CHAPTER 4

THE RESEARCH METHODOLOGY

4.1 Tax policy

In the first period, the regulator sets the emission tax rate (T_1) where tax rate equals to marginal abatement benefit equals to expected marginal abatement cost, $T_1 = B'(q_1) = E\left[C_q(q_1, \Theta)\right]$. Firm chooses an abatement level (q_1^j), in response to the tax rate in order to minimize total cost (abatement cost plus regulatory cost).

In the second period, the regulator observes the abatement level from two firms and believes that firm chooses abatement level where marginal abatement cost equals to tax rate, $C_q(q_1, \theta) = T_1$. Afterwards, the regulator infers that the actual value of Θ is θ^R. After that, the regulator sets the emission tax rate (T_2) according to an observed abatement level in the first period. The abatement level in the first period responds to tax rate in the first period. Therefore, the tax rate in the second period is a function of the abatement level in the first period which responds to tax rate in the first period, $T_2 = T_2(q_1|T_1)$. The tax rate in the second period is $T_2 = B'(q_2) = C_q(q_2, \theta^R) = T_2(q_1|T_1)$. Firms will choose their abatement level again.

The total cost of firm *j* in the first period is $TC_1^j = T_1 \cdot (e_1^j - q_1^j) + C^j(q_1^j, \theta)$. In the second period is $TC_2^j = T_2 \cdot (e_2^j - q_2^j) + C^j(q_2^j, \theta)$. However, as firms are strategic, they know that tax rate in the second period is $T_2 = T_2(q_1|T_1) = T_2(q_1^j + q_1^{-j}|T_1)$ where q_1^{-j} is the abatement level of firm *j's* component. The objective of firm *j* is to minimize the individual long term total cost. The problem of firm *j* is

$$\min_{\{q_1^j, q_2^j\}} TC^j = T_1 \cdot (e_1^j - q_1^j) + C^j(q_1^j, \theta) + \delta\left\{T_2(q_1^j + q_1^{-j}|T_1) \cdot (e_2^j - q_2^j) + C^j(q_2^j, \theta)\right\}$$

where δ = discount factor between period.

The term $\delta\left\{T_2\left(q_1^j+q_1^{-j}\middle|T_1\right)\cdot\left(e_2^j-q_2^j\right)+C^j\left(q_2^j,\theta\right)\right\}$ shows that firm *j* is strategic.

If firm *j* is nonstrategic, it can lead to the optimal situation.

The objectives of firm *j* are

$$\min_{q_1^j} TC_1^j = T_1\cdot\left(e_1^j-q_1^j\right)+C^j\left(q_1^j,\theta\right) \quad \text{and} \quad \min_{q_2^j} TC_2^j = T_2\cdot\left(e_2^j-q_2^j\right)+C^j\left(q_2^j,\theta\right).$$

After that, it is to find the solution equations and find the firms abatement behavior by mathematical proof in order to find whether the abatement level is over or under-abatement.

4.2 Permit policy

Assumed a_t is a total number of tradable emission permit allocated by the regulator in period *t*, and a_t^j is a number of permit allocated to firm *j* in period *t*

Denotes,

Super script *L* = Firm that has more influence or the permit price maker = leader

Super script *F* = Firm that has less influence or the permit price taker = follower

Assumption,

$$a_t^L + a_t^F = a_t \quad \text{and} \quad a_t^L = \alpha a_t \quad \text{and} \quad a_t^F = (1-\alpha)a_t$$

where α is a fraction of permit to leader ; $0 \le \alpha \le 1$

- Firms can trade permit but cannot bank them for future use.
- In each period, total emission must not exceed total permit.

$$e_t^L + e_t^F - \left(q_t^L + q_t^F\right) \le a_t^L + a_t^F \quad \text{or} \quad e_t - q_t \le a_t$$

In the first period, the regulator sets the emission permit (a_1) where marginal abatement benefit equals to expected marginal abatement cost, $B'(e_1-a_1)=E\left[C_q\left(e_1-a_1,\Theta\right)\right]$. Firms choose their abatement level and trade permit at the market permit price (p_{a_1}).

In the second period, the regulator observes the market permit price and believes that firms trade permit where the marginal abatement cost equals to the market permit price, $C_q(e_1 - a_1, \theta) = p_{a_1}$. Afterwards, the regulator infers that the actual value of Θ is θ^R. After that, the regulator sets the emission permit in the second period ($a_2 = a_2\left(p_{a_1} \middle| a_1\right)$) where $B'(e_2 - a_2) = C_q(e_2 - a_2, \theta^R)$. Firms choose their abatement level and trade permit at the permit price market (p_{a_2}).

In the second period, the follower (price taker) chooses abatement level $\left(q_2^F\right)$ in order to minimize total cost by using an allocation of permit (a_2^F) and permit price in the second period (p_{a_2}).

The follower's objective in the second period is

$$\min_{q_2^F} \left\{ p_{a_2} \cdot \left(e_2^F - a_2^F - q_2^F\right) + C^F\left(q_2^F, \theta\right) \right\}$$

The leader chooses the permit price (p_{a_2}) in order to minimize total cost by using an allocation of permits (a_2^L) and the follower's reaction of permit price, $q_2^F(p_{a_2})$.

The leader's objective in the second period is

$$V_2^L(a_2; \alpha) = \min_{p_{a_2}} \left\{ p_{a_2} \cdot \left(e_2^L - a_2^L - q_2^L\right) + C^L\left(q_2^L, \theta\right) \right\}$$

subject to $\quad e_2 - q_2^L - q_2^F\left(p_{a_2}\right) = a_2 \quad$ (1)

$$a_2 = a_2^L + a_2^F \quad (2)$$

$$e_2 = e_2^L + e_2^F \quad (3)$$

; α = fraction of permits to leader

The condition (2) means that all permits in each period are allocated to two firms. Thus, the summation of permit from two firms must equal to all permits in that period.

The condition (3) means that emission in each period is created by two firms only. Thus, the summation of emission from two firms must equal to all emission in that period.

The condition (1) is the total emission in each period that cannot exceed the total permit in that period, and firms cannot bank permit from one period for future use. Hence, two firms will use all permit that they received.

In the first period, because the regulator sets emission permit by using permit price as information, the follower, which is a permit price taker, cannot manipulate the regulator.

Therefore, the follower's objective in the first period is

$$\min_{q_1^F}\left\{p_{a_1}\cdot\left(e_1^F-a_1^F-q_1^F\right)+C^F\left(q_1^F,\theta\right)\right\}.$$

However, for the leader who sets a permit price and has strategic behavior, the leader knows that the regulator allocates emission permit in the second period where $a_2=a_2\left(p_{a_1}\middle|a_1\right)$, and tries to minimize long term total cost.

Therefore, the leader's problem in the first period is

$$V_1^L\left(a_1^L,a_1^F\right)=\min_{p_{a_1}}\left\{p_{a_1}\cdot\left(e_1^L-a_1^L-q_1^L\right)+C^L\left(q_1^L,\theta\right)+\delta V_2^L\left(a_2\left(p_{a_1}\middle|a_1\right);\alpha\right)\right\}$$

subject to $e_1-q_1^L-q_1^F\left(p_{a_1}\right)=a_1$ (1)

$a_1=a_1^L+a_1^F$ (2)

$e_1=e_1^L+e_1^F$ (3)

The term $\delta V_2^L\left(a_2\left(p_{a_1}\middle|a_1\right);\alpha\right)$ shows that leader has strategic behavior.

If the leader is nonstrategic, the problem of the leader in the first period is

$$V_1^L\left(a_1;\alpha\right)=\min_{p_{a_1}}\left\{p_{a_1}\cdot\left(e_1^L-a_1^L-q_1^L\right)+C^L\left(q_1^L,\theta\right)\right\}$$

After that, it is to find the solution equations and find the firms abatement behavior by mathematical proof in order to find whether the abatement level is over or under-abatement.

4.3 Safety valve policy

In this policy, the regulator sets tax rate and allocates permit by using information from observed abatement level or permit price in order to infer firms' abatement cost function. Firms can always trade their permit but cannot bank permit.

The total emission in each period can exceed the total permit, but firms must pay for excess emission by paying tax that the regulator set.

In the first period, the regulator sets emission tax rate (T_1) and allocates emission permit (a_1) where marginal abatement benefit equals to expected marginal abatement cost, $B'(q_1) = E\left[C_q(q_1, \Theta)\right]$. Firms choose their abbatement level and trade permit at the market permit price (p_{a_1}).

In the second period, the regulator observes the abatement level and market permit price and believes that firms trade permit and set abatement level where the marginal abatement cost equals to marginal of non-abatement cost. The regulator infers that realization of Θ is θ^R. After that, the regulator sets emission tax rate and allocates emission permit where $B'(q_2) = C_q(q_2, \theta^R)$. The regulator sets emission tax rate (T_2) and emission permit (a_2) in the second period by using an observed abatement level or observed permit price in the first period. Abatement level and permit price in the first period respond to tax rate and permit in the first period. Therefore, tax rate and permit in the second period are as the following functions: $T_2 = T_2\left(q_1, p_{a_1} \middle| a_1, T_1\right)$ and $a_2 = a_2\left(q_1, p_{a_1} \middle| a_1, T_1\right)$ respectively. Firms choose their abatement level and trade permit at the market permit price.

In safety valve policy, the total cost of firm is a component of taxation cost, permit buying cost, and abatement cost.

Total Cost = Taxation cost + Buy permits cost + Abatement cost

= $T\cdot$(pollution after trade) + Price$\cdot$(number of buy permit) + $C(q_t, \theta)$

There are two firms: permit seller (*s*) and permit buyer (*b*)

Denotes, d_s = Number of permit that seller needs to sell

d_b = Number of permit that buyer needs to buy

$$\begin{array}{lll} d_s > 0 \ ; & d_s = a_s + q_s - e_s & \therefore e_s - a_s - q_s = -d_s \\ d_b < 0 \ ; & d_b = a_b + q_b - e_b & \therefore e_b - a_b - q_b = -d_b \end{array} \Bigg\} -d_b \geq d_s$$

The cost equation of both seller and buyer are,

$$Cost_s = T\cdot(e_s - a_s - q_s + d_s) + p_a\cdot(-d_s) + C(q_s,\theta_s)$$
$$= p_a\cdot(-d_s) + C(q_s,\theta_s) \qquad ; d_s = a_s + q_s - e_s$$
$$Cost_s = p_a\cdot(e_s - a_s - q_s) + C(q_s,\theta_s)$$

$$Cost_b = T\cdot(e_b - a_b - q_b - d_s) + p_a\cdot(d_s) + C(q_b,\theta_b)$$

Seller does not pay tax because seller has permit enough to sell.

To consider these two cases, leader is a permit seller which is akin to **monopoly market**, and leader is a permit buyer which is akin to **monopsony market**.

4.4.1 Monopoly market

Leader's objective equation

$$V_1^L(T_1,a_1;\alpha) = \underset{p_{a_1},q_1^L}{Min}\left\{p_{a_1}\cdot\left(e_1^L - a_1^L - q_1^L\right) + C^L\left(q_1^L,\theta\right) + \delta V_2^L\left(T_2\left(q_1,p_{a_1}\right),a_2\left(q_1,p_{a_1}\right);\alpha\right)\right\}$$

$$V_2^L\left(T_2\left(q_1,p_{a_1}\right),a_2\left(q_1,p_{a_1}\right);\alpha\right) = \underset{p_{a_2},q_2^L}{Min}\left\{p_{a_2}\cdot\left(e_2^L - a_2^L - q_2^L\right) + C^L\left(q_2^L,\theta\right)\right\}$$

Follower's objective equation

$$V_1^F(T_1,a_1;\alpha) = \underset{q_1^F}{Min}\left\{T_1\cdot\left(e_1^F - a_1^F - q_1^F - d_1^L\right) + p_{a_1}\cdot\left(d_1^L\right) + C^F\left(q_1^F,\theta\right) + \delta V_2^F\left(T_2\left(q_1,p_{a_1}\right),a_2\left(q_1,p_{a_1}\right);\alpha\right)\right\}$$

$$V_2^F\left(T_2\left(q_1,p_{a_1}\right),a_2\left(q_1,p_{a_1}\right);\alpha\right) = \underset{q_2^F}{Min}\left\{T_2\cdot\left(e_2^F - a_2^F - q_2^F - d_2^L\right) + p_{a_2}\cdot\left(d_2^L\right) + C^F\left(q_2^F,\theta\right)\right\}$$

4.4.2 Monopsony market

Leader's objective equation

$$V_1^L(T_1,a_1;\alpha) = \underset{p_{a_1},q_1^L}{Min}\left\{T_1\cdot\left(e_1^L - a_1^L - q_1^L - d_1^F\right) + p_{a_1}\cdot\left(d_1^F\right) + C^L\left(q_1^L,\theta\right) + \delta V_2^L\left(T_2\left(q_1,p_{a_1}\right),a_2\left(q_1,p_{a_1}\right);\alpha\right)\right\}$$

$$V_2^L\left(T_2\left(q_1,p_{a_1}\right),a_2\left(q_1,p_{a_1}\right);\alpha\right) = \underset{p_{a_2},q_2^L}{Min}\left\{T_2\cdot\left(e_2^L - a_2^L - q_2^L - d_2^F\right) + p_{a_2}\cdot\left(d_2^F\right) + C^L\left(q_2^L,\theta\right)\right\}$$

Follower's objective equation

$$V_1^F(T_1,a_1;\alpha) = \underset{q_1^F}{Min}\left\{p_{a_1}\cdot\left(e_1^F - a_1^F - q_1^F\right) + C^F\left(q_1^F,\theta\right) + \delta V_2^F\left(T_2\left(q_1,p_{a_1}\right),a_2\left(q_1,p_{a_1}\right);\alpha\right)\right\}$$

$$V_2^F\left(T_2\left(q_1,p_{a_1}\right),a_2\left(q_1,p_{a_1}\right);\alpha\right) = \underset{q_2^F}{Min}\left\{p_{a_2}\cdot\left(e_2^F - a_2^F - q_2^F\right) + C^F\left(q_2^F,\theta\right)\right\}$$

After that, it is to find the solution equations and find the firms abatement behavior by mathematical proof in order to find whether the abatement level is over or under-abatement.