra time.

SPECIAL REQUIREMENTS

A slide rule or a calculator may be used to do the calculations. A digital computer may be desirable to forecast a large number of variables.

DESCRIPTION OF TOOL

SUPPLEMENTAL DEFINITIONS

1) The *smoothing constant* is a fraction between 0 and 1 that indicates the degree of confidence placed on the most recent datum. It is denoted by "a" in [1].

2) x_t and x_{t-1} are data values observed at time t and t-1 respectively.

3) S_t is the smoothed estimate of the average value of the variable for the time period t.

4) A_t is an estimate for the linear trend for period t.

5) T is the forecast lead time, or the number of periods into the future for which the forecast is being made.

6) a and β denote the smoothing constants whose values lie between 0 and 1.

7) x_{t+1} and x_{t+T} are forecasted estimates of the variable x calculated at time t for the next period and for T periods ahead, respectively.

REQUIRED INPUTS

Some understanding of the variable being forecast helps in estimating any trend that may be observable in past data. The data may be plotted on a graph against time (see figure 1). The variation of the data is shown in these plots. Visual inspection indicates the presence of an increasing trend in the plot shown in figure 1b.

Selecting a smoothing constant is also necessary before forecasting can be done. The function of the smoothing constant is to control the amount of importance given to the past data. The constant is greater than 0 and is usually less than 0.3. The smaller the smoothing constant, the greater is the importance given to past data, signaling confidence that the past behavior of the variable will continue. On the other hand, a large smoothing constant (but always less than 1) gives more importance to the current datum. However, a large smoothing constant may lead to large errors in the forecasts. A value of 0.2 is recommended for most applications.

Initial values for the smoothed estimates are needed before forecasting. The starting smoothed value can be taken from the plot of the past data. When the graph indicates that the data have no trend, only the smoothed value for the average needs to be estimated. This can be taken as the height of the horizontal line drawn through the data (see figure 1a). If the variable appears to follow a trend, starting values for both an average value and a trend component are required.

TOOL OUTPUT

The output is a short-term forecast of the future values of the variable. The forecast is computed from the estimate of the average value or from the estimates of the average value and the trend component, whichever is appropriate.

IMPORTANT ASSUMPTIONS

The model presented here is based on the assumption that the process which produced the behavior of the forecasted variable does not change with time. That is, the variable is assumed to have either a constant average or a constant linear trend. Higher order exponential smoothing models may be used if these assumptions are not valid for a variable (see Brown, 1965).

METHOD OF USE

GENERAL PROCEDURE

Forecasting by exponential smoothing is done in two steps:

1) Updating the smoothed values.
2) Obtaining a forecast from the smoothed values.

In the case of a variable with constant average, step 1 will be used to update the smoothed value for the average. In case of a variable with a trend, step 1 will be used to estimate the trend and the average value by smoothing.

Variable with Constant Average

1. **Update smoothed values.**
   The smoothed value for the average is obtained by the following equation:
   $$S_t = S_{t-1} + a(x_t - S_{t-1}) \qquad [1]$$
   The difference between the new datum and the old smoothed value gives an idea of the error in the forecast. A fraction, a, of this error is added to the old smoothed value to obtain a new smoothed value.

2. **Obtain the forecast.**
   The forecast is given by the following equation:
   $$x_{t+1} = S_t \qquad [2]$$
   This equation is used since the variable is assumed to have a constant average estimated by the smoothed value and no trends. When datum for the next period, x_{t+1}, is obtained, the new smoothed value becomes the old smoothed value for the next period, and forecasting is continued by computing the smoothed value for that period. These calculations are repeated.

Variable with Trend

1. **Update smoothed values.**
   Obtain a smoothed value for the average and an estimate for the trend. The equations used for the two are similar. Equation [1] is used to estimate the average. The equation used for estimating the trend is:
   $$A_t = A_{t-1} + \beta[(S_t - S_{t-1}) - A_{t-1}] \qquad [3]$$

2. **Obtain the forecast.**
   Add the trend estimate to the smoothed value that has been newly calculated. The forecast is given by the following equation:
   $$x_{t+1} = S_t + A_t \qquad [4]$$
   If forecasting is to be done for more than one time period ahead, the increase (or decrease) due to the trend component needs to be accounted for. The forecast for T time periods in the future is given by:
   $$x_{t+T} = S_t + T(A_t) \qquad [5]$$
   Note that in case of a variable with no trend component, the forecast for one period ahead and the forecast for many periods ahead is the same.

   As before, when a new datum is available, the new smoothed value and the trend estimate become the old smoothed value and the old trend estimate for the next period. The calculations are repeated in order to update smoothed values.

Worksheet

A worksheet can be used to facilitate the calculation of forecasts on a regular basis (see figure 2). The columns in figure 2 represent different stages in the calculation, with the forecast given in the final column and the datum given in the first column. Each row corresponds to a time period.

EXAMPLE

It is necessary to forecast the number of births in a district each month in order to procure child immunization medicine. Exponential smoothing is used to forecast the number of births. A slight increasing trend is assumed to be present in the data (see figure 2 for the calculations).

The worksheet shows that the forecast for births in February is 207. The S_{t-1} and A_{t-1} values for March can be written in the row corresponding to March.

THEORY

The basic smoothing equation in exponential smoothing is [1], which can be rewritten as:
$$S_t = a(x_t) + (1 - a)S_{t-1} \qquad [6]$$

FIGURE 2
Work Sheet for Exponential Smoothing

	1 Data x_t	2 Old Smoothed Value S_{t-1}	3 Difference $(x_t - S_{t-1})$ (1) − (2)	4 Fraction Added α ( ) α (3)	5 New Smoothed Value S_t (2) + (4)	6 Old Trend Estimate A_{t-1}	7 Change in Smoothed Value $(S_t - S_{t-1})$ (5) − (2)	8 Difference [− A_{t-1}] (7) − (6)	9 Fraction Added β [ ] β (8)	10 New Trend Estimate A_t (6) + (8)	12 Forecast x_{t+1} (5) + (10)
January	205	200	5	1	201	3	1	-2	-.2	2.8	203.8 ~204
February	210	201	9	1.8	202.8 ≅203	2.8	2	-.8	-.08	2.72 ~2.7	205.7 ~206
March		203				2.7					

The smoothed value for the previous time period* was given by:

$$S_{t-1} = a(x_{t-1}) + (1 - a)S_{t-2} \qquad [7]$$

and the smoothed value for the period before is

$$S_{t-2} + a(x_{t-2}) + a(1 - a)S_{t-3} \qquad [8]$$

and so on. By repeated substitution for S_{t-1} into [6] and then for S_{t-2} into [7], we obtain:

$$S_t = a(x_t) + a(1 - a)x_{t-1} + \ldots + a(1 - a)^n x_{t-n} \qquad [9]$$

From this equation it can be seen that past data (x_{t-1}, x_{t-2}, x_{t-3}, etc.) have decreasing exponential weights. Hence the name exponential smoothing. The weights of the past data decrease with the age of the data because a is less than 1. This makes intuitive sense as more recent data are given more weight. The actual weights depend on the smoothing constant. The earlier discussion on the value of the smoothing constant follows directly from [8].

*t = present time, therefore t-1 is one time period prior, t-2 is two time periods prior, etc.

Similar formulations of exponential smoothing have been developed to forecast data that have a cyclic variation or a combination of a trend and cyclic variation. Montgomery (1968) and Brown (1965) discuss these cases at length.

BIBLIOGRAPHY

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