CHAPTER 3

THEORETICAL BACKGROUND

3.1 Coastal Area

The general view of the territories for coastal region is showed in Fig. 3.1. In this study, the study area is mainly on the surf zone, which is defined as the shallow water where waves break. This area is the area with the most intense sediment transport because of high intensity of the turbulence due to the breaking of waves, which makes agitation of sediment from bottom easy.

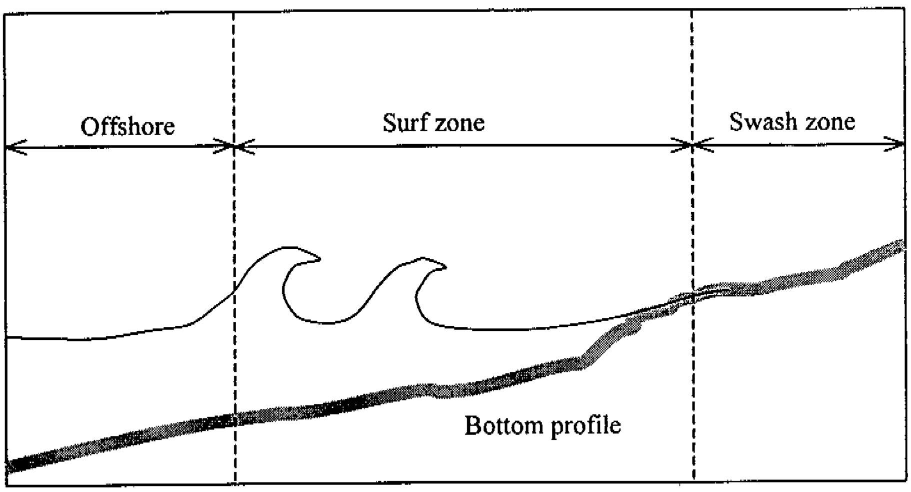

Fig. 3.1 The general view of the territories for coastal region

The shore has natural defenses protected from attack of waves, currents and storms. The first of these defenses is the sloping of beach, which causes the wave to break. The breaking wave may re-form to break again, and may repeat breaking until reaching the beach.

3.2 Characteristics of Waves

In this study, the regular wave has been simulated through out the characteristics of sinusoidal wave (Fig. 3.2). To analyze the waves in nature, which behave irregular. Applying the superposition of a large number of sinusoidal waves with different frequencies and directions can stimulate and model the irregular waves. Therefore, using of linear wave theory is an effective stepping–stone to irregular wave.

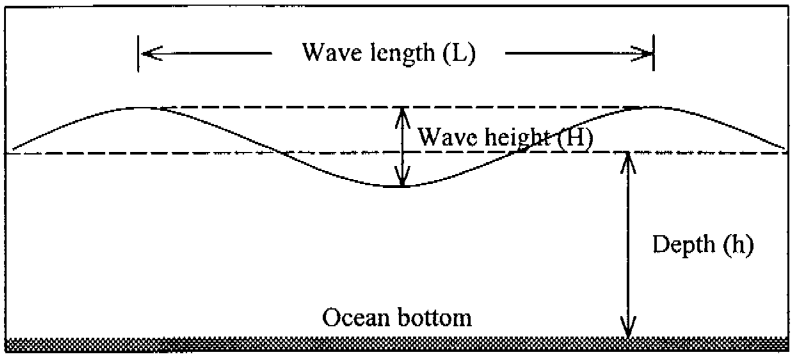

Fig. 3.2 A two-dimensional schematic of sinusoidal wave propagating in x-direction

3.3 Linear Wave Theory

By assuming the constant depth for two-dimensional flow in x-z plane, the governing equation and boundary conditions assumed small values of surface elevation (η), and velocity in x-direction (u) can be obtained as follow:

3.3.1 The governing differential equation

The governing equation to describe wave mechanics is continuity equation as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{3.1}$$

Wave kinematics is usually described by potential theory requiring the fluid to be inviscid and irrational. In this case, a potential ϕ can be introduced, which is related to the velocity field by

$$u=-\frac{\partial \phi}{\partial x} \tag{3.2}$$

$$v=-\frac{\partial \phi}{\partial y} \tag{3.3}$$

$$z=-\frac{\partial \phi}{\partial z} \tag{3.4}$$

where x and y are horizontal coordinates, and z is the vertical coordinate. u, w and v are the velocity components in the x, y and z direction. After applying this concept to the continuity equation, the Laplace equation can be obtained as

$$\frac{\partial^2 \phi}{\partial x^2}+\frac{\partial^2 \phi}{\partial y^2}+\frac{\partial^2 \phi}{\partial z^2}=0 \tag{3.5}$$

Considering only two dimensional of flow, then the governing equation can be expressed as

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial z^2}=0 \qquad \text{for} \qquad 0<x<L, -h<z<\eta \quad (3.6)$$

3.3.2 Boundary conditions

There are three basic boundary conditions: kinematics, dynamic and lateral boundary conditions. The description of each boundary condition is illustrated in Fig. 3.3 and is described as follow:

1. Kinematic boundary conditions

At any surface or fluid interface, there must be no flow across the interface; otherwise, there would be no interface.

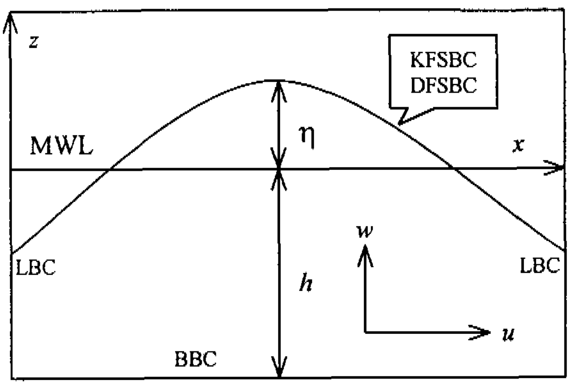

Fig. 3.3 Definition sketch of Boundary condition

- At the seabed, $z=-h(x)$, the velocity perpendicular to the bed is zero. The surface equation for the bottom is

$$w=-\frac{\partial\phi}{\partial z}=0 \quad \text{at} \qquad z=-h \qquad (3.7)$$

- A fluid particle located at the free surface , $z=\eta(x,t)$, must remain at the free surface giving

$$\frac{\partial\eta}{\partial t}=-\frac{\partial\phi}{\partial z} \quad \text{at} \qquad z=0 \qquad (3.8)$$

where h is mean water depth and η is the surface elevation.

2. Dynamic boundary conditions

The pressure at the water surface must be equal to the atmospheric pressure and can be set to zero, applying the Bernoulli equation with the constant p_η is applied on the free surface as illustrated in Fig. 3.3, $z=\eta(x,t)$.

$$-\frac{\partial \phi}{\partial t}+\frac{1}{2}\left(u^2+w^2\right)+\frac{p_\eta}{\rho}+g\eta \; = C(t) \qquad (3.9)$$

or

$$-\frac{\partial \phi}{\partial t}+\frac{1}{2}\left[\left(\frac{\partial \phi}{\partial x}\right)^2+\left(\frac{\partial \phi}{\partial z}\right)^2\right]+\frac{p_\eta}{\rho}+g\eta \; = C(t) \qquad (3.10)$$

where C is a function of t and p_η is a constant which usually taken as a gauge pressure and equal to 0 then equation becomes

$$-\frac{\partial \phi}{\partial t}+g\eta \; = C(t) \quad \text{on} \quad z=0 \qquad (3.11)$$

3. Lateral boundary conditions

For waves that are periodic in space and time, the boundary condition is expressed as a periodic condition in space and time (see Fig. 3.3),

$$W(x,t)=W(x+L,t) \qquad (3.12)$$

$$W(x,t)=W(x,t+T) \qquad (3.13)$$

where W is any wave variable.

After solving the equations by assuming that waves are periodic in space and in time, the solution of the linear wave theory can be obtained as

$$\phi=-\frac{Hg\cosh k(h+z)}{2\sigma\cosh kh}\sin(kx-\sigma t) \qquad (3.14)$$

By introducing the dispersion relationship, $\sigma^2=gk\tanh kh$, this equation can be rewritten as

$$\phi=-\frac{H}{2}c\frac{\cosh k(h+z)}{\sinh kh}\sin(kx-\sigma t) \qquad (3.15)$$

The water surface elevation (η), the horizontal velocity (u) and the vertical velocity (w) are given by

$$\eta(x,t)=\frac{H}{2}\cos(kx-\sigma t) \qquad (3.16)$$

$$u=-\frac{\partial \phi}{\partial x}=\frac{H}{2}\sigma\frac{\cosh k(h+z)}{\sinh kh}\cos(kx-\sigma t) \qquad (3.17)$$

$$w=-\frac{\partial \phi}{\partial z}=\frac{H}{2}\sigma\frac{\sinh k(h+z)}{\sinh kh}\sin(kx-\sigma t) \qquad (3.18)$$

3.4 Wave Transformation Across Shore

In coastal area, waves are brought from the offshore and are propagated over shallow water. Due to the effect of coastal bottom to water motion, waves are deformed according to the variation of the bottom topography. There are several types of wave deformations, which will be described as follows:

3.4.1 Wave energy

Wave energy consists of two parts, potential energy and kinetic energy. The potential energy arises because water below the mean water level is moved above the mean water level, so for linear wave the potential energy per unit area is given

$$E_P = \frac{1}{L}\int_0^L \frac{\rho g \eta^2}{2}\,dx = \frac{1}{16}\rho g H^2 \tag{3.19}$$

The kinetic energy for a linear wave is

$$E_K = \frac{1}{2}\rho g \frac{1}{L}\int_0^L \left[\int_0^{h+\eta} \left(u^2 + w^2\right) dz\right] dx = \frac{1}{16}\rho g H^2 \tag{3.20}$$

The energy is transported in the direction of wave propagation. The total average energy per unit surface area or the energy density is

$$E = E_P + E_K = \frac{1}{8}\rho g H^2 \tag{3.21}$$

and the total energy per unit width is

$$E_L = \frac{1}{8}\rho g H^2 L \tag{3.22}$$

3.4.2 Energy flux

Linear waves do not transmit mass as they propagate across a fluid but they do transmit energy. The rate at which the energy is transferred called the energy flux, F_E, which is the product of energy and velocity.

$$F_E = \int_{-h}^{\eta}\left(p + \frac{\rho}{2}(u^2 + w^2) + \rho g z\right) u\,dz \tag{3.23}$$

The average of energy flux over a wave period can be expressed as

$$\overline{F}_E = \frac{1}{T}\int_t^{t+T}\int_{-h}^{\eta}\left(p + \frac{\rho}{2}(u^2 + w^2) + \rho g z\right) u\,dz\,dt \tag{3.24}$$

Since $\frac{\rho}{2}(u^2 + w^2)u$ is small compare with others, then the average energy flux over a wave period can be expressed as

$$\overline{F}_E = \frac{1}{T}\int_{t}^{t+T}\int_{-h}^{\eta}(p + \rho g z)u\,dz\,dt \tag{3.25}$$

$$\overline{F}_E = \left(\frac{1}{8}\rho g H^2\right)\frac{\sigma}{k}\left[\frac{1}{2}\left(1 + \frac{2kh}{\sinh 2kh}\right)\right] \tag{3.26}$$

$$\overline{F}_E = Ecn = Ec_g \tag{3.27}$$

where $n = \frac{1}{2}\left(1 + \frac{2kh}{\sinh 2kh}\right)$ and $c_g = cn$ is group velocity.

3.4.3 Wave shoaling

As waves approach the shallow water region, waves suffer the effect of coastal bottom. The change in wave height due to varying depth is called wave shoaling and usually calculated by the energy flux conservation as

$$(Ec_g)_1 = (Ec_g)_2 \tag{3.28}$$

or the relation can be written as

$$\frac{\partial Ec_g}{\partial x} = 0 \tag{3.29}$$

where $E = \frac{1}{8}\rho g H^2$ is the wave energy density

$c_g = cn =$ group velocity

$n = \frac{1}{2}\left(1 + \frac{2kh}{\sinh 2kh}\right)$

The cause of variation in wave height is the variation in the speed of energy propagation with water depth.

3.4.4 Breaking waves

The Eq. (3.29) indicates that the wave height will increase and approach infinity in very shallow water, which clearly is unrealistic. At some depth, a wave will become unstable and break, dissipating energy in the form of turbulence and heat. Due to the strong energy dissipation, the wave height decreases toward the shore in the surf zone. The change in wave height inside the surf zone is usually calculated by using energy flux concept, which is expressed as

$$\frac{\partial Ec_g}{\partial x} = -D_B \tag{3.30}$$

where D_B is the energy dissipation rate which is zero outside the surf zone.

The main difficulty of energy flux conservation approach is how to compute the energy dissipation rate, D_B, inside the surf zone. Since the phenomenon of wave breaking is very complicated, the formula for computing D_B depends on empirical relation. Widely used formulas have been proposed for computing energy dissipation rate. They can be classified into several concepts but the most widely used are bore concept and stable energy concept, which already presented in the literature review.

3.4.5 Breaking location

When wave propagates to the nearshore zone, wave profile is steepen and eventually break and it will induce strong turbulence. At the present stage, the knowledge of breaking wave is not enough to describe the detail of breaking process. Therefore, empirical relation must be used to predict the breaking location and wave transformation after breaking.

Consider wide range of data, Goda (1970) proposed an empirical breaking index diagram of breaking wave height diagram and breaking depth diagram. However, if breaking wave height diagram is used together with linear wave theory, the predicted breaking point, in some cases, will shift on shore-ward of the real one. In those cases, linear wave theory gives under estimation of wave height just before the breaking point. To avoid this problem, Watanabe et al. (1984) used linear wave theory to convert breaking depth diagram of Goda (1970) to be the diagram of particle velocity-celerity ratio ($\hat{u}/c$) and used it to determine the breaking point. For the convenience of numerical calculation, the diagram of Watanabe et al. (1984) was approximated by Isobe (1987) as

$$(\hat{u}/c)_b = 0.53 - 0.3\exp\left[-3\sqrt{\frac{h_b}{L_o}}\right] + 5m_b^{3/2}\exp\left[-45\left(\sqrt{\frac{h_b}{L_o}} - 0.1\right)^2\right] \tag{3.31}$$

where $\hat{u}$ is the amplitude of horizontal water particle velocity at the mean water level, c is the wave celerity, h is the water depth, L_o is the deep water wave length, m_b is the bottom slope and subscript b denote the quantity at breaking point. The variables $\hat{u}$ and c are calculated based on linear wave theory. From Goda breaking depth diagram, the Goda breaking index can be used to compute the location of wave breaking, which can be expressed as

$$H_b = \frac{L_o}{\pi \coth^2(k_b h_b)}\left(0.53 - 0.3\exp\left[-3\sqrt{\frac{h_b}{L_o}}\right] + 5m_b^{3/2}\exp\left[-45\left(\sqrt{\frac{h_b}{L_o}} - 0.1\right)^2\right]\right) \tag{3.32}$$

where k_b is the wave number at breaking point and m_b is the bottom slope.

3.5 Mean Water Level

The height of the mean water level ($\overline{\eta}$) can be determined from the mean momentum balance. Following the derivation of Longuet-Higgins and Stewart (1963) which is written as

$$\frac{d\overline{\eta}}{dx} = -\frac{1}{\rho g h}\frac{dS_{xx}}{dx} \tag{3.33}$$

where $S_{xx} = (\frac{1}{2} + \frac{2kh}{\sinh 2kh})(\frac{1}{8}\rho g H^2)$. Then the mean water depth can be calculated by

$$h = h_o + \overline{\eta} \tag{3.34}$$

Since S_{xx} increases as the wave moves from deep water to shallow water, the mean water level ($\overline{\eta}$) decreases so called set-down. After breaking, energy is dissipated and the wave height decreases therefore S_{xx} decreases and $\overline{\eta}$ increases with a resultant set-up of the water level.