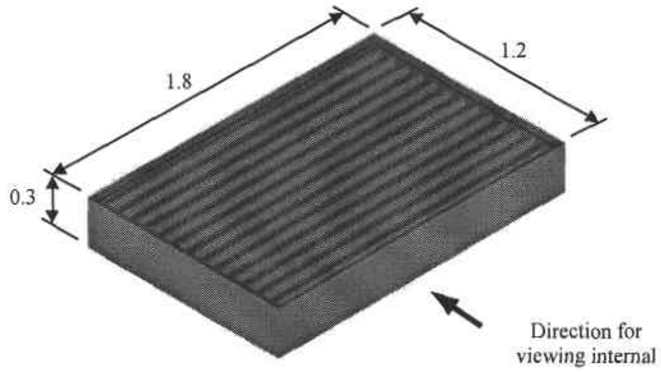
Chapter 3

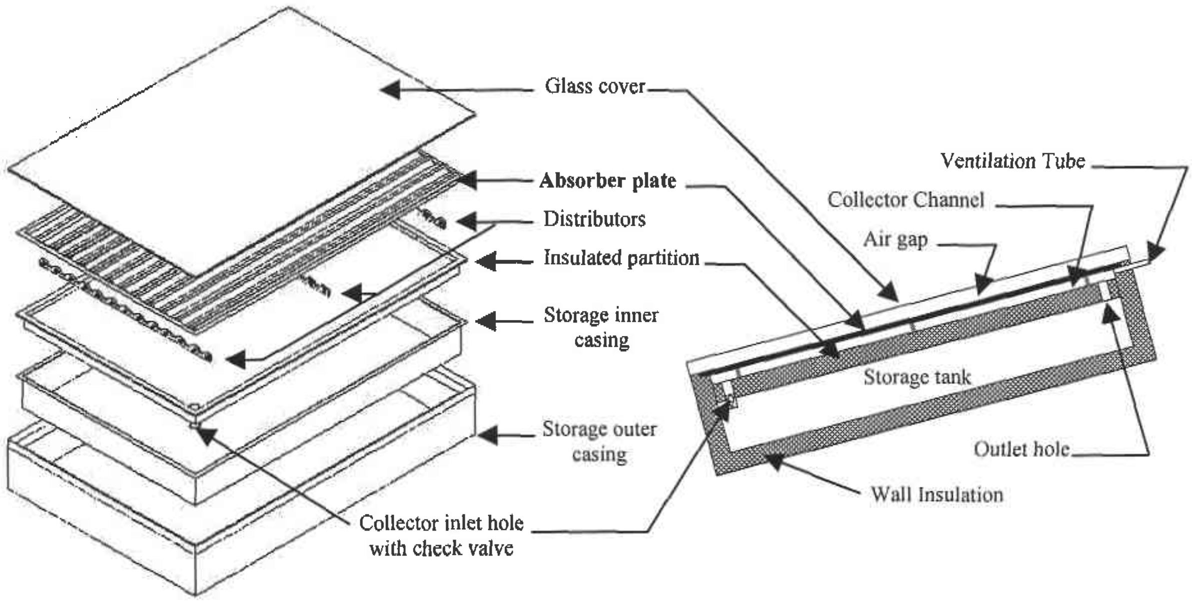
System Design and Modeling

3.1 System Configuration

The disadvantage of a simple built-in-storage solar water heater is that it has significant heat losses during off-sunshine periods due to (i) the reverse flows of warm water from the storage to cool down at the collector and (ii) the uninsulated wall between the storage and the collector. This study is therefore aimed at designing a new built-in-storage solar water heater which can prevent or reduce these heat losses.



(a) Overall dimensions



(b) Assembly of main components

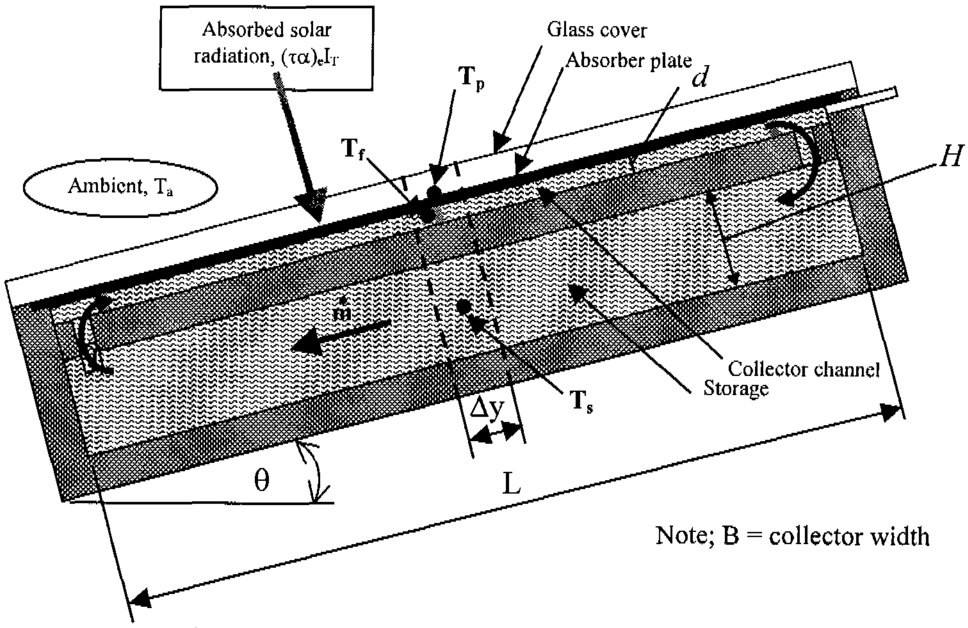
(c) Internal structure viewing from the direction shown in (a)

Fig. 3.1 Configuration of the developed built-in-storage solar water heater.

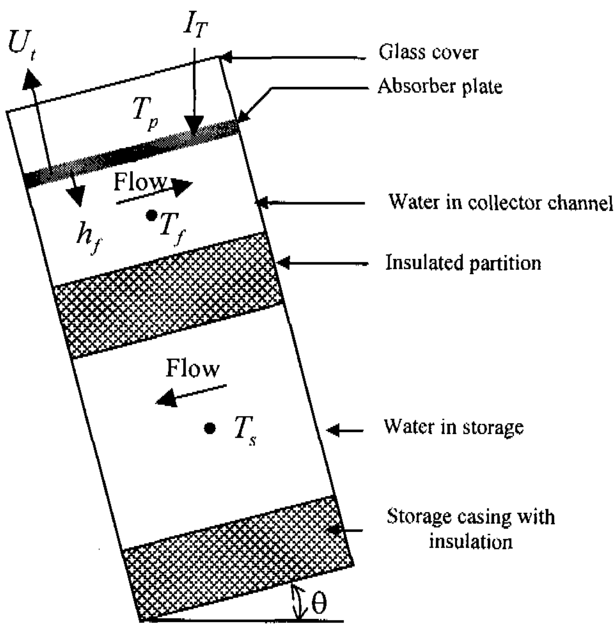
Figure 3.1 shows the newly-designed and constructed built-in-storage solar water heater. It consists of a solar absorber plate of 2.01-m^2 area which is integrated into a rectangular storage tank of 273-liter capacity. It is placed facing south with a tilt angle of 15°. The rectangular tank is made of stainless steel sheets. A corrugated absorber plate is made of a copper sheet which is painted with black matte. It is fixed over the tank and then covered by a 3-mm glass sheet. An air gap of 30 mm is left between the glass sheet and absorber plate. The sides and bottom of the tank is thermally insulated with 50-mm thick glass-fiber boards. An insulated partition is fixed between the absorber and storage tank. The partition is made of stainless steel sheets covering over a 25-mm thick styrofoam board. The gap between the absorber plate and insulated partition forms a collector channel of 25 mm in depth for water flowing passage. Three distributors are fixed along the water flowing passage inside the collector channel; 50 mm near the lower end, at the middle and at 50 mm near the upper end of the collector channel. Each distributor is mad of plastic plate. Its upper part is shaped to fit with the corrugated absorber and 16 holes of 12.5-mm in diameter are made for water flowing. The purpose of these distributors is to enhance a uniform flow distribution across the width of the collector channel. At the lower and upper ends of the partition, a hole of 25-mm diameter is provided at one corner of each end for water circulation between the collector channel and the tank. A commercial check-valve, the compressing spring of which is removed, is fixed to the hole at the lower end of the insulated partition. The valve is arranged so that it allows the water to flow only from the storage tank to the collector channel whereas the reversed flow is prevented. This arrangement forms a thermal diode. When the water in the collector channel (between the absorber and insulated partition) is heated in the daytime, its density decreases. A pressure difference is built up between the storage tank and the collector channel. It lifts the valve lid to open and the cooler water flows out from the tank to the collector for solar heating. During the night the water in the collector channel cools down, and a pressure difference is established between the warm water in the storage tank and the cold water in the collector channel causing the valve lid closed thus preventing cold water to flow from the collector channel to the storage tank.

3.2 Mathematical Model Formulation

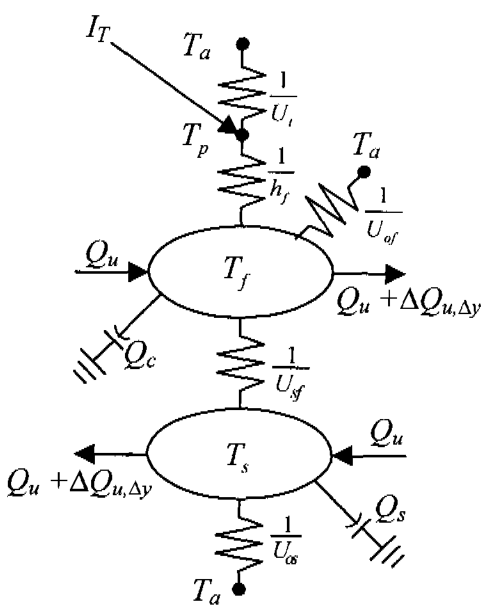
The thermal performance of the developed built-in-storage solar water heater can be described by the energy balances on three main components, i.e. the absorber plate, the water in the collector channel and the water in the storage tank as shown in Fig. 3.2.



(a) Schematic diagram of the three main components



(b) Enlarged schematic diagram of the three main components for an elementary portion Δy



(c) Heat transferred network connecting the three main components

Fig. 3.2 Main components of the built-in-storage solar water heater for model formulation

Various assumptions are made in order to simplify the thermal analysis of the system based on the three main components. They can be explained as in the following.

1) The temperatures of the absorber plate, water inside the collector channel, and water inside the storage are represented by T_p, T_f and T_s respectively.
2) The heat capacities of the glass cover and the absorber plate are very small and negligible in comparison with that of water either in the collector channel or in the storage.
3) The heat capacities of metal casing of insulated partition and metal distributor plates are very small, hence the heat capacity water capacity of the collector channel is represented by that of the water inside the collector channel which is assumed to be at T_f. Similarly, for storage, the heat capacity of the storage is assumed to that of the water inside the storage which is assumed to be at T_s.
4) The water mass flow rate is assumed to be uniform throughout the system.
5) Thermophysical properties (i.e. absorbtivity, emissivity and thermal conductivity) of materials (i.e. glass cover, absorber plate, metal wall and insulation) are constant and not function of temperature.
6) The convective heat coefficient between the absorber plate and the water in the collector channel is assumed to be average for each portion.
7) Thermophysical properties (density, viscosity, specific heat, heat conductivity and thermal expansion coefficient) of water in the collector channel and in the storage tank are dependent on their respective water temperatures.

Based on the above assumptions, the three governing equations for an elementary portion Δy as shown in Figs. 3.2(a) and 3.2(b), on the absorber plate, the water in collector channel and in storage tank can be developed for characterizing the thermal performance of the built-in-storage solar water heater operated under the specified input conditions of solar irradiance, ambient temperature and wind speed as follows:

3.2.1 Absorber Plate

The energy balance on the absorber plate, the heat capacity of which is assumed to be negligible, can be described by

$$Q_A - Q_{TL} = Q_{p-c}$$

or

$$(\tau\alpha)_e I_T - U_t(T_p - T_a) = h_f(T_p - T_f) \quad (3.1)$$

where Q_A = absorbed total solar radiation on the absorber plate (W/m^2),

Q_{TL} = total heat losses from the top of the absorber plate to the ambient air (W/m^2),

Q_{p-c} = total heat transfer from the absorber plate to the water in collector channel (W/m^2),

$(\tau\alpha)_e$ = effective transmittance-absorptance product of the collector,

I_T = total solar irradiance falling on the collector surface (W/m^2),

U_t = overall top loss coefficient between the absorber plate and ambient temperature taken into account the effect of glass cover (W/m^{2}K),

T_p = average temperature of the absorber plate (°C),

T_a = ambient temperature (°C),

h_f = convective heat transfer coefficient between the absorber and water in the channel (W/m^{2}K), and

T_f = average temperature of water in the channel (°C).

From eq. (3.1), one gets

$$T_p = \frac{(\tau\alpha)_e I_t + h_f T_f + U_t T_a}{h_f + U_t} \quad (3.2)$$

The effective transmittance-absorbtance product can be approximated by (Duffie and Beckman, 1980)

$$(\tau\alpha)_e = 1.01\,\tau_g \alpha_p \quad (3.3)$$

where τ_g = transmittance of glass cover, and

α_p = absorptance of absorber plate.

The overall top loss coefficient can be calculated by the following equation (Klein, 1979).

$$U_t = \left[\frac{N}{\frac{C}{T_p^*}\left(\frac{T_p^*-T_a^*}{N+f}\right)^e} + \frac{1}{h_v} \right]^{-1} + \frac{\sigma(T_p^* + T_a^*)(T_p^{*2} + T_a^{*2})}{(\varepsilon_p + 0.00591Nh_v)^{-1} + \frac{2N+f-1+0.133\varepsilon_p}{\varepsilon_g} - N} \quad (3.4)$$

where N = number of glass cover,

T_p^* = absolute absorber plate temperature (K),

T_a^* = absolute ambient temperature (K),

ε_g = emittance of glass cover,

ε_p = emittance of absorber plate,

σ = Stefan-Boltzmann constant (5.6697 x 10^{-8} W/m^{2}K^4),

h_v = wind heat transfer coefficient = 5.7 + 3.8v_{wind} (W/m^2°C, [McAdams, 1954], where v_{wind} = wind speed across the front cover of the collector, m/s),

e = $0.43(1-\frac{100}{T_p^*})$,

f = $(1 + 0.089 h_v - 0.1166 h_v \varepsilon_p)(1 + 0.07866 N)$, and

C = $520(1-0.000051\theta^2)$ for $0° < \theta < 70°$, θ = collector tilt angle (degree).

Note that U_t is calculated based on projected area of the absorber plate (Hottel and Whillier, 1958; Whillier, 1977; and Bliss, 1959).

The convective heat transfer coefficient between the absorber and water in the collector channel can be calculated for the inclined plate facing downward with approximately constant heat flux as follows (Fujii and Imura, 1972):

$$h_f = \overline{Nu_e} \frac{k_w}{\Delta y} \quad (3.5)$$

where $\overline{Nu_e}$ = average Nusselt number = $0.56(Gr_e \, Pr_e \cos\theta)^{1/4}$ $\theta < 88°$; $10^5 < Gr_e \, Pr_e \cos\theta < 10^{11}$

Gr_e	$= g\rho\beta(T_p - T_f)L^3/\mu$,
g	= gravitational constant (m/s^2),
ρ	= density of water (kg/m^3),
β	= thermal expansion coefficient of water (K^{-1}),
μ	= viscosity of water (kg/m-s),
k_w	= thermal conductivity of water (W/mK),
Δy	= length of the portion (m).

Note that all properties of water except β, using to calculated the average Nusselt number, are evaluated at a reference temperature $T_{ref1} = T_p - 0.25(T_p - T_f)$; β is evaluated at a temperature of $T_{ref2} = T_f + 0.5(T_p - T_f)$.

3.2.2 Collector Channel

The energy balance of the water in the collector channel for each portion *dy* at any instant is described by the following equation.

$$Q_c = Q_{p-c} - Q_{c-s} - Q_{c-a} - Q_u$$

or

$$\rho_f d C_{pf} \frac{dT_f}{dt} = h_f(T_p - T_f) - U_{sf}(T_f - T_s) - U_{of}(\frac{2d}{B})(T_f - T_a) - \frac{\dot{m}C_{pf}}{B}\frac{dT_f}{dy} \qquad (3.6)$$

where

Q_c	= total energy stored in the water in the collector channel at any instant (W/m^2),
Q_{c-s}	= energy transfer between water in the collector channel and the storage tank by passing through the insulated partition (W/m^2),
Q_{c-a}	= energy loss from the collector channel to the ambient air (W/m^2),
Q_u	= useful energy in the water flowing out from the collector channel (W/m^2),
ρ_f	= density of the water in the collector channel (kg/m^3),
d	= collector channel depth (m),
B	= width of collector channel (m),

C_{pf} = specific heat of water in the collector channel (kJ/kgK),

$\dot{m}$ = mass flow rate of water flowing through the system (kg/s),

T_f = water temperature in the collector channel (°C),

T_s = water temperature in the storage tank (°C), and

U_{sf} = heat transfer coefficient between water in the channel and the storage tank (W/m^{2}K) = k_{sf}/l_{sf}, where k_{sf} = heat conductivity of the insulated partition and l_{sf} = thickness of the insulated partition,

U_{of} = heat losses coefficient from water in the collector channel through the side insulation wall to the ambient air (W/m^{2}K) = k_{of}/l_{of}, where k_{of} = heat conductivity of the wall insulation partition and l_{of} = thickness of the wall insulation.

After substituting an expression of T_p into eq. (3.6) one has

$$\rho_f d C_{pf} \frac{dT_f}{dt} = \frac{h_f[(\tau\alpha)_e I_T - U_t(T_f - T_a)]}{h_f + U_t} - U_{sf}(T_f - T_s) - U_{of}(\frac{2d}{B})(T_f - T_a) - \frac{\dot{m} C_{pf}}{B}\frac{dT_f}{dy} \qquad (3.7)$$

Since the partial differential form of eq. (3.7) is not easy to solve analytically, a finite difference numerical method is used. If the collector channel is represented by n portions along its flow direction as shown in Fig. 3.3. The temperature of the j^{th} portion at the next time step Δt can be written as follows:

$$T_{f_j}^{t+\Delta t} = T_{f_j}^t + \frac{\Delta t}{\rho_f^t d C_{pf}^t}\{\frac{h_f^t[(\tau\alpha)_e I_T^t - U_t^t(T_{f_j}^t - T_a^t)]}{h_f^t + U_t^t} - U_{sf}(T_{f_j}^t - T_{s_j}^t) - U_{of}(\frac{2d}{B})(T_{f_j}^t - T_a^t) - \frac{\dot{m}^t C_{pf}^t}{B\Delta y}(T_{f_j}^t - T_{f_{j-1}}^t)\} \qquad (3.8)$$

where T_f^t = water temperature in the collector channel at time t (°C),

$T_f^{t+\Delta t}$ = water temperature in the collector channel at time $t+\Delta t$ (°C),

T_s^t = water temperature in the storage tank at time t (°C),

T_a^t = ambient temperature at time t (°C),

ρ_f^t = density of the water in the collector channel at time t (kg/m^3),

C_{pf}^t = specific heat of water in the collector channel at time t (kJ/kgK),

I_T^t = total solar irradiance falling on the collector surface at time t (W/m^2),

h_f^t = convection heat transfer coefficient between the absorber and water in the channel at time t (W/m^{2}K),

U_t^t = overall top loss coefficient between the absorber plate and ambient temperature taken into account the effect of glass cover at time t (W/m^{2}K), and

$\dot{m}^t$ = mass flow rate of water flowing through the system at time t (kg/s).

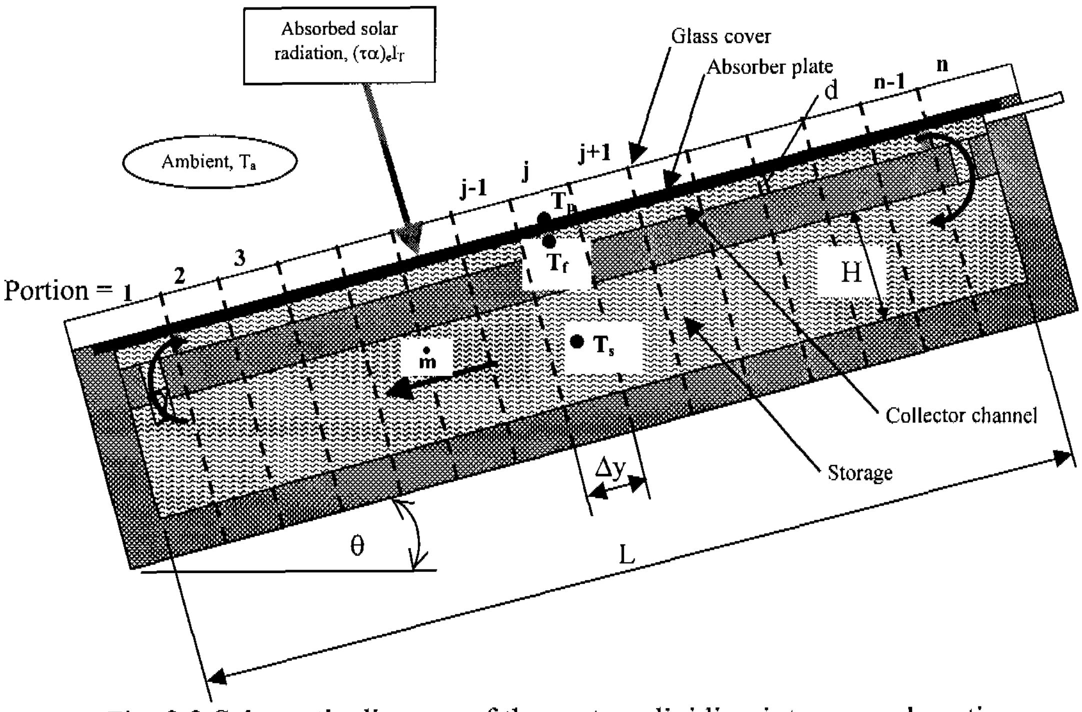


Fig. 3.3 Schematic diagram of the system dividing into n equal portions.

Note that, for the bottom and the top end portions of the collector channel, there is an additional heat loss from the water through the end side to the ambient air. Hence the 3rd heat loss term on the right hand side of eq. (3.8) must be replaced by $U_{of}(\frac{1}{\Delta y}+\frac{2}{B})d(T_{f_j}^t - T_a^t)$.

3.2.3 Storage Tank

The energy balance for an elementary portion *dy* of the water in the storage tank can be described by the following equation.

$$Q_s = Q_u + Q_{c-s} - Q_{s-a}$$

or

$$\rho_s H C_{ps} \frac{dT_s}{dt} = \frac{\dot{m} C_{ps}}{B}\frac{dT_s}{dy} + U_{sf}(T_f - T_s) - U_{os}(1+\frac{2H}{B})(T_s - T_a) \qquad (3.9)$$

where Q_s = total energy stored in the water in the storage at any instant (W/m^2),

Q_{s-a} = energy loss from the storage to the ambient air (W/m^2),

ρ_s = density of the water in the storage tank (kg/m^3),

H = storage tank depth (m),

C_{ps} = specific heat of water in the storage tank (kJ/kgK),

U_{os} = heat loss coefficient between the storage tank and ambient (W/m^{2}K); $U_{os} = k_{os}/l_{os}$, where k_{os} = heat conductivity of tank wall insulation (W/mK) and l_{os} = thickness of tank wall insulation (m).

If the tank is represented by *n* portions in the same way as the collector channel as shown in Fig. 3.3. The temperature at the next period of time for the j^{th} node can be written as

$$T_{s_j}^{t+\Delta t} = T_{s_j}^t + \frac{\Delta t}{\rho_s^t H C_{ps}^t}[\frac{\dot{m}^t C_{ps}^t}{B\Delta y_j}(T_{s_{j-1}}^t - T_{s_j}^t) + U_{sf}(T_{f_j}^t - T_{s_j}^t)$$

$$-U_{os}(1+\frac{2H}{B})(T_{s_j}^t - T_a^t)] \qquad (3.10)$$

where $T_s^{t+\Delta t}$ = water temperature in the storage tank at time $t + \Delta t$ (°C),

ρ_s^t = density of water in of storage tank at time *t* (kg/m^3),

C_{ps}^t = specific heat of water in the storage tank at the time *t* (kJ/kgK).

Note that, for the bottom and the top end portions of the storage, there is an additional heat loss from the water through the end side to the ambient air. Hence the 3rd

heat loss term on the right hand side of eq. (3.10) must be replaced by $U_{os}(1+\frac{2H}{B}+\frac{H}{\Delta y})(T_{s_j}^t - T_a^t)$.

3.2.4 Thermosyphon Flow Rate

a) *Thermosyphon Head*

The thermosyphon built-in-storage solar water heater and its density-head diagram can be shown in Fig. 3.4. The thermosyphon head at any instant is created by the difference between the total pressure head along the path 123 and that along the path 143.

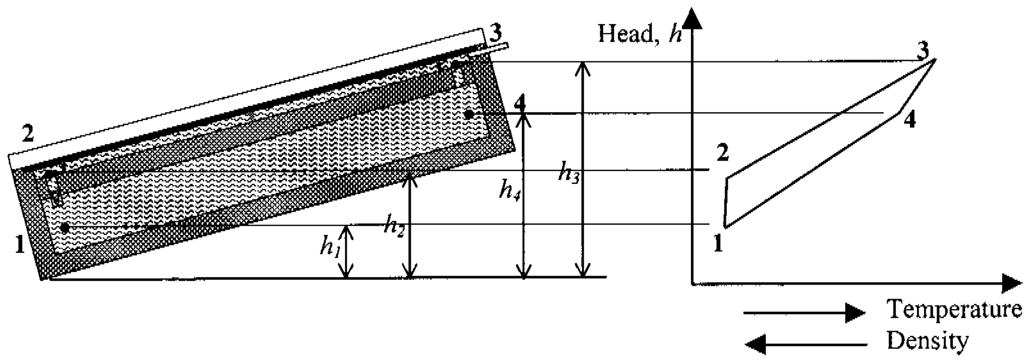


Fig. 3.4 The density head diagram of the system.

The thermosyphon head (H_T) can be described by

$$H_T = \int_1^2 \rho g dh + \int_2^3 \rho g dh - \int_1^4 \rho g dh - \int_4^3 \rho g dh \tag{3.11}$$

where h = the vertical height (m).

Since the temperature variation of water inside the system is small, the water temperature is assumed to be linearly dependent on its density i.e. $T = C\rho g$, where C is a constant. The thermosyphon head can then be expressed as:

$$H_T = C_1(h_2-h_1)(T_{mb}-T_{sb}) + C_1(h_3-h_2)(T_{fm}-T_{sb}) - C_1(h_4-h_1)(T_{sm}-T_{sb})$$

$$- C_1(h_3-h_4)(T_{mt}-T_{sb}) \tag{3.12}$$

where C_1 = constant

T_{fm} = average temperature of the water in the collector channel (°C),

T_{sm} = average temperature of the water in the storage tank (°C),

T_{mb} = average temperature of the water at the bottom, $(T_{sb}+T_{fb})/2$, (°C),

T_{mt} = average temperature of the water at the top, $(T_{ft}+T_{st})/2$, (°C),

T_{sb} = temperature of the water at the bottom of the storage (°C),

T_{fb} = temperature of the water at the bottom of the collector channel (°C),

T_{ft} = temperature of the water at the top of the collector channel (°C),

T_{st} = temperature of the water at the top of the storage (°C).

b) *Friction Head*

The total friction head that occurs in the built-in-storage solar water heater due to the friction of water flowing through the entrance, exit, distributor and check valve in the flow circuit can be described by:

$$H_F = C_2 V^2 = C_3 \dot{m}^2 \tag{3.13}$$

where C_2, C_3 = constant,

V = velocity of flow (m/s).

Note that the friction due to the viscosity of water along the collector channel is relatively small in comparison with the above-mentioned friction. However it can be assumed that this small effect is already included in eq (3.13).

c) *Mass Flow Rate*

Equating eqs. (3.12) and (3.13), the mass flow rate in passing through the collector channel can be obtained as;

$$C_3\dot{m}^2 = C_1\{(h_2-h_1)(T_{mb}-T_{sb}) + (h_3-h_2)(T_{fm}-T_{sb}) - (h_4-h_1)(T_{sm}-T_{sb}) - (h_3-h_4)(T_{mt}-T_{sb})\} \tag{3.14}$$

or

$$\dot{m} = K_f\{(h_2-h_1)(T_{mb}-T_{sb}) + (h_3-h_2)(T_{fm}-T_{sb}) - (h_4-h_1)(T_{sm}-T_{sb})$$

$$-(h_3\text{-}h_4)(T_{mt}\text{-}T_{sb})\}^{(1/2)} \quad (3.15)$$

where K_f = the overall flow coefficient due to the friction along the water flow passage through distributor, inlet and outlet hole of the collector channel and check valve (kg $s^{-1}\{m\ K\}^{-1/2}$).

3.2.5 Thermal Performance Parameters

The useful energy is calculated based on heat stored in the storage tank as the following.

$$Q_u = M_s C_p (T_2 - T_1) \quad \text{(kJ)} \quad (3.16)$$

where M_s = mass capacity of water in the storage tank (kg),

T_1 = average water temperature in the storage tank at sunrise (°C),

T_2 = average water temperature in the storage tank at sunset (°C).

The collection efficiency η_c during the day is defined as the ratio of amount of heat stored in the storage tank during the daytime to the energy absorbed by the collector as follows;

$$\eta_c = \frac{Q_u}{A_c G_T} \quad (3.17)$$

where G_T = total solar energy falling on the collector (kJ/m^2).

The storage efficiency η_s during the cool-down period at night is determined based on the remaining heat content of the storage tank before sunrise of the next morning to the maximum possible heat losses from the storage tank as follow;

$$\eta_s = \frac{T_3 - T_{a,night}}{T_{s,\max} - T_{a,night}} \quad (3.18)$$

where T_3 = average water temperature in the storage tank before sunrise of the next morning (°C),

$T_{s,\max}$ = maximum average water temperature in the storage tank (°C),

$T_{a,night}$ = average ambient temperature during the night (°C).

The system efficiency during 24 hours η_{24hour} is defined as amount of heat stored in the storage before sunrise of the next morning to the energy falling on the collector as follows;

$$\eta_{24hour} = \frac{M_s C_p (T_3 - T_1)}{A_c G_T} \tag{3.19}$$

Amount of energy stored in the storage tank before sunrise of the next morning can be calculated as

$$Q_{morning} = M_s C_p (T_3 - T_{a,morning}) \quad \text{(kJ)} \tag{3.20}$$

where $T_{a,morning}$ = ambient temperature at the same time of T_3 (°C).