CHAPTER 3

THEORETICAL MODEL

This chapter introduces two important model; the Kemp-Jones model and the comparative advantage index (CAI) developed by Saito (1999). The former explains the theoretical foundation for the CAI index. The latter introduces the way to construct the index.

3.1 <u>The Kemp-Jones Model</u>

In this study, the Kemp-Jones model is based on Ruffin (1984)[1]. The model assumes that technological differences and perfect capital mobility are introduced into the Heckscher-Ohlin model. Indeed, it verifies that analysis of labor productivity among countries can indicate the comparative advantage among them by considering differences in technological progress[2].

To show technological progress in terms of labor productivity change, assume that a_{Li} (L_i/X_i) and a_{Ki} (K_i/X_i) are the labor and capital requirements for good i, respectively. In an economy, there are labor intensive goods (X_1) and capital intensive good (X_2). Given that competitive pricing holds, we obtain the following conditions:

$$\begin{aligned} a_{L1}w + a_{K1}r &= 1 \\ a_{L2}w + a_{K2}r &= p \end{aligned} \qquad (3.1)$$

where w, r and p denote the wage rate, rental price and the price of X_2, respectively.

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[1] The Kemp-Jones model was developed by Kemp and Jones.

[2] Additionally, the model shows that "trade patterns in such a world is Ricardian since they reflect technology difference rather than factor endowments" (Saito, 1999).

Let θ_{ji} represent the cost share of input j into goods i, i.e. $\theta_{L1} = a_{L1}w$ and $\theta_{L2} = (a_{L2}w)/p$. By totally differentiating (3.1) and letting ^ denote the proportionate change in a variable, for example, $\hat{w} = \partial w / w$. Then we obtain:

$$\begin{aligned} \theta_{L1}\hat{w} + \theta_{K1}\hat{r} &= \pi_1 \\ \theta_{L2}\hat{w} + \theta_{K2}\hat{r} &= \pi_2 + \hat{p} \end{aligned} \qquad (3.2)$$

where $\pi_1 = -(\theta_{L1}\hat{a}_{L1} + \theta_{K1}\hat{a}_{K1})$ and $\pi_2 = -(\theta_{L2}\hat{a}_{L2} + \theta_{K2}\hat{a}_{K2})$. If there is technological progress (the percentage change in factor requirements, $\hat{a}_{ji}$, is negative), then π_i takes a positive value. Specifically, π_i represents the percentage reduction in production cost of X_i due to decreasing in the factor requirements ($\hat{a}_{ji}$) at constant factor price[3].

Solving equation (3.2) for $\hat{w}$ and $\hat{r}$, then obtain following condition:

$$\begin{aligned} \hat{w} &= \frac{\theta_{K2}\pi_1 - \theta_{K1}(\pi_2 + \hat{p})}{\theta} \\ \hat{r} &= \frac{\theta_{L1}(\pi_2 + \hat{p}) - \theta_{L2}\pi_1}{\theta}; \ \theta = \theta_{K2} - \theta_{K1} = \theta_{L1} - \theta_{L2} > 0 \end{aligned} \qquad (3.3)$$

From equation (3.3), it gives us three important points. Firstly, if there is no technological progress (no cost reduction) in both sectors, $\pi_i = 0$ for i = 1 and 2, then the Stolper-Samuelson theorem adheres and equation (3.3) becomes as follows:

$$\begin{aligned} \hat{w} &= -\theta_{Ki}\hat{p} < 0 \\ \hat{r} &= \frac{\theta_{L1}}{\theta}\hat{p} \ \ or \ \ \frac{\hat{r}}{\hat{p}} = \frac{\theta_{L1}}{\theta} > 1 \end{aligned}$$

[3] The Hick-Neutral is assumed in the model, and then it necessarily implies that the rate of technological progress in both labor and capital are the same. Thus we have following condition: $-(\theta_{L2}\hat{a}_{L2} + \theta_{K2}\hat{a}_{K2}) = -\hat{a}_{L2}(\theta_{L2} + \theta_{K2}) = -\hat{a}_{L2} = \pi_2$.

Secondly, given technological progress occurs in both sectors, the p-r curve will shift to the right ($\hat{r} > 0$). See Figure 1.B in Appendix B[4] if

$$\theta_{L1}\pi_2 - \theta_{L2}\pi_1 > 0\,. \qquad (3.4)$$

If we assume Hicksian-uniform technical progress, then we obtain following condition: $\hat{r} = \pi_1 = \pi_2$. The shift in the p-r curve (equation 3.4) due to technical progress could be expressed as:

$$1 > \frac{\theta_{L2}/(1+\pi_2)}{\theta_{L1}/(1+\pi_1)}$$

where $\theta_{Li}/(1+\pi_i)$ denotes the share of labor costs in sector i discounted by technological progress.

Thirdly, given that there are two countries, home (denoted as h) and a foreign (denoted as f) country, the two are assumed to be different in their respective rates of technological progress. Additionally, suppose that both countries' rates of return on capital are the same as equilibrium rate (r*) in the world market. If the relative price of a capital intensive good in home country (p^h) is lower than that of foreign country (p^f), then, for any positive technical progress ($\pi_i > 0$), a comparative advantage in the production of the capital intensive good in the home country needs to be satisfied in terms of the following condition (see Figure 2.B in Appendix B):

$$\frac{\theta_{L2}^h/(1+\pi_2^h)}{\theta_{L1}^h/(1+\pi_1^h)} < \frac{\theta_{L2}^f/(1+\pi_2^f)}{\theta_{L1}^f/(1+\pi_1^f)} \qquad (3.5)$$

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[4] Consider the case of $\hat{p} = 0$ and the capital intensive good sector. Figure 1.B, 2.B and 3.B were including in Saito, 1999.

Initially, the Kemp-Jones model suggests that labor cost share analysis could explain international comparative advantage. Indeed, the model assumes that factor price equalization holds here. Thus the source of a reduction in labor cost shares is only from technological progress. However, different wage rates are actually observed across sectors in each country. It is probably due to resistance in labor mobility across sectors within country. Under this circumstance, sector-specific wage rates can change at different rates (Saito, 1999). If there is any change in the sector-specific wage-rentals ratio (due to differences in sector-specific wage rates), then the capital-labor ratio will change over time in each sector. When the capital-labor ratio increases, the capital cost share also increases while the labor cost share decreases in each sector. In the presence of different wage rates across sectors, technological progress is not necessarily the only source of a reduction in the labor cost share.

To show labor market distortion effects on the labor cost share, assume that there is no change in technological progress in the labor-intensive good sector ($\pi_1 = 0$) and the world interest rate ($\hat{r} = 0$), see Figure 3.B in Appendix B. These assumptions imply that there is no change in wage rates ($\hat{w} = 0$), see equation (3.2). In other words, factor prices are assumed to be constant. Due to the constant factor prices, we could see a reduction in the labor cost share caused by a change in the labor input requirement.

Figure 3.B shows the unit isoquant for the capital-intensive good sector. Given that point 0 denotes the initial choice of the capital-labor ratio, the labor cost share at this point is $\theta_{L2} = (a_{L2}w)/p$. If there is a cost reduction in this sector, then the unit isoquant will shift to the origin (moving from 0 to A). At point A, the labor cost share is $\theta_{L2}^{A} = (a_{L2}^{A}w)/p$ and is lower than at point 0. In case of imperfect labor mobility without a cost reduction, the choice of the capital-labor ratio takes place at point B. It implies that the sector-specific wage rate is higher than w at this point. The labor cost share at this point is $\theta_{L2}^{B} = (a_{L2}^{B}w^{B})/p$, which could be lower than θ_{L2}. According to Figure 3.B, we can observe that a reduction in the labor cost share is caused by technological progress and resistance in labor mobility across sectors within country. However, the change in the labor cost share that we observe in the data cannot distinguish between the two effects.

In conclusion, the Kemp-Jones model shows that technological progress has an influence on productivity. The model also suggests an alternative analysis of comparative advantage among two countries by considering the relative labor cost shares. Finally, the model points out the two sources of a reduction of production cost: technological progress and resistance in labor market mobility (the choice of capital-labor ratio). In the next section, we will focus on all details of the comparative advantage index (CAI) introduced by Saito 1999.

3.2 The Comparative Advantage Index (CAI)

Under the Kemp-Jones model, Saito (1999) developed new measurement (called the CAI index) based on labor cost share and technological progress analysis. In this section, there are three important parts. The first part introduces the specification of production technology and describes assumptions in the production function. Then, the second part illustrates the method of computing and decomposing the CAI index. The final part is the interpretation of the CAI index.

3.2.1 Production Technology

Indeed, the CAI index is directly computed from the production function (called "the specification of production technology") which has labor and capital inputs. The production technology could capture effects of technical progress and choice of capital-labor ratio on productivity. As a result, the estimation of the production technology allows us to decompose the CAI index. This section focuses on the characteristics of production technology in order to explain the foundations of CAI index computation.

The production technology for all countries and all tradable sectors are defined under the following assumptions (Saito, 1999).

Under the first assumption, all countries are supposed to have access to the same production technology[5]. In other words, it is assumed that "all countries have the same underlying aggregate production function[6] F (.), sometimes referred to as a meta-production function, but may operate on different parts of it" (Boskin and Lau, 1992). This production function in terms of "efficient-equivalent" quantities of outputs and inputs can be expressed as: for all country m = 1,….,M, and all tradable good sector i = 1,….,I, at time t = 1,….,T:

$$X_{it}^{m*} = F_{it}(k_{it}^{m*}, l_{it}^{m*}, K_{it}^{*}) \qquad (3.6)$$

where X_{it}^{m*}, k_{it}^{m*}, and l_{it}^{m*} refer to quantities of output, capital and number of labor for each sector i in country m, respectively. On the other hand, K_{it}^{*} is the efficiency equivalent level of world aggregate capital in each sector i. It also represents capital mobility across countries. In other words, it allows the external economies of scale from the worldwide size of capital in each industry to exist[7].

However, in fact, the efficiency equivalent of outputs and inputs are not directly observed. The second assumption consequently assumes that the efficiency equivalent quantities of outputs and inputs, for each country, are assumed to be linked to the measured quantities of outputs and inputs, through time-varying, country- and commodity specific augmentation factors $A_i(t)$[8]. The commodity augmentation factors are assumed to have a constant exponential form with respect to time. Thus we obtain following equations:

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[5] It is the basic assumption of "the meta-production function approach" introduced by Boskin and Lau (1990). This approach is useful for analyzing productivity and technological progress by using the direct econometric estimation of the meta-production function. See more detail in Boskin and Lau (1992).

[6] In the long run, the underlying aggregate production function is assumed to change due to only technological progress (the reduction of inputs requirement). This production function could thus capture the effect of technical progress.

[7] Interestingly, the production technology can identify the separate effect of the degree of return to scale and the rate of technological progress in each sector i, see equation (3.8) and (3.9).

[8] Specifically, the commodity augmentation factors are introduced to capture the difference in technological progress across countries.

$$\begin{aligned} X_{it}^{m*} &= X_{it}^{m} \\ k_{it}^{m*} &= A_i^m \exp(\lambda_i^m \cdot t) k_{it}^m \\ l_{it}^{m*} &= A_i^m \exp(\lambda_i^m \cdot t) l_{it}^m \\ K_{it}^{*} &= K_{it} \end{aligned} \tag{3.7}$$

In this study, X_{it}^{m*} and K_{it}^{*} are assumed to be the same as directly observable. In contrast, the efficient equivalent of both inputs is adjusted by the initial efficiency level of input, A_i^m, and the rate of technical progress, λ_i^m. Notice that the rate of technical progress in labor and capital input are the same in each industrial sector because of the existence of Hick-neutral technical progress[9].

The third assumption supposes that the production function $F_{it}(.)$ has a transcendental logarithmic (translog) functional form. The translog functional form was introduced by Christensen et al, 1973. Under this assumption, the production function in equation (3.6) takes the form as follows:

$$\begin{aligned} \ln X_{it}^{m*} = {} & \ln X_{0i} + \alpha_{ki} \ln k_{it}^{m*} + \alpha_{li} \ln l_{it}^{m*} + \alpha_{Ki} \ln K_{it}^{*} \\ & + \beta_{kki} \frac{(\ln k_{it}^{m*})^2}{2} + \beta_{lli} \frac{(\ln l_{it}^{m*})^2}{2} + \beta_{KKi} \frac{(\ln K_{it}^{*})^2}{2} \\ & + \beta_{kli} (\ln k_{it}^{m*})(\ln l_{it}^{m*}) + \beta_{kKi} (\ln k_{it}^{m*})(\ln K_{it}^{*}) + \beta_{lKi} (\ln l_{it}^{m*})(\ln K_{it}^{*}) \end{aligned} \tag{3.8}$$

From equation (3.8), it allows us to test some standard assumptions of technological progress parameters such as tests of constant returns to scale.

[9] In terms of efficiency equivalent units, the production functions are identical across countries. However, in terms of measured quantities of outputs and input, the production functions of any two countries are not necessarily the same (Boskin and Lau, 1992). Thus the rate of technical progress needs not to be the same in any two countries.

Finally, the last assumption is that the only source of increasing return to scale is through external economies of scale from the worldwide size of capital in each industry. In other words, it assumes that firms are in competitive markets and their production function is a constant return to scale[10]. This assumption leads to the following conditions for each commodity sector i.

$$\alpha_{ki} + \alpha_{li} = 1$$
$$\beta_{kki} + \beta_{kli} = \beta_{lli} + \beta_{kli} = \beta_{kKi} + \beta_{lKi} = 0$$

By substituting equation (3.7) into equation (3.8) while holding the above conditions, the production function is as follows: for country m and tradable commodity sector i, at time t:

$$\ln\frac{X_{it}^m}{l_{it}^m} = \ln X_{0i} + \ln A_i^m + \lambda_i^m \cdot t + \alpha_{ki}\ln\frac{k_{it}^m}{l_{it}^m} + \alpha_{Ki}\ln K_{it} + \frac{\beta_{kki}}{2}(\ln\frac{k_{it}^m}{l_{it}^m})^2 + \frac{\beta_{KKi}}{2}(\ln K_{it})^2 + \beta_{kKi}(\ln\frac{k_{it}^m}{l_{it}^m})(\ln K_{it}) \quad (3.9)$$

[10] In this study, a CRS assumption must be tested. Actually, this assumption has a benefit for regression estimation. Since this assumption holds, then it can help to reduce the number of parameters in the model. Furthermore, this assumption also avoids the ambiguous effect of external economies of scale from world wide of capital which is caused by technological progress. Given that the production function is not CRS, if there is technological progress, the economies of scale effect will be taken from domestic capital inputs and worldwide of capital inputs in each industry. In this case, we can not capture exactly the effect of economies of scale from external economies of scale from the worldwide size of capital.

3.2.2 The CAI Decomposition

The Kemp-Jones model and the four assumptions in the production function are the main basis of the CAI index. This section describes the method of computing the index and also shows the way to decompose the index into four components: productivity effect, technology effect, endowment effect, and wage effect[11].

The CAI index is basically derived from the Ricardian measure which represents the law of comparative advantage. Saito (2004) showed the way to draw the index from the Ricardian measure as following.

Recall to the Ricardian measure, assume that p_{it}^{m} is the price of a good in a given country, where superscript m (or n) refers to country and subscript i (or j) represents the commodity sector. Under the law of comparative advantage, if the relative price of the commodity in the two countries satisfies:

$$\frac{p_{it}^{m}}{p_{jt}^{m}} < \frac{p_{it}^{n}}{p_{jt}^{n}} \qquad (3.10)$$

then country m has a comparative advantage in producing commodity i and will export commodity i at time t. In the Ricardian model, it is assumed that labor input is only one factor in the production of both goods. Given that c_{it}^{m} is an unit cost of producing commodity i in country m, if $p_{it}^{m} = c_{it}^{m}$, then the price of commodity i can be expressed as:

$$p_{it}^{m} = c_{it}^{m} = w_{it}^{m} \frac{l_{it}^{m}}{X_{it}^{m}}$$

[11] In decomposition of the index, the productivity effect can be decomposed into technology effect and endowment effect. In other words, total technology effect and endowment effect is equal to productivity effect. Moreover technology effect is actually defined as Hicksian measure (HICKS), see Saito (2004).

where w_{it}^{m}, l_{it}^{m}, X_{it}^{m} are the wage rate, labor input, and output, respectively. Therefore the Ricardian measure can express an alternative condition as follows.

$$\frac{w_{it}^{m}\dfrac{l_{it}^{m}}{X_{it}^{m}}}{w_{jt}^{m}\dfrac{l_{jt}^{m}}{X_{jt}^{m}}} < \frac{w_{it}^{n}\dfrac{l_{it}^{n}}{X_{it}^{n}}}{w_{jt}^{n}\dfrac{l_{jt}^{n}}{X_{jt}^{n}}} \qquad (3.11)$$

Under the Ricardian model, the labor input requirement is denoted by a constant a_{Li}^{m} (or l_{i}^{m} / X_{i}^{m}). This model also supposes that the wage rates are the same in both sectors of each country, for example, $w_{i}^{m} = w_{j}^{m}$. Then equation (3.11) will be expressed as.

$$\frac{a_{Li}^{m}}{a_{Lj}^{m}} < \frac{a_{Li}^{n}}{a_{Lj}^{n}} \qquad (3.12)$$

For equation (3.12), it says that the difference in labor productivity can indicate the comparative advantage across any two countries. However, in fact, factor price are not necessarily equalized in the real world. Saito (1999) defined and computed the new measurement in terms of the Ricardian measure of technology differences (called the CAI index) as follows[12].

$$CAI_{ijt}^{mn} = \ln\frac{w_{it}^{m}\dfrac{l_{it}^{m}}{X_{it}^{m}}}{w_{jt}^{m}\dfrac{l_{jt}^{m}}{X_{jt}^{m}}} - \ln\frac{w_{it}^{n}\dfrac{l_{it}^{n}}{X_{it}^{n}}}{w_{jt}^{n}\dfrac{l_{jt}^{n}}{X_{jt}^{n}}} \qquad (3.13)$$

[12] The CAI index is expressed in the terms of a natural logarithm in order to see the different determinants of comparative advantage (from decomposition of the index).

The CAI index is thus represented by equation (3.13). To show the index's components, we need to follow two steps of decomposition. Firstly, the Ricardian measure (in term of the CAI index) needs to be decomposed into two components: a relative labor productivity component (or the labor productivity effect: PE) and a relative wage component (or the wage effect: WE). By rearranging equation (3.13), then it can be expressed as:

$$\begin{aligned} CAI_{ijt}^{mn} &= PE_{ijt}^{mn} + WE_{ijt}^{mn} \\ &= (\Delta \ln \frac{X_{jt}}{l_{jt}} - \Delta \ln \frac{X_{it}}{l_{it}}) + (\Delta \ln w_{it} - \Delta \ln w_{jt}) \end{aligned} \tag{3.14}$$

where $\Delta \ln(X_{it} / l_{it}) = \ln(X_{it}^{m} / l_{it}^{m}) - \ln(X_{it}^{n} / l_{it}^{n})$ is the cross-country difference in labor productivity in sector i, and $\Delta \ln w_{it} = (\ln w_{it}^{m} - \ln w_{it}^{n})$ is the cross-country difference in wage rate in sector i.

The second step is to decompose the labor productivity component[13] into two terms: a Hicksian measure (the direct technology effect, TE_{ijt}^{mn}) and an endowment measure (the indirect endowment effect, EE_{ijt}^{mn}). Therefore, the index's decomposition can be written as:

$$\begin{aligned} CAI_{ijt}^{mn} &= PE_{ijt}^{mn} + WE_{ijt}^{mn} \\ &= TE_{ijt}^{mn} + EE_{ijt}^{mn} + WE_{ijt}^{mn} \end{aligned} \tag{3.15}$$

[13] In equation (3.11), note that the labor cost share can represent the labor productivity in each sector. Under the Kemp-Jones model, technical progress is the only source of a reduction in the labor cost share (Saito, 1999). However, in the real world, the assumption of factor price equalization does not exist. Thus a reduction of the labor cost share is not only due to technical progress, but due to labor market distortions between two sectors (or the endowment effect).

In order to compute the CAI index, the productivity effect and the wage effect can be estimated directly by using the formula in the equation (3.14). In contrast, the technology effect and the endowment effect cannot be estimated directly. These two effects must be derived from the production function (recall equation (3.9)). For country m and n within sector i, the production functions are defined as follows, where c = m or n:

$$\ln\frac{X_{it}^{c}}{l_{it}^{c}} = \ln X_{0i} + \ln A_{i}^{c} + \lambda_{i}^{c}\cdot t + \alpha_{ki}\ln\frac{k_{it}^{c}}{l_{it}^{m}} + \alpha_{Ki}\ln K_{it}$$
$$+\frac{\beta_{kki}}{2}(\ln\frac{k_{it}^{c}}{l_{it}^{m}})^{2} + \frac{\beta_{KKi}}{2}(\ln K_{it})^{2} + \beta_{kKi}(\ln\frac{k_{it}^{c}}{l_{it}^{m}})(\ln K_{it}) + u_{it}^{c} \tag{3.16}$$

where $u_{it}^{c} = \rho_{i}^{c}u_{it-1}^{c} - v_{it}^{c}$ (the disturbance term in the production function).

In equation (3.16), notice that it has two country-specific technological parameters; A_i^c and λ_i^c. The two parameters represent the technology effect (Hicksian measure) among countries. " A_i^c captures differences in the initial of technology across counties (relative to that of a particular country X_{0i}), and λ_i^c captures difference in the rate of technical progress countries" (Saito, 2004). Additionally, there are two assumptions for the two technology parameters. Firstly, technological progress is Hicks neutral across factors within each sector[14]. Secondly, technological progress is not necessarily Hicks neutral across sectors. This assumption implies that $A_i^c \neq A_j^c$ and $\lambda_i^c \neq \lambda_j^c$.

[14] Hicks neutral technology implies that the shift in technological progress are common in both sectors; labor and capital inputs.

From equation (3.16), the cross-country difference in the labor productivity for each sector i at time t, $\ln(X_{it}^{m}/l_{it}^{m})-\ln(X_{it}^{n}/l_{it}^{n})$, or $\Delta\ln(X_{it}/l_{it})$, can be expressed as follows[15]:

$$\Delta\ln\frac{X_{it}}{l_{it}}=\Delta\ln A_i+\Delta\lambda_i\cdot t+\alpha_{ki}\Delta\ln\frac{k_{it}}{l_{it}}+\alpha_{Ki}\Delta\ln K_{it}$$
$$+\frac{\beta_{kki}}{2}\Delta(\ln\frac{k_{it}}{l_{it}})^2+\frac{\beta_{KKi}}{2}\Delta(\ln K_{it})^2+\beta_{kKi}\Delta(\ln\frac{k_{it}}{l_{it}})(\ln K_{it})+\Delta u_{it} \quad (3.17)$$

where $\Delta\ln\lambda_i=\ln\lambda_i^m-\ln\lambda_i^n$,

$\Delta\ln(k_{it}/l_{it})=\ln(k_{it}^m/l_{it}^m)-\ln(k_{it}^n/l_{it}^n)$

$\Delta(\ln(k_{it}/l_{it}))^2=(\ln(k_{it}^m/l_{it}^m))^2-(\ln(k_{it}^n/l_{it}^n))^2$

$\Delta u_{it}=u_{it}^m-u_{it}^n$.

However, two terms, $\alpha_{Ki}\Delta\ln K_i$ and $\beta_{KKi}\Delta(\ln K_i)^2$, are canceled out because both countries m and n have the same world aggregate level of industrial capital, K_i.

Therefore, the cross-country difference in labor productivity for each sector i can be decomposed into two components as follows:

$$\Delta\ln\frac{X_{it}}{l_{it}}=TE_{it}^{mn}+EE_{it}^{mn} \quad (3.18)$$

where $TE_{it}^{mn}=\Delta\ln A_i+\Delta\lambda_i\cdot t$, and

$$EE_{it}^{mn}=\alpha_{ki}\Delta\ln(k_{it}/l_{it})+(\beta_{kki}/2)\Delta(\ln k_{it}/l_{it})^2+(\beta_{kKi})\Delta\ln(k_{it}/l_{it})\ln K_i.$$

Similarly, for sector j, $\Delta\ln(X_{jt}/l_{jt})$ is also the cross country difference in labor productivity which can be decomposed into two components, which are:

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[15] Note that the translog functional form, in this model, assumes that ln X_{oi} for both countries m and n are the same. Thus the difference in that disappears.

$$\Delta \ln \frac{X_{jt}}{l_{jt}} = TE_{jt}^{mn} + EE_{jt}^{mn} \tag{3.19}$$

where $TE_{jt}^{mn} = \Delta \ln A_j + \Delta\lambda_j \cdot t$, and

$EE_{jt}^{mn} = \alpha_{kj}\Delta \ln(k_{jt}/l_{jt}) + (\beta_{kkj}/2)\Delta(\ln k_{jt}/l_{jt})^2 + (\beta_{kKj})\Delta \ln(k_{jt}/l_{jt})\ln K_j$.

In conclusion, the CAI index firstly must be decomposed into PE_{ijt}^{mn} and WE_{ijt}^{mn} (see equation 3.14), and then decompose the labor productivity effect into TE_{ijt}^{mn} and EE_{ijt}^{mn} (see equation 3.15). To compute the CAI index, these steps are easily expressed as follows:

$$\begin{aligned} CAI_{ijt}^{mn} &= PE_{ijt}^{nm} + WE_{ijt}^{mn} \\ &= TE_{ijt}^{mn} + EE_{ijt}^{mn} + WE_{ijt}^{mn} \end{aligned} \tag{3.20}$$

where

$$\begin{aligned} PE_{ijt}^{mn} &= PE_{jt}^{mn} - PE_{it}^{mn} \\ &= \Delta \ln(X_{jt}/l_{jt}) - \Delta \ln(X_{it}/l_{it}) \end{aligned}$$

$$\begin{aligned} TE_{ijt}^{mn} &= TE_{jt}^{mn} - TE_{it}^{mn} \\ &= (\Delta \ln A_j - \Delta \ln A_i) + (\Delta\lambda_j - \Delta\lambda_i) \cdot t \end{aligned}$$

$$\begin{aligned} EE_{ijt}^{mn} &= EE_{jt}^{mn} - EE_{it}^{mn} \\ &= \left[\alpha_{kj}\Delta \ln(k_{jt}/l_{jt}) - \alpha_{ki}\Delta \ln(k_{it}/l_{it})\right] + \\ &\quad \left[(\beta_{kkj}/2)\Delta(\ln k_{jt}/l_{jt})^2 - (\beta_{kki}/2)\Delta(\ln k_{it}/l_{it})^2\right] + \\ &\quad (\beta_{kKj})\Delta \ln(k_{jt}/l_{jt})\ln K_j - (\beta_{kKi})\Delta \ln(k_{it}/l_{it})\ln K_i \end{aligned}$$

$$WE_{ijt}^{mn} = \Delta \ln w_{it} - \Delta \ln w_{jt}.$$

3.2.3 The Interpretation of CAI

This final section introduces an interpretation of the CAI index and the index's decomposition. Firstly, the CAI index can indicate a country's comparative advantage in a given production. Given that $CAI_{ijt}^{mn} < 0$, then it says that country m has a comparative advantage in tradable good sector i. On the other hand, assuming that $CAI_{ijt}^{mn} > 0$, it indicates that country m has a comparative dis-advantage in tradable commodity sector i.

The CAI index has three decompositions capturing different effects on the international comparative advantage pattern between two countries. Firstly, the technology effect refers to the effect of different technological progress in labor productivity. Secondly, the endowment effect represents the effect of differences in the capital labor ratio on labor productivity. The first two components have impacts on the labor cost share because of improvement in the labor productivity. Thirdly, the wage effect is the impact of differences in relative wage rates on comparative advantage.

The source of comparative advantage in production can be indicated by the index's composition. If the composition is negative (positive), then it is the source of comparative advantage (comparative dis-advantage) in production. For instance, the productivity effect is assumed to be of negative value. It means that country m has comparative advantage in the productivity effect (relative to country n).