Appendix A. Proof of Adjacency Condition.

Let $c(ij)$ be the cost of sub-sequence $\{i,j\}$ and $c(ji)$ be the cost of inverse sub-sequence $\{j,i\}$

- There are six cases, which may occur for job i and j in the sequence as follows:
- Both jobs are early in both positions.
- One job is early in both positions, but another is early in the first position and tardy in the second position.
- Both jobs are early in the first position and tardy in the second position.
- One job is tardy in both positions, but another is early in both positions.
- One job is tardy in both positions, but another is early in the first position and tardy in the second position.
- Both jobs are tardy in both positions

The illustrations of all cases are shown in Figure A.1. The proof is similar to that in Ow and Morton (1989) with an addition of sequence-dependent setup cost.

Case 1. Both jobs are early in both positions.

Position of jobs i and j can be interchanged without affecting the cost of other jobs in the sequence. If adjacency condition holds, then $c(ij) \leq c(ji)$.

$$TSC_{ij} = SC_{xi} + SC_{ij} + SC_{jy} \tag{A.1}$$

$$c(ij) = h_i(d_i - t - p_i) + h_j(d_j - t - p_i - p_j) + TSC_{ij} \tag{A.2}$$

$$c(ij) = h_i(d_i - t - p_i) + h_j(d_j - t - p_j) - h_j p_i + TSC_{ij} \tag{A.3}$$

$$TSC_{ij} - h_j p_i = c(ij) - h_i(d_i - t - p_i) - h_j(d_j - t - p_j) \tag{A.4}$$

Similarly,

$$TSC_{ji} = SC_{xj} + SC_{ji} + SC_{iy} \tag{A.5}$$

$$c(ji) = h_j(d_j - t - p_j) + h_i(d_i - t - p_i - p_j) + TSC_{ji} \tag{A.6}$$

$$c(ji) = h_j(d_j - t - p_j) + h_i(d_i - t - p_i) - h_i p_j + TSC_{ji} \tag{A.7}$$

$$TSC_{ji} - h_i p_j = c(ji) - h_i(d_i - t - p_i) - h_j(d_j - t - p_j) \tag{A.8}$$

Since job i is early in both positions (d_i - t - $p_i \geq p_j$) thus, $\Omega_{ij} = p_j$. Similarly, job j is early in both positions (d_j - t - $p_j \geq p_i$) thus, $\Omega_{ji} = p_i$. Substituting these values into adjacency condition gives:

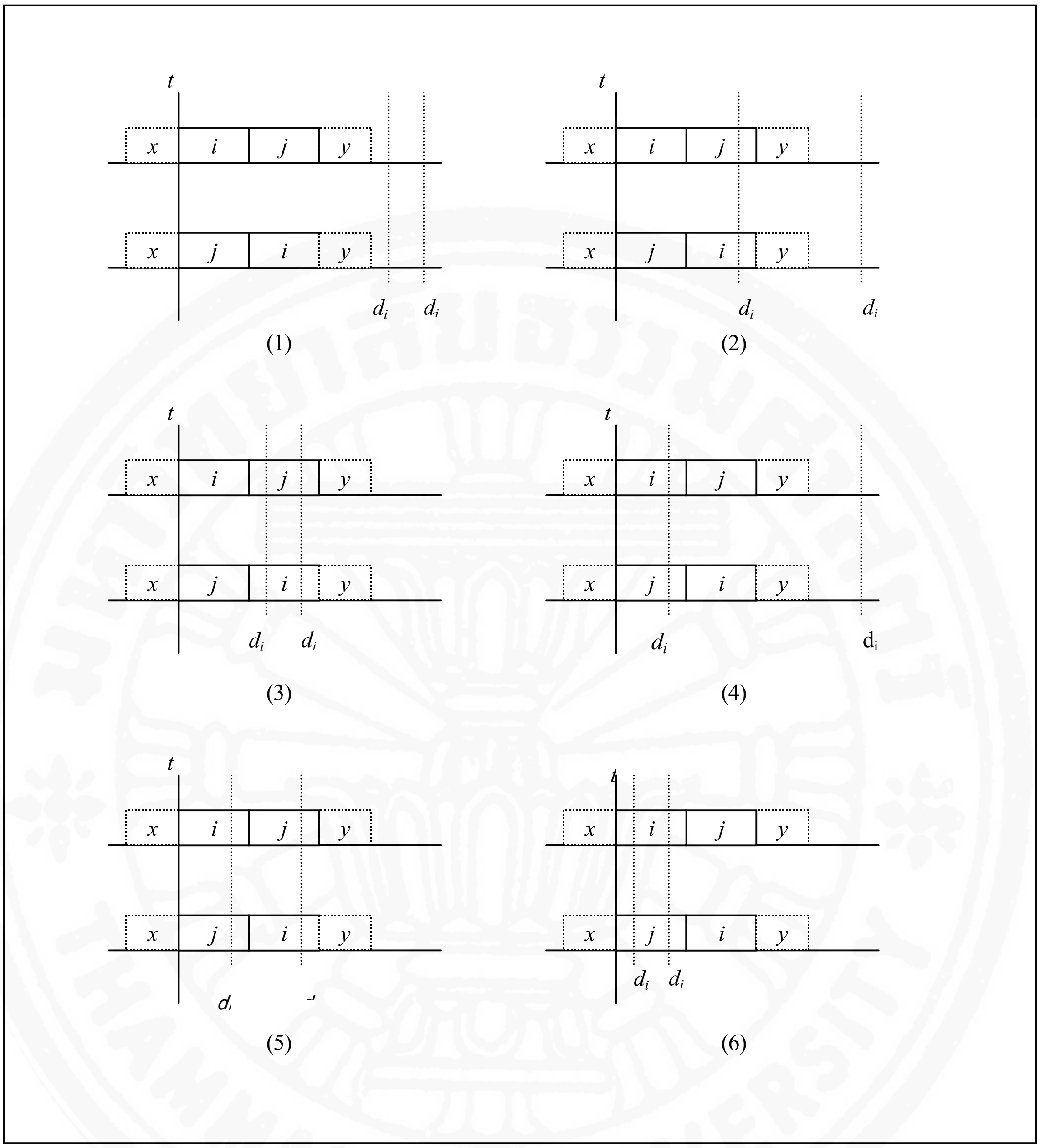

Figure A.1 Illustration of adjacency condition cases 1 – 6

$$TSC_{ji} + w_i p_j - p_j (w_i + h_i) \geq w_j p_i - p_i (w_j + h_j) + TSC_{ij} \tag{A.9}$$

By simplifying,

$$TSC_{ji} - h_i p_j \quad \geq \quad - h_j p_i + TSC_{ij} \tag{A.10}$$

Substituting (A.4) and (A.8) into (A.10) gives:

$$c(ij) \quad \leq \quad c(ji) \tag{A.11}$$

Case 2. One job is early in both positions, but another is early in the first position and tardy in the second position.

$$c(ij) = h_i(d_i - t - p_i) + h_j(d_j - t - p_i - p_j) + TSC_{ij} \quad \text{A.12)}$$

$$c(ij) = h_i(d_i - t - p_i) + h_j(d_j - t - p_j) - h_j p_i + TSC_{ij} \quad \text{(A.13)}$$

$$TSC_{ij} - h_j p_i + h_i(d_i - t - p_i) = c(ij) - h_j(d_j - t - p_j) \quad \text{(A.14)}$$

and,

$$c(ji) = h_j(d_j - t - p_j) + w_i(t + p_i + p_j - d_i) + TSC_{ji} \quad \text{(A.15)}$$

$$c(ji) = h_j(d_j - t - p_j) - w_i(d_i - t - p_i) + w_i p_j + TSC_{ji} \quad \text{(A.16)}$$

$$TSC_{ji} + w_i p_j - w_i(d_i - t - p_i) = c(ji) - h_j(d_j - t - p_j) \quad \text{(A.17)}$$

Since job *i* is early in the first position only ($s_i < p_j$), thus $\Omega_{ij} = s_i$. While job *j* is early in both positions ($s_j \geq p_i$), thus $\Omega_{ji} = p_i$. Substitute these values in adjacency condition and then simplify the expression as presented in formula (A.18).

$$TSC_{ji} + w_i p_j - s_i(w_i + h_i) \geq w_j p_i - p_i(w_j + h_j) + TSC_{ij} \quad \text{(A.18)}$$

Substituting $s_i = d_i - t - p_i$ gives:

$$TSC_{ji} + w_i p_j - (d_i - t - p_i)(w_i + h_i) \geq w_j p_i - w_j p_i - h_j p_i + TSC_{ij} \quad \text{(A.19)}$$

By simplifying

$$TSC_{ji} + w_i p_j - w_i(d_i - t - p_i) \geq -h_j p_i + h_i(d_i - t - p_i) + TSC_{ij} \quad \text{(A.20)}$$

Similarly, substituting (A.14) and (A.17) into (A.20) gives:

$$c(ij) \leq c(ji) \quad \text{(A.21)}$$

The same procedure is repeated for the remaining cases of *i* and *j*.