

APPENDIX A

Calculation for Problem 5.1 by the Mathematica Program

In[1]: = (*****
 (*This is the program for problem 5.1 with two cracks*)
 (*****)

(*This is to construct the matrix before static condensation*)

$$K11 = \frac{2 \cdot A \cdot E}{L}$$

$$K12 = \left(\frac{A \cdot E}{L}, -\frac{A \cdot E}{L}, -\frac{A \cdot E}{L}, \frac{A \cdot E}{L} \right)$$

$$K21 = \text{Transpose}[K12]$$

$$K22 = \begin{pmatrix} \frac{A \cdot (E+H)}{L} & -\frac{A \cdot (E+H)}{L} & 0 & 0 \\ -\frac{A \cdot (E+H)}{L} & \frac{A \cdot (E+H)}{L} & 0 & 0 \\ 0 & 0 & \frac{A \cdot (E+H)}{L} & -\frac{A \cdot (E+H)}{L} \\ 0 & 0 & -\frac{A \cdot (E+H)}{L} & \frac{A \cdot (E+H)}{L} \end{pmatrix}$$

$$\text{Out[1]} = \frac{2AE}{L}$$

$$\text{Out[2]} = \left\{ \left\{ \frac{AE}{L}, -\frac{AE}{L}, -\frac{AE}{L}, \frac{AE}{L} \right\} \right\}$$

$$\text{Out[3]} = \left\{ \left\{ \frac{AE}{L} \right\}, \left\{ -\frac{AE}{L} \right\}, \left\{ -\frac{AE}{L} \right\}, \left\{ \frac{AE}{L} \right\} \right\}$$

$$\text{Out[4]} = \left\{ \left\{ \frac{A(E+H)}{L}, -\frac{A(E+H)}{L}, 0, 0 \right\}, \left\{ -\frac{A(E+H)}{L}, \frac{A(E+H)}{L}, 0, 0 \right\}, \right. \\ \left. \left\{ 0, 0, \frac{A(E+H)}{L}, -\frac{A(E+H)}{L} \right\}, \left\{ 0, 0, -\frac{A(E+H)}{L}, \frac{A(E+H)}{L} \right\} \right\}$$

In[5]: = (*This is to construct the matrix after static condensation*)

$$Kcr = \text{Simplify}\left[K22 - \frac{K21 \cdot K12}{K11}\right]$$

$$\text{Out[5]} = \left\{ \left\{ \frac{A(E+2H)}{2L}, -\frac{A(E+2H)}{2L}, \frac{AE}{2L}, -\frac{AE}{2L} \right\}, \left\{ -\frac{A(E+2H)}{2L}, \frac{A(E+2H)}{2L}, -\frac{AE}{2L}, \frac{AE}{2L} \right\}, \right. \\ \left. \left\{ \frac{AE}{2L}, -\frac{AE}{2L}, \frac{A(E+2H)}{2L}, -\frac{A(E+2H)}{2L} \right\}, \left\{ -\frac{AE}{2L}, \frac{AE}{2L}, -\frac{A(E+2H)}{2L}, \frac{A(E+2H)}{2L} \right\} \right\}$$

In[6]: = (*This is to compute eigenvalues and eigenvectors of Kcr*)

$$\text{Simplify}[\text{Eigensystem}[Kcr]]$$

$$\text{Out[6]} = \left\{ \left\{ 0, 0, \frac{2AH}{L}, \frac{2A(E+H)}{L} \right\}, \{(0, 0, 1, 1)\}, \{(1, 1, 0, 0)\}, \{(1, -1, -1, 1)\}, \{-1, 1, -1, 1\} \right\}$$

In[7]: = (*This is to construct the right-hand side force*)

$$R1 = \frac{E \cdot A \cdot U}{L}$$

$$R2 = \begin{pmatrix} 0 \\ 0 \\ -\frac{E \cdot A \cdot U}{L} \\ \frac{E \cdot A \cdot U}{L} \end{pmatrix}$$

$$R = \text{Simplify}\left[R2 - \left(\frac{K21 \cdot R1}{K11}\right)\right]$$

$$\text{Out[7]} = \frac{AEU}{L}$$

$$\text{Out[8]} = \left\{ (0), (0), \left\{-\frac{AEU}{L}\right\}, \left\{\frac{AEU}{L}\right\} \right\}$$

$$\text{Out[9]} = \left\{ \left\{-\frac{AEU}{2L}\right\}, \left\{\frac{AEU}{2L}\right\}, \left\{-\frac{AEU}{2L}\right\}, \left\{\frac{AEU}{2L}\right\} \right\}$$

In[10]: = (*This is to construct the matrix after constrain*)

$$Kcrhat = \text{Simplify}\left[\left(\left(Kcr[[1, 1]] - Kcr[[1, 2]], Kcr[[1, 3]] - Kcr[[1, 4]]\right), \left(Kcr[[3, 1]] - Kcr[[3, 2]], Kcr[[3, 3]] - Kcr[[3, 4]]\right)\right)\right]$$

$$\text{Out[10]} = \left\{ \left\{\frac{A(E+2H)}{L}, \frac{AE}{L}\right\}, \left\{\frac{AE}{L}, \frac{A(E+2H)}{L}\right\} \right\}$$

In[11]: = (*This is to compute eigenvalues and eigenvectors of Kcrhat*)

$$\text{Simplify}\left[\text{Eigensystem}\left[Kcrhat\right]\right]$$

$$\text{Out[11]} = \left\{ \left\{\frac{2AH}{L}, \frac{2A(E+H)}{L}\right\}, \left\{(-1, 1), (1, 1)\right\} \right\}$$

In[7]:= (*****
 (*This is the program for problem 5.1 with one crack*)
 (*****)

(*This is to construct the matrix before static condensation*)

$$K_{11} = \frac{2 * A * E}{L}$$

$$K_{12} = \left(\frac{A * E}{L}, -\frac{A * E}{L} \right)$$

$$K_{21} = \text{Transpose}[K_{12}]$$

$$K_{22} = \begin{pmatrix} \frac{A * (E + H)}{L} & -\frac{A * (E + H)}{L} \\ -\frac{A * (E + H)}{L} & \frac{A * (E + H)}{L} \end{pmatrix}$$

Out[7]= $\frac{2 A E}{L}$

Out[8]= $\left\{ \left\{ \frac{A E}{L}, -\frac{A E}{L} \right\} \right\}$

Out[9]= $\left\{ \left\{ \frac{A E}{L} \right\}, \left\{ -\frac{A E}{L} \right\} \right\}$

Out[10]= $\left\{ \left\{ \frac{A (E + H)}{L}, -\frac{A (E + H)}{L} \right\}, \left\{ -\frac{A (E + H)}{L}, \frac{A (E + H)}{L} \right\} \right\}$

In[11]:= (*This is to construct the matrix after static condensation*)

$$K_{cr} = \text{Simplify}\left[K_{22} - \frac{K_{21} \cdot K_{12}}{K_{11}} \right]$$

Out[11]= $\left\{ \left\{ \frac{A (E + 2 H)}{2 L}, -\frac{A (E + 2 H)}{2 L} \right\}, \left\{ -\frac{A (E + 2 H)}{2 L}, \frac{A (E + 2 H)}{2 L} \right\} \right\}$

In[12]:= (*This is to compute eigenvalues and eigenvectors of Kcr*)

Simplify[Eigensystem[Kcr]]

Out[12]= $\left\{ \left\{ 0, \frac{A (E + 2 H)}{L} \right\}, \left\{ \{1, 1\}, \{-1, 1\} \right\} \right\}$

In[13]:= (*This is to construct the right-hand side force*)

$$R_1 = \frac{E * A * u}{L}$$

$$R_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$R = \text{Simplify}\left[R_2 - \left(\frac{K_{21} * R_1}{K_{11}} \right) \right]$$

Out[13]= $\frac{A E u}{L}$

Out[14]= $\{\{0\}, \{0\}\}$

Out[15]= $\left\{ \left\{ -\frac{A E u}{2 L} \right\}, \left\{ \frac{A E u}{2 L} \right\} \right\}$