CHAPTER 6

CONCLUSION

In this study, the smeared crack model of the finite element method is used as an analysis tool to solve problems involving cracking localization. In the analysis of the cracking localization, consideration of stability and bifurcation of equilibrium states is one of the tasks to be done. To consider stability and bifurcation of irreversible processes, the stationary condition of the energy of the system with respect to irreversible parameters have to be examined. This requires expression of the energy in terms of the irreversible parameters. For crack problems, the irreversible parameters can be the crack opening displacement variables in the discrete crack approach or the crack strain variables in the smeared crack approach. In the discrete approach, the crack opening displacement variables are usually discretized along crack paths and treated as the degrees of freedom in the analysis. The energy of the system is expressed in terms of these degrees of freedom. Computing the first and second variations of the energy with respect to the crack opening displacement degrees of freedom can be done easily. This is because having the energy written as a function of discrete variables allows differentiation of the energy expression with respect to these irreversible variables. As a result, we will be able to consider the stability and bifurcation of equilibrated solutions by employing just the ordinary calculus. On the contrary, if the smeared crack approach is employed, the energy of the system will be expressed in terms of the irreversible crack strain variables. These crack strain variables are functions of position. To compute the first and second variations of the energy with respect to these crack strain functions, complex mathematics involving the calculus of variations must be employed.

This fact implies that the discrete crack approach in the finite element method may be more suitable for the cracking localization analysis than the smeared crack approach. Nevertheless, the discrete crack approach may not perform best when there are many cracks. In the cracking localization analysis, there will be many cracks in the domain. Having many cracks in the domain leads to more degrees of freedom, and the mesh topology of the problems may have to be changed drastically. Moreover, the singularity problem of the system stiffness equation may also appear. These problems can be mostly avoided if the smeared crack approach is employed. In the smeared crack models, no increase in the degrees of freedom or change in the mesh topology is required. Although, the smeared crack models may also face the singularity problem of the system in case of softening materials, the problem is less serious than that of the discrete crack models.

In this study, a special consideration on the smeared crack finite element analysis is proposed. The proposed scheme makes the consideration of the cracking localization possible even when the smeared crack models are used. In the proposed method, a discrete irreversible variable related to the crack strains is introduced in the smeared crack models. This discrete variable allows the consideration of stability and bifurcation of the equilibrated solution to be done easily by considering the variations of the energy with respect to the proposed discrete variable. However, the proposed variable is not used to obtain the equilibrium solutions. The original smeared crack models are still used for that purpose.

In this study, the incremental formulation is employed. The derivation starts from the expression of the total energy increment. The total energy increment is divided into 2 parts, i.e., the mechanical potential energy increment and the dissipated energy increment. In the expression for the total energy increment, the irreversible variable that has to be considered in the stability analysis is the crack strain increment. The first variation of the total energy with respect to this crack strain increment results in the equilibrium path. The second variation gives the information on the stability condition of the obtained equilibrium path. Since the total energy increment is a functional of the crack strain increment function, the calculus of variations is required. To avoid this difficulty, a parameter called crack displacement increment is introduced. The first variation of the total energy with respect to this crack displacement increment leads to the equilibrium equation. The second variation can be considered for the stability condition. It can be shown that the information on the stability condition can be obtained from the eigenvalue analysis of the stiffness in the stiffness equation that is written only in terms of the crack displacement increment. If all eigenvalues are positive, the equilibrium state is stable. Otherwise, the equilibrium state is unstable and bifurcation occurs.

From the result, it can be seen that the proposed method is applicable to the consideration of the cracking localization. It is found that the responses of the cases with localization consideration are different from those without the localization consideration. Therefore, it can be concluded that the consideration of localization is really necessary for fracture problems. The introduction of the new discrete crack displacement variable to the smeared crack approach allows the judgement of the stability of the equilibrium path to be done perfectly. However, it is also found that, in case of many stable solutions, there is still no basic principle to definitely judge which solution should be selected. In many cases, it is clear that any stable solution is justified. However, it can not be confirmed that this conclusion is always true. Further study is still necessary.