Appendix B. Proof of Non-adjacency Condition

Similar to the proof of adjacency condition, there are also six cases which can occur for jobs *i* and *j* in the sequence. Non-adjacency condition can be applied only when the processing time of non-adjacent pairs of jobs are equal. Therefore, changing the position of jobs *i* and *j* are not affecting the cost of other jobs in the sequence. If non-adjacency condition holds, then $c(ij) \leq c(ji)$.

Case 1. Both jobs are early in both positions as shown in Figure B.1.

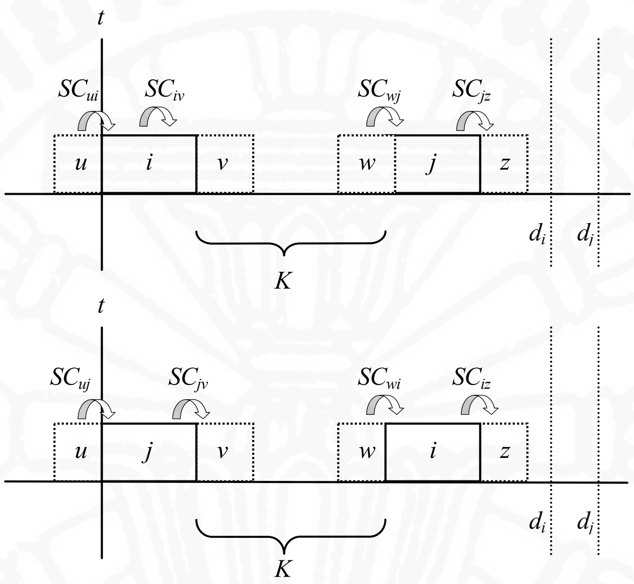

Figure B.1 Illustration of non-adjacency condition case 1

$$TSC_{ij} = SC_{ui} + SC_{iv} + SC_{wj} + SC_{jz} \tag{B.1}$$

$$c(ij) = h_i(d_i - t - p_i) + h_j(d_j - t - p_i - K - p_j) + TSC_{ij} \tag{B.2}$$

$$c(ij) = h_i(d_i - t - p_i) + h_j(d_j - t - p_j) - h_j(p_i + K) + TSC_{ij} \tag{B.3}$$

$$TSC_{ij} - h_j(p_i + K) = c(ij) - h_i(d_i - t - p_i) - h_j(d_j - t - p_j) \tag{B.4}$$

and,

$$TSC_{ji} = SC_{uj} + SC_{jv} + SC_{wi} + SC_{iz} \tag{B.5}$$

$$c(ji) = h_j(d_j - t - p_j) + h_i(d_i - t - p_j - K - p_i) + TSC_{ji} \tag{B.6}$$

$$c(ji) = h_j(d_j - t - p_j) + h_i(d_i - t - p_i) - h_i(p_j + K) + TSC_{ji} \tag{B.7}$$

$$TSC_{ji} - h_i(p_j + K) = c(ji) - h_i(d_i - t - p_i) - h_j(d_j - t - p_j) \tag{B.8}$$

Since job i is early in both positions (d_i - t - $p_i \geq p_j + K$), thus, $\Delta_{ij} = p_j + K$. Similarly, job j is early in both positions (d_j - t - $p_j \geq p_i + K$) thus, $\Delta_{ji} = p_i + K$. Substituting these values into non-adjacency condition gives:

$$TSC_{ji} + w_i(p_j + K) - (p_j + K)(w_i + h_i) \geq w_j(p_i + K) - (p_i + K)(w_j + h_j) + TSC_{ij} \quad \text{(B.9)}$$

This can be reduced to:

$$TSC_{ji} - h_i(p_j + K) \quad \geq \quad - h_j(p_i + K) + TSC_{ij} \quad \text{(B.10)}$$

Similarly, substituting (B.4) and (B.8) into (B.10) gives:

$$c(ij) \quad \leq \quad c(ji) \quad \text{(B.11)}$$

The same procedure can be repeated for the remaining cases of jobs i and j.