CHAPTER 5

CONCEPTS AND MODEL FOR STRENGTH RATIO OF CONCRETE

5.1 General

In this chapter, the relationship between strength development and degree of hydration is determined. The w/b ratio and γ are used as the main parameters to describe the development of the strength. There are many approaches using degree of hydration concept and they show some correlation between strength development and degree of hydration. Unfortunately, they mostly fail to predict the strength accurately and precisely. This might be explained by the fact that the strength of the concrete depends not only on the reaction of the cementitious material but also on others such as microstructure formation and porosity during the reaction proceeds.

For simplicity of the model, we define new parameter called "strength ratio", which represents the relative strength at any age to the 28-day compressive strength of the same concrete as an important index for determining the compressive strength at other ages.

The compressive strength development of concrete depends on time(t), degree of hydration(α_{av}), curing temperature of concrete(T), water to cement ratio(w/c), the paste volume to void volume of compacted aggregate(γ) and porosity. All parameters are used to find out a mathematical model for predicting compressive strength of concrete. This model is divided into two parts. The first part considers strength development of concrete cured at room temperature while the second one takes into account the effect of elevated curing temperature.

5.2 Strength ratio of Concrete

Tangtermsirikul et al. (1998) proposed a model for strength development which took into account the effects of ages of concrete, water to binder ratio and SiO_2/CaO. This model had the assumption based on sufficient amount of $Ca(OH)_2$ and SiO_2 for pozzolanic reaction. Even though it gave good results of prediction, it could not explain some physical phenomena of strength development of concrete. For example, the rate of hydration reaction will change as the time goes by, which results in the dynamic capillary porosity change.

In this chapter, the development of the strength is discussed. The effect of pore structure change during the reaction is the main idea of this chapter. The degree of hydration is used as the parameter indicating the age of concrete. The temperature, w/b and γ are considered as the main parameters affecting the pore structure of the paste matrix.

To develop the model, the new parameter called 'strength ratio' is introduced as the ratio between the strength at any age of the concrete and its 28-day compressive strength. In strength ratio model, the average degree of hydration is considered as a part of strength ratio function (ϕ) which appears in term of hydration ratio(α_{nav}). The strength ratio is the compressive strength at any time compared with the compressive strength at 28 days.

$$\phi\,(t, T) = \frac{f(t,T)}{f(28,30°C)} \tag{5.1}$$

where

ϕ (t, T) : the compressive strength ratio of concrete at any considered age t and cured at any constant temperature based on 28-day compressive strength

f(t,T) : the compressive strength of concrete at any considered age tand cured at any constant temperature (MPa)

f(28,30°C) : the 28-day compressive strength of concrete cured at room temperature (MPa)

In the model, degree of hydration is one of the main parameters and also represents age of concrete. Since the 28-day compressive strength is considered as the basis of this study. It is reasonable to define the new term to link between degree of hydration and time, and to have the same value at 28 days of all mixes. For this propose, the hydration ratio is defined as the ratio between the degree of hydration at any age of the concrete and the degree of hydration at 28 days and is represented by the symbol '$\alpha_{nav}(t,T)$'.

$$\alpha_{nav}(t,T) = \frac{\alpha_{av}(t,T)}{\alpha_{av}(28,T)} \tag{5.2}$$

where

$\alpha_{nav}(t,T)$: the hydration ratio at any considered age t and cured at any constant temperature based on degree of hydration at 28 days.

$\alpha_{av}(t,T)$: the average degree of hydration at any considered age t and cured at any temperature(%)

$\alpha_{av}(28,T)$: the average degree of hydration at 28 days at any constant curing temperature (%)

To construct the model, it is much more simple if $\phi(t,T)$ and $\alpha_{nav}(t,T)$ are based on the compressive strength and the average degree of hydration at 1 year. But it is deliberated that most concrete construction work takes 28-day compressive strength into account in the analysis and design. Furthermore, the 28-day compressive strength model has already been established. These make 28-day compressive strength of concrete more reasonable and reliable to be used as based strength than compressive strength at any other ages. It can be seen that as the hydration proceeds; in term of ratio; concrete will develop its strength as shown in Fig. 5.1.

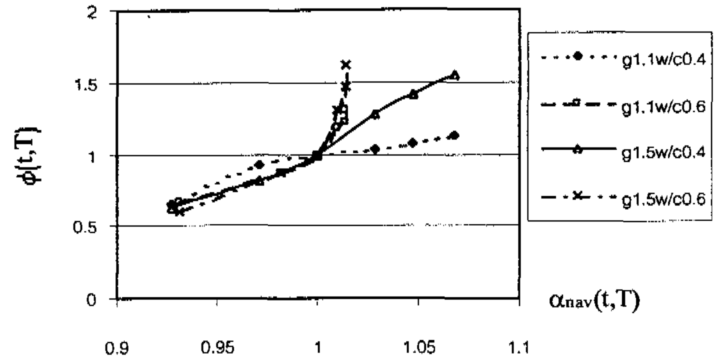

Fig. 5.1 The effect of hydration ratio on the strength ratio

The strength development of concrete depends upon many factors. Those factors are described in this section.

5.2.1 Hydration reaction

Strength development is an indirect result from hydration reaction. When hydration reaction proceeds, concrete will develop its strength as shown in Fig. 5.2. The degree of hydration in Fig. 5.2 was computed from Eq. (4.9) in Chapter 4. As described in chapter 4, the degree of hydration directly assumes the effect of time, w/c and temperature. It can imply that those parameters also have effects on strength development.

It is also found that the more rapid hydration leads to the lower quality of the hardened concrete in long term, since non-uniform gel and more porous structure of hydration products can be established, leading to weaker hydrated structure.

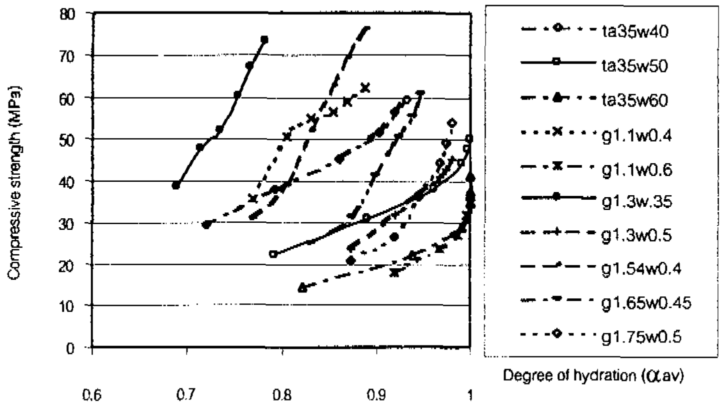

Fig. 5.2 Relationship between the Compressive Strength and Degree of Hydration

5.2.2 Water to cement ratio

The water to binder ratio is the main factor affecting the strength development. The water to binder ratio affects both the degree of hydration and pore structure of the cement paste matrix. The water content in cement paste controls the porosity of the cement paste matrix. The water to binder ratio directly influences microstructure formation by affecting the mean inter-particle distance and the volume of capillary porosity. The higher w/c ratio means a higher fraction of total porosity residing in the cement paste matrix. This effect causes the lower density in concrete resulting in the lower compressive strength as shown in Fig. 5.3. Therefore, it can imply that the water to binder ratio have strong relationship with the pore-structure.

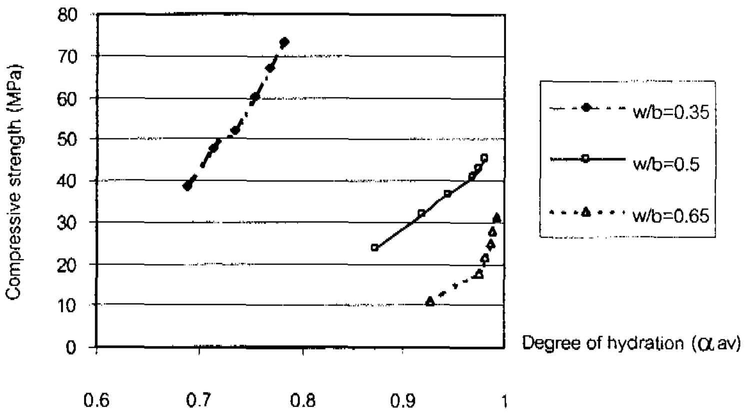

Fig. 5.3 Effect of the Water to Binder Ratio on the Development of the Strength (obtained from the concrete with γ =1.3)

Furthermore, the water to binder ratio also affects the hydration reaction inside the cement paste. The overall hydration level, which can be achieved in concrete, increases with the higher water to binder ratio as shown in Fig. 4.1. This means that, with enough water to binder ratio, the hydration reaction almost proceed to complete hydration reaction. While, in the concrete with low water to binder ratio, the hydration reaction seems not complete due to the lack of the water available for the reaction. The maximum degree of hydration increases when water to binder ratio of the concrete increases.

Although the more water to binder ratio increases the maximum degree of hydration, the condition of pore structure of the cement paste becomes weaker due to corresponding porosity. It is obvious that, at the maximum degree of hydration, the strength of the concrete with low water to binder ratio is greater than the strength of concrete with higher water to binder ratio though the maximum degree of hydration is lower.

5.2.3 Paste

In this study, the effect of paste is considered in terms of the paste volume to void volume of compacted aggregate (γ). From the data available, the effect of γ on strength development seemed to couple with effect of water to binder ratio as shown in Fig. 5.4 and Fig. 5.5. In other words, the effect of paste content is also related to the porosity of the cement paste.

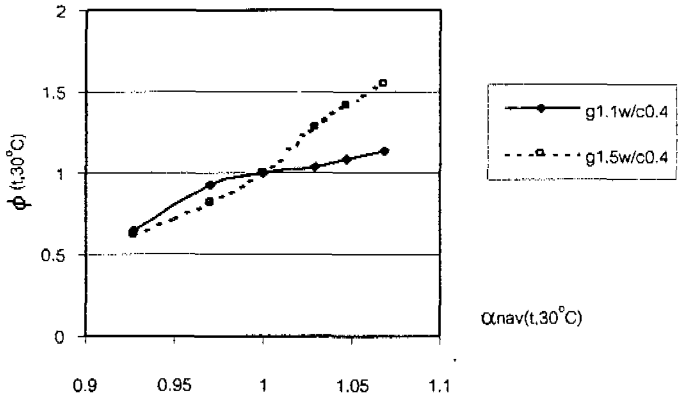

Fig. 5.4 Effect of paste on strength development when concrete has low water to cement ratio

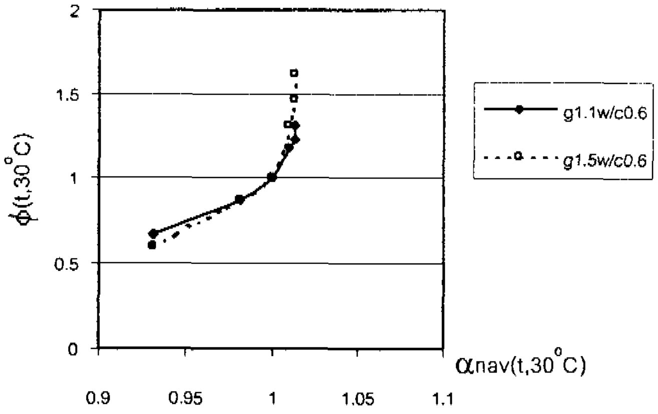

Fig. 5.5 Effect of paste on strength development when concrete has high water to cement ratio

The facts about this phenomenon are still not so obvious. In case of low water to binder ratio, the higher paste concrete seems to develop better strength than the lower one. One of the suitable reasons is that the higher paste content gives higher possibility of the contact between water and the cement particles; while, the lower paste gives less possibility. However, for high water content concrete, this phenomenon is disappears due to the sufficient amount of water surrounding the cement particles.

5.2.4 Temperature

The effect of temperature is considered in the calculation of degree of hydration. Higher rates of hydration due to higher temperatures would lead to fast microstructure and strength development at the early ages. Due to higher reaction rates at higher temperature, the hydration products are produced rapidly and precipitate near the reactant particle surfaces owing to high product concentrations.

This results in not only a higher pore volume but also a coarser pore size distribution. Consequently, the lower compressive strength development is presented in long term when compared with concrete cured at room temperature as shown in Fig. 5.6.

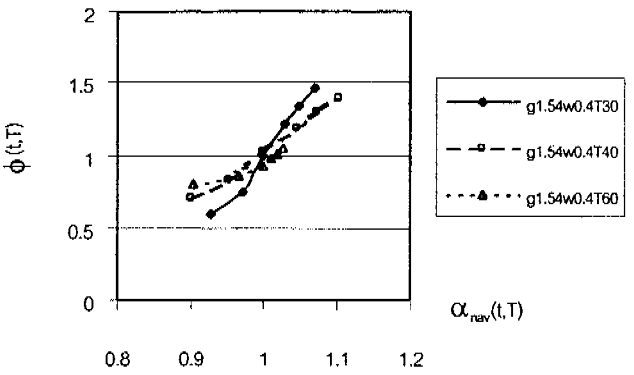

Fig. 5.6 Effect of curing temperature on compressive strength development of concrete

5.2.5 Pore structure

The physical properties of concrete such as strength, durability, shrinkage, creep, and permeability are directly influenced or controlled by the quantity and the characteristics of the various types of pores in concrete. The air void and void in aggregate are excluded in this study. Other void space, which results from excess water content or poor mix proportioning, is taken into consideration in this section.

The overall microstructure is subdivided into three basic components. The first one is the capillary pore that is actually empty spaces left between partially hydrated cement grains. Second one is the gel pore that is formed as an internal geometrical structure of the CSH grains which deposit around a hydrating cement grain. Third one is the physically fixed interlayer moisture that is actually a structural part of the CSH gel. In this study, only capillary porosity is taken into consideration to have the effect on strength development of concrete since it represents the space where new hydration products can be formed. A higher capillary porosity leads not only to a lower strength of concrete, but also to a lower durability of concrete, too.

However, in this study, it is difficult to directly measure the amount and volume of the capillary porosity distribution, since the capillary pores undergo dynamic changes, such as filled up with new hydrated product, as the hydration proceeds. By back analysis of strength development, effect of capillary pore is determined in terms of the relative pore structure.

5.3 Strength Ratio Model

The objective of the development of this model is to obtain the mathematical equation to predict the compressive strength of the conventional concrete at any considered time t. Parameters described in section 5.2 are used in constructing the strength ratio model. This model is subdivided into two parts in order to describe and consider the physical effect of curing concrete at room temperature and at elevated temperature.

5.3.1 Strength ratio of concrete cured at room temperature

In this section, the effect of both w/c and γ is discussed through the steps to achieve the model. The model is developed based on data from Tahir (1998), Deatpan (1999) and Sujjavanich (1999).

At first, it is assumed that hydration reaction gives an important role to develop strength of concrete. Then the strength ratio is also expressed in terms of w/c and γ as shown in Eq. (5.3).

$$\phi\ (t,30^{\circ}C)=f\ (\ \alpha_{nav}(t,30^{\circ}C),\ w/c,\ \gamma\ ) \qquad (5.3)$$

The strength ratio represents the ratio between compressive strength of concrete at any time, t, and the 28-day compressive strength of concrete cured at room temperature and also implies the rate of strength development in each concrete mix. In Fig. 5.7, the relationship between the strength ratio and degree of hydration is shown. The steeper slope of the graph means the faster strength development. Among various concrete mixes, the higher strength ratio does not mean the higher strength due to the different 28-day compressive strength of different mixes. In conclusion, the strength ratio describes only the rate of development of compressive strength based on 28-day compressive strength of each concrete mix.

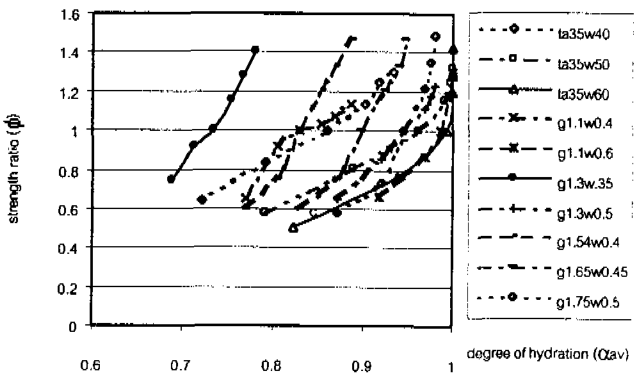

Fig. 5.7 Relationship between the Strength Ratio and Degree of Hydration (the same data set as in Fig. 5.2)

In Fig. 5.8, the relationship between the strength ratio and hydration ratio is shown. It is noted that the strength ratio and hydration ratio are always one at the age of 28 days. From Fig. 5.8, for the part where hydration ratio is less than one, the behavior of all concrete mixes is the rather similar. However, for the later ages, the large differences are noticed. This is considered to be the effect of the condition of pore structure. The effect is resulted from the effect of water to binder ratio, paste volume to void volume ratio (γ) and also the age of the concrete itself.

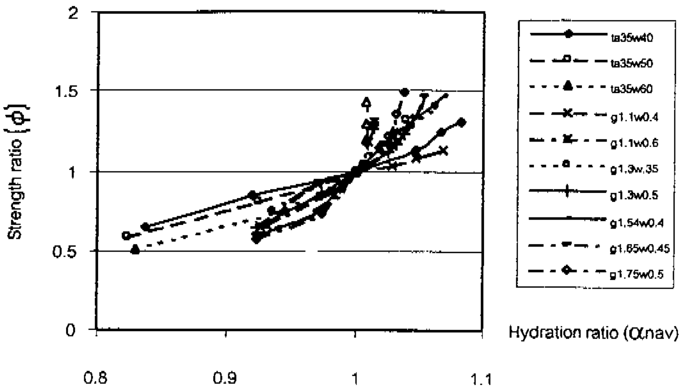

Fig. 5.8 Relationship between the Strength Ratio and Hydration Ratio
(the same data set as in Fig. 5.2 and Fig. 5.7)

The effect of water to binder ratio and the strength ratio seems to be coupled as discussed in section 5.2.3. Fig. 5.1 shows the effect of w/b ratio and γ on the development of the strength. The behavior of the strength development for the range of hydration ratio larger than one is discussed. For the concrete with high water to cement ratio, it is obvious that the effect of γ is negligible; while, for the concrete with low water to binder ratio, the effect of γ on the strength ratio is obvious. Even though the fact of this effect is still not clarified. Some of the assumptions are made about this behavior. This follows the behavior of the concrete discussed in section 5.2.3. The increase of the γ increases the possibility of the cement to be completely surrounded by the water resulting in higher compressive strength for concrete with low w/b ratio. The strength ratio model in this study was developed as the empirical formula.

The relationship between strength ratio and hydration ratio as shown in Fig. 5.1 illustrates that it has non-linear relationship, which causes a difficulty in constructing the model. The model is divided into two ranges. The first range is used when degree of hydration ratio ($\alpha_{nav}(t,T)$) is less than 1. On the other hand, the second range is used when hydration ratio is greater than or equal to 1. The equations of strength ratio of concrete cured at room temperature are shown in Eq. (5.4).

$$\phi\,(t,30°C) = [4\times\alpha_{nav}(t,30°C)-3]-a\times[\alpha_{nav}(t,30°C)-1] \qquad (5.4)$$

when $\alpha_{nav}(t,30°C) < 1$

$$a = (277.46\times\gamma^2-685.21\times\gamma+408.22)\times(w/c)+(-143.95\times\gamma^2+352.99\times\gamma-207.92) \qquad (5.5)$$

when $\alpha_{nav}(t,30°C) \geq 1$

$$a = [-0.5579\times\exp\{7.541\times(w/c)\}]\times\gamma^2+[0.531\times\exp\{9.35\times(w/c)\}]\times\gamma \\ -0.0562\times\exp\{12.96\times w/c\} \qquad (5.6)$$

where

a : function that take into account the effect of w/c and γ

5.3.2 Strength ratio of concrete cured at elevated temperature

The effect of the changing in pore structure of concrete (β) is introduced in this section in order to take into account the effect of curing at elevated temperature. It is assumed that bis equal to 0 for the standard curing temperature (30±3°C). By back analysis, β is indirectly derived by comparing ϕ (t,T) with ϕ (t,30°C). The back analysis method is shown in Eq. (5.7). From Eq. (5.7), it can be implied that it is possible to predict ϕ (t,T) if β is known and appeared in function form.

$$\beta = \phi(t,T) - \phi(t,30°C) \qquad (5.7)$$

$$\phi(t,T) = \phi(t,30°C) + \beta(\alpha_{nav}(t,T), w/c \text{ and } T) \qquad (5.8)$$

$$\beta = f(\alpha_{nav}(t,T), w/c \text{ and } T) \qquad (5.9)$$

where β: the relative pore structure function

Fig. 5.9 and Fig. 5.10 show the effect of temperature on the relative pore structure development. Both figures illustrate that the higher curing temperature gives a good pore structure in early ages, but worse in long term when compared with concrete cured at room temperature.

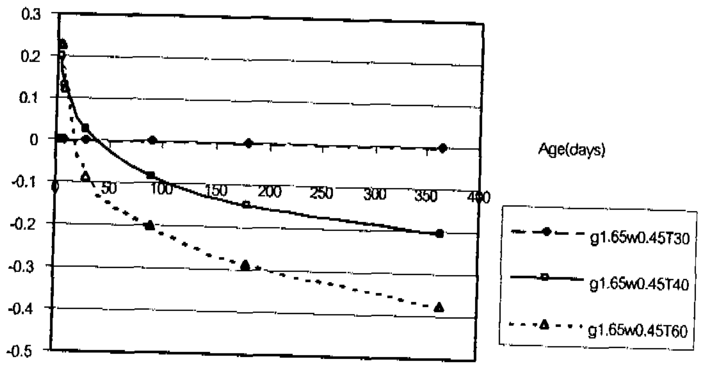

Fig. 5.9 The relative pore structure at elevated temperature compare with pore structure at room temperature considered in terms of time

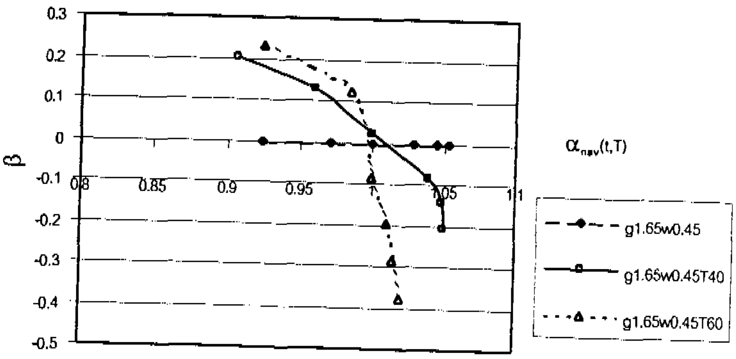

Fig. 5.10 The relative pore structure at elevated temperature compared with pore structure at room temperature considered in terms of hydration ratio

As discussed earlier, it is also possible to express the porosity of hardened paste as a direct function of degree of hydration that implicitly takes into account the water to cement ratio as well as the temperature. These two effects are shown in Fig. 5.11.

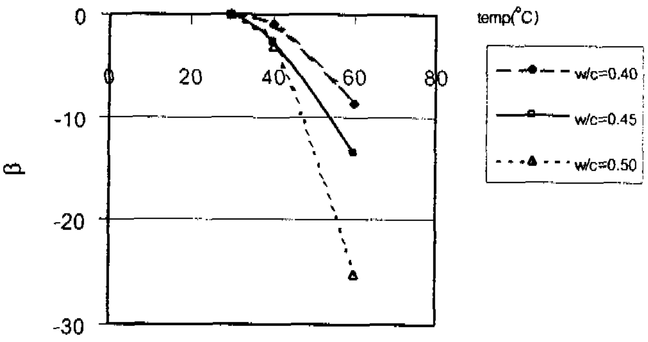

Fig. 5.11 The effects of temperature and w/c on relative pore structure for concrete with $\gamma = 1.54$

The effect of relative pore structure shown in Fig. 5.10 has non-linear relationship with hydration ratio. This effect makes it more convenient to divide the relative pore structure function into two functions as shown in Eq. (5.10) and Eq. (5.11).

when $\alpha_{nav}(t,30°C) < 1$

$$\beta = [(-0.432\times(w/c)+0.1208)\times100\times(T/100)^{0.4}+(0.1812\times e^{5.4654\times(w/c)})] \times [\alpha_{nav}(t,T)-1] \tag{5.10}$$

when $\alpha_{nav}(t,30°C) \geq 1$

$$\beta = [(-5.761\times(w/c)+2.0434)\times100\times(T/100)^{(-0.025\times T-2\times w+2.9)}+(0.136\times e^{10.61\times(w/c)})] \times [\alpha_{nav}(t,T)-1] \tag{5.11}$$

5.4 Summary of Equations

This section summarizes all equations that are used for predicting the compressive strength of concrete at any ages and any curing temperature as illustrated in Eq. (3.1) as:

$$f(t,T) = f(28,30°C)\times\phi(t,T)$$

Since this study is aimed at investigating the compressive strength of cement-only concrete. Eq. (A.2) in Appendix A will be used for predicting 28-day compressive strength of cement-only concrete under normal curing temperature. This equation (Eq. A.2) is more practical to be used to predict compressive strength of cement concrete which has the range of unit CaO about 150-320 kg/m^3 of concrete.

$$f'_c(28,30°C) = [(-79.095\times(w/c)+63.73)\times\log(CaO)+(93.94\times(w/c)-64.886)] \times(-0.3074\times\gamma+1.355)$$

Strength ratio at any considered time t and temperature is considered as shown in Eq. (3.2) as:

$$\phi(t,T) = \phi(t, 30°C)+\beta$$

For obtaining the strength ratio, $\phi(t,T)$, Eq. (5.4), Eq. (5.5), Eq. (5.10) and Eq. (3.2) are applied when $t < 28$ days and Eq. (5.4), Eq. (5.6), Eq. (5.11) and Eq. (3.2) are applied when $t \geq 28$ days.

The verification of this model is shown in chapter 8.