APPENDIX D

The estimated shift parameters are however base on the data of price indices. As discussed in the chapter 4, the f.o.b. prices indices using as a proxies of exportable prices over-estimates their producer prices; the required price data in our model. Hence the estimated shift coefficients might have some bias from the actual values which will be discussed below. Recall our model, equation (4.2), in the chapter 4.

(4.2) $$\log(P_h/P_x) = a + b\log(P_m/P_x) - \mu_t$$

Let us simplify it as,

(D.1) $$Y_t = a + bX_t + \mu_t$$

where $Y_t = \log(P_h/P_x)$ = true (but unobservable) value of the dependent variable

$X_t = \log(P_m/P_x)$ = true (but unobservable) value of the explanatary variable

In our sample we observed the over value of P_x. It means that we employed the lower value of both Y and X which are related to the true value by the following relationships.

(D.2) $$Y_t^* = Y_t - V_t$$

(D.3) $$X_t^* = X_t - V_t$$

Where Y_t^* and X_t^* are the observable values of Y_t and X_t, and V_t is random variable representing the error of measurment in Y_t and X_t

Before we expore the implications of this model, it is necessory to specify more precisely the behavior patterns of the errors μ_t and V_t.[1]

We assume that both μ_t and V_t sastisfy the usual set of assumptions of random variables, that is

(1) $E(\mu_t) = 0$ and $E(V_t) = 0$, the error terms have zero means

(2) $E(\mu_t^2) = \sigma_\mu^2$ and $E(V_t^2) = \sigma_V^2$

(3) $E(\mu_t \mu_{t-1}) = 0$ and $E(V_t V_{t-1}) = 0$

(4) The error in measuring the variables is independent from the values of these variables.

$$E(V_t Y_t) = 0 \qquad E(V_t X_t) = 0$$

(5) The error μ_t is independent from the true value X; $E(\mu_t X_t) = 0$

(6) The two error terms are not connected in any way; $E(\mu_t V_t) = 0$

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[1]See errors in variables in Walters, A.A., An Introduction to Econometrics. (New York : WW Norton & Company, INC, 1970), pp.108-114 and Koutsoyiannis, A., Theory of Econometrics : An Introductory Exposition of Econometric Methods. (London and Busing Stoke : the Macmillan Press, 1977), pp.258-275.

Then substituting the observed values Y_t^* and X_t^* into the true relationship, we find

(D.4) $$Y_t^* + V_t = a + b(X_t^* + V_t) + \mu_t$$

(D.5) $$Y_t^* = a + b(X_t^* + V_t) + (\mu_t - V_t)$$

(D.6) $$Y_t^* = a_1 + b_1(X_t^*) + U_t$$

Where $U_t = (\mu_t + bV_t - V_t)$ is the error term in the new model (D.6).

Thus in practice we have a set of observed values Y_t^* and X_t^* which include errors. Y_t^* includes stochastic errors as well as measurement errors, while X_t^* includes only errors of measurement. From the observed values of Y_t^* and X_t^*, we can obtain estimates of b_1 by the method of classical least squares as follows,

(D.7) $$b_1 = \frac{\Sigma x^* y^*}{\Sigma x^*} = \frac{\Sigma(X^*-\bar{X}^*)(Y^*-\bar{Y}^*)}{\Sigma(X^*-\bar{X}^*)^2}.$$

Substitute $Y^* = Y_t - V_t$, $\bar{Y}^* = \bar{Y}_t - \bar{V}_t$

$X^* = X_t - V_t$, $\bar{X}^* = \bar{X}_t - \bar{V}_t$

and obtain

(D.8) $$b_1 = \frac{\Sigma(X_t-\bar{X}_t)(Y_t-\bar{Y}_t) + \Sigma(Y_t-\bar{Y}_t)(V_t-\bar{V}_t) + \Sigma(X_t-\bar{X}_t)(V_t-\bar{V}_t) + \Sigma(V_t-\bar{V}_t)^2}{\Sigma(X_t-\bar{X}_t)^2 + 2\Sigma(X_t - \bar{X}_t)\ (V_t-\bar{V}_t) + \Sigma(V_t-\bar{V}_t)^2}$$

Taking into account that the error of measurment (V_t) is independent from the true of the variable [by the assumption (4)], the middle two terms of the numerator and the middle term of the denominator are zero asymptotically (as $n \rightarrow \infty$). Therefore in the limit

$$\text{(D.9)} \qquad \operatorname{plim}(b_1) = \frac{\Sigma(X_t - \bar{X}_t)(Y_t - \bar{Y}_t) + n\sigma_v^2}{\Sigma(X_t - \bar{X}_t)^2 + n\sigma_v^2}$$

Multiplying $\dfrac{\Sigma(X_t-\bar{X}_t)^2}{\Sigma(X_t-\bar{X}_t)^2}$ to equation (D.9), we get

$$\text{(D.10)} \qquad \operatorname{plim}(b_1) = \frac{[\Sigma(X_t-\bar{X}_t)(Y_t-\bar{Y}_t)/\Sigma(X_t-\bar{X}_t)^2] + [n\sigma_v^2/\Sigma(X_t-\bar{X}_t)^2]}{1 + [n\sigma_v^2/\Sigma(X_t-\bar{X}_t)^2]}$$

$$= \frac{b + \sigma_v^2/\sigma_x^2}{1 + \sigma_v^2/\sigma_x^2}$$

Since $0 < b < 1$ and $\sigma_v^2/\sigma_x^2 > 0$ (being the ratio of squares, plim $(b_1) \leq b$. That is b_1 may be biased asymptotically. If b is not equal to one, the OLS b_1 then under estimates the true parameter b.

However the error of measurement in our model occurs in both dependent and explanatory variables in the same size and manner.[2] Moreover the model is in the logarithmic form, the scale of the bias, thereby, is reduced about ten times. Hence the error might not effect our estimates much.

But if the error of measurement in Y_t and X_t is not random, the OLS b_1 then is exactly the same as b. That

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[2]See the figure 4.1-4.4 in the chapter 4.

is there is no error in our estimation. It can be explicitly shown by reexamining equation (D.8), given V_t as constant term. Then the last three terms of the numerator and the last two terms of the denominator are zero asymptotically. Therefore we obtain the probability limit of b_1,

$$\text{(D.11)} \qquad \text{plim}(b_1) = \frac{\Sigma(X_t - \bar{X}_t)(Y_t - \bar{Y}_t)}{\Sigma(X_t - \bar{X}_t)^2} = b$$

Thus the OLS b_1 is equal to the true coeffient b.