CHAPTER 2

LITERATURE REVIEW

2.1 Fracture Mechanics of Concrete

It is commonly known that the Linear Elastic Fracture Mechanics (LEFM) is inapplicable to concrete as realized by Shah and McGarry (1971). The main reason is that the nonlinear fracture process zone, composed of microcracking and bridging zones, ahead of a newly formed crack tip cannot be taken into account. Moreover, in the linear elastic concept, stress at crack tip approaches infinity that is not true in reality. Hence, the behaviors of cracking concrete are more accurately predicted by the concept of the nonlinear fracture mechanics that considers the fracture process zone.

The models incorporating the fracture process zone can be classified into two main groups, i.e., fictitious crack approach and effective-elastic crack approach, based on different energy dissipation mechanism. There are two main models of fictitious crack approach, i.e., fictitious crack model of Hillerborg (1976) and crack band model of Bazant and Oh (1983). Hillerborg (1976) first proposed a fictitious crack model for fracture of concrete. In his study, the newly formed crack surfaces and the corresponding fracture process zones are simply simulated by a cohesive zone located in the front of the initial crack tip. When the transmitted tensile stress in the cohesive zone decreases to zero, the cohesive crack will become a real crack. The parameter used to explain the behavior in the fracture process zone in this model is the relationship between the crack opening displacement and the transmitted tensile stress called tension-softening relationship. This model is applicable to the discrete crack model in the finite element analysis effectively. The other model is known as crack band model and was proposed by Bazant and Oh (1983). In the crack band model, concrete also exhibits gradual strain-softening due to microcracking in the fracture process zone. However, in this model, the fracture process zone is modeled as a band of micro-cracks distributed in the band. The parameter to describe the fracture process zone is the constitutive relationship between the crack strain and the transmitted tensile stress. This model is well applicable to the smeared crack model in the finite element analysis. The other main approach is the effective-elastic crack approach. This approach models the fracture process zone by using an equivalent, traction-free elastic crack. The effective-elastic crack is governed by the criterion from LEFM, and the equivalence between the actual and the corresponding effective crack is prescribed explicitly in each model. Most of the effective-elastic crack models use two fracture parameters to define the elastic fracture process and to govern crack propagation. Jenq and Shah (1985) proposed a two-parameter fracture model based on elastic fracture responses of structures. The responses are composed of elastic and plastic fracture responses. The two parameters in the model are critical stress intensity factor and critical crack tip opening displacement. Both parameters are found to be independent of size but material. Moreover, these parameters can be used as critical fracture properties.

2.2 Finite Element Method in Cracking Problems

The finite element method is used in this study. The basic concept of the finite element method is that the structure is subdivided into elements connected by nodes. At the element interface, the conditions of equilibrium and compatibility are satisfied. In the field of fracture mechanics, there are two main methods to model crack by the finite element method, i.e., discrete crack model and smeared crack model. These two concepts will be briefly reviewed.

2.2.1 Discrete crack model

The discrete crack model was first proposed by Ngo and Scordelis (1967). In this model, crack is modeled by introducing real displacement discontinuities into the finite element mesh. This model seems to be attractive physically because it reflects the localized nature of crack. However, there are many obstacles arising in using the discrete crack model. For example, its crack path usually has to be assumed in advance and mesh has to be subsequently arranged so that the crack path coincides with boundaries between elements. Therefore, it may not be suitable for some problems such as the problem with many simultaneous cracks. The main disadvantage of the discrete crack model is that it consumes long computational time due to remeshing. Even though automatic remeshing procedure (Ingreffea and Saouma 1984) have been developed for such remeshing, they are still not appropriate for large-scale analysis.

Nanakorn and Horii (1995) developed a finite element with embedded displacement discontinuity. The displacement discontinuity is embedded into the cracked element by adjusting the interpolation function of the element. The interpolation function is obtained from the consideration of the relative displacement induced by the rigid translations and the rigid rotation between two crack surfaces separated by the discontinuity.

2.2.2 Smeared crack model

The smeared crack model, introduced by Rashid (1968), is based on the concept of replacing crack by a continuous medium with degraded material properties. The weak point of the model is that the size of damage zone will affect the accuracy. However, many methods have been developed to solve this problem (Bazant and Oh 1983 and Oliver 1989).

Bazant and Oh (1983) introduced the single fixed crack model that a linear elastic material tensor was coupled via a static constraint with a local matrix, which described the softening-crack opening relation. The cracking behavior is defined on the principal tensile plane. Although the principal stress may rotate during the analysis, the orientation of this plane is fixed due to the complete loss of material resistance.

The single fixed crack approach has the problem of accuracy when stresses are rotating due to change in the load pattern or shift in the principal stress direction. Therefore, de Borst and Nauta (1985) proposed a new concept called multidirectional fixed crack approach. Every single crack plane in this approach holds the same condition as the single crack plane in the single fixed crack approach. However, after the first crack initiates, the following crack plane initializations are decided on the trespassing of a threshold value in intensity and/or orientation. This approach is applicable to conditions where the fracture starts in tension and subsequently proceeds in tension-shear.

Both single and multidirectional fixed crack models include the explicit shear term for the fixed plane, but shear does not provide much insight in the behavior of structures. Thus, many researchers include the shear term in terms of principal stress. However, principal stress axes continuously rotate after cracking. Therefore, the stress-strain relation in the rotating principal coordinate system, instead of the fixed crack system, is required. However, it is found that the stress-strain relation in the rotating principal coordinate system requires the principal stress and strain to be coaxial. It has been shown that this coaxial can be achieved only via an implicit shear term (Willam et al. 1987). Rots (1989) indicated that when smeared crack approaches were used, the results would suffer from stress locking. The coaxial rotating crack concept and the fixed crack concept with negligible shear retention provide the results that suffer least from stress locking.

An important problem of the smeared crack model besides stress locking is the characteristic length or crack band width. Bazant and Oh (1983) estimated the characteristic length from the experiment as the ratio between the energy dissipated per unit area and the energy dissipated per unit volume of the material, and the characteristics length was approximated as three times of the maximum aggregate size. However, when the finite element mesh is irregular, this characteristic length is not applicable. Oliver (1989) proposed a method that could estimate the characteristic length of the irregular finite element mesh. The idea of his work is that crack is modeled as a limiting case of two singular lines which tend to coincide with each other. Across the two lines, the displacements are continuous but the displacement gradients are discontinuous. His method shows the objective estimation of the characteristic length of the irregular mesh.

2.3 Localization and Bifurcation

In the problem of crack initiation and crack propagation, many models seem to give good results in certain cases even without the consideration of the cracking localization. However, it is clear that this is not always true as already shown in Figs.1.1 and 1.2. The problem is how to incorporate this localization phenomenon into the analysis.

In the consideration of the post-peak behavior of materials like concrete, the equilibrated solution is not always unique. A bifurcation of the equilibrated solution occurs. Therefore, tracing for the equilibrated solution is not enough. The analysis of stability of the possible solution to select the stable one is needed. It turns out that the stable solution is a localized one. Therefore, many researchers have tried to propose methods or models to consider the cracking localization.

De Borst (1987) studied the post-bifurcation and post-failure behavior of strain-softening solids. It is shown that the bifurcation point that occurs in strain-softening solids can be located by eigenvalue analysis, as the bifurcation point appears whenever the lowest eigenvalue of the tangent stiffness of the structure become slightly negative. Furthermore, the post-bifurcation response can be also traced successfully. In his consideration, it is indicated that the bifurcation path will not in general be orthogonal to the fundamental path. When the eigenmode that belongs to the vanishing eigenvalue is added to the incremental field of the fundamental path, the orthogonality condition will maximize the possibility to which the bifurcation branch can be converged. This procedure can be repeated until the negative eigenvalue disappears and the lowest bifurcation path is finally obtained.

Nguyen (1987) proposed a mathematical model based on the minimum potential energy to consider bifurcation and post-bifurcation behaviors of rate independent systems. The model was tried with simple models of fracture, brittle damage and metal plasticity. The result of the consideration shows the applicability of the model. Moreover, the model implies that the localization can be considered by using the minimum total potential energy approach to determine the stationary condition, and by investigating the sign of eigenvalues of the stiffness with respect to irreversible parameters to check for the stability condition.

Bazant (1989) applied the second law of thermodynamics to study damage localization and propagation of damage zones by considering path bifurcation. It was concluded that the energy surface that determined the stability of inelastic structures was unsmooth as it exhibited lines of curvature discontinuity. The discontinuity implied that the state on all the branches emanating from the bifurcation point can be stable or unstable, and the stable states of one branch beyond the first bifurcation point could not be reached by a continuous loading. Moreover, it was also found that (1) the propagation of damage led to bifurcation equilibrium path (2) thermodynamics stability analysis could identify which path would actually occur and (3) checking for path stability was required to be included in finite element programs for damage investigation.

Valente (1992) developed the cohesive crack model to make it possible to detect and analyze the possible bifurcation of the equilibrium path by considering the positive definiteness of the stiffness of the structure or eigenvalue analysis. It was concluded that the criterion used to determine the stable solution is that the state involving a smaller quantity of stored elastic energy is more stable. Horii and Okui (1995) employed the theory of the thermomechanics to propose an approach to investigate the stability of the homogenous solution by considering the sign of the eigenvalues of the stiffness with respect to irreversible parameters. The theory was used to effectively analyze simple problems. Later, Brocca (1997) applied the concept of Horii and Okui (1995) to solve slightly more complicated structures by applying the finite element method with the discrete crack model.