

CHAPTER V

Mathematical Model

A simple mathematical model for calculating the performance of the experimental DAR is presented in this chapter. The model is intended for use as a primary tool for analysis of the system. It is used to determine the maximum performance for given operating conditions. When comparing the calculated results with actual values, it shows how losses occurred in different devices. It helps in modifications of the system. To calculate the model, some operating conditions are required. They can be obtained from the experimental results. This chapter provides concepts of the calculation model and calculation procedures.

Figure 5.1 shows a schematic diagram of the experimental refrigerator used in the analysis. Knowing that the DAR is a complex system, which requires specific information in various areas. It is complicated for studying in all details. To simplify the calculation model, some assumptions must be included. They are listed as the followings:

- Temperatures and pressure of working fluid were obtained, based on actual values, from the experimental refrigerator.
- The system was operated in a steady state at the specified working conditions.
- There was no heat loss, no condensation of the vaporized working solution while it was flowing through the pump tube. Therefore, all vaporized liquid would cause a pumping effect in the bubble pump.
- The entire system had an equalized pressure. Pressure drops occurring in tubes, heat exchanger, and system components were negligible.
- The internal heat load as well as mass transfer due to the auxiliary gas was not considered.

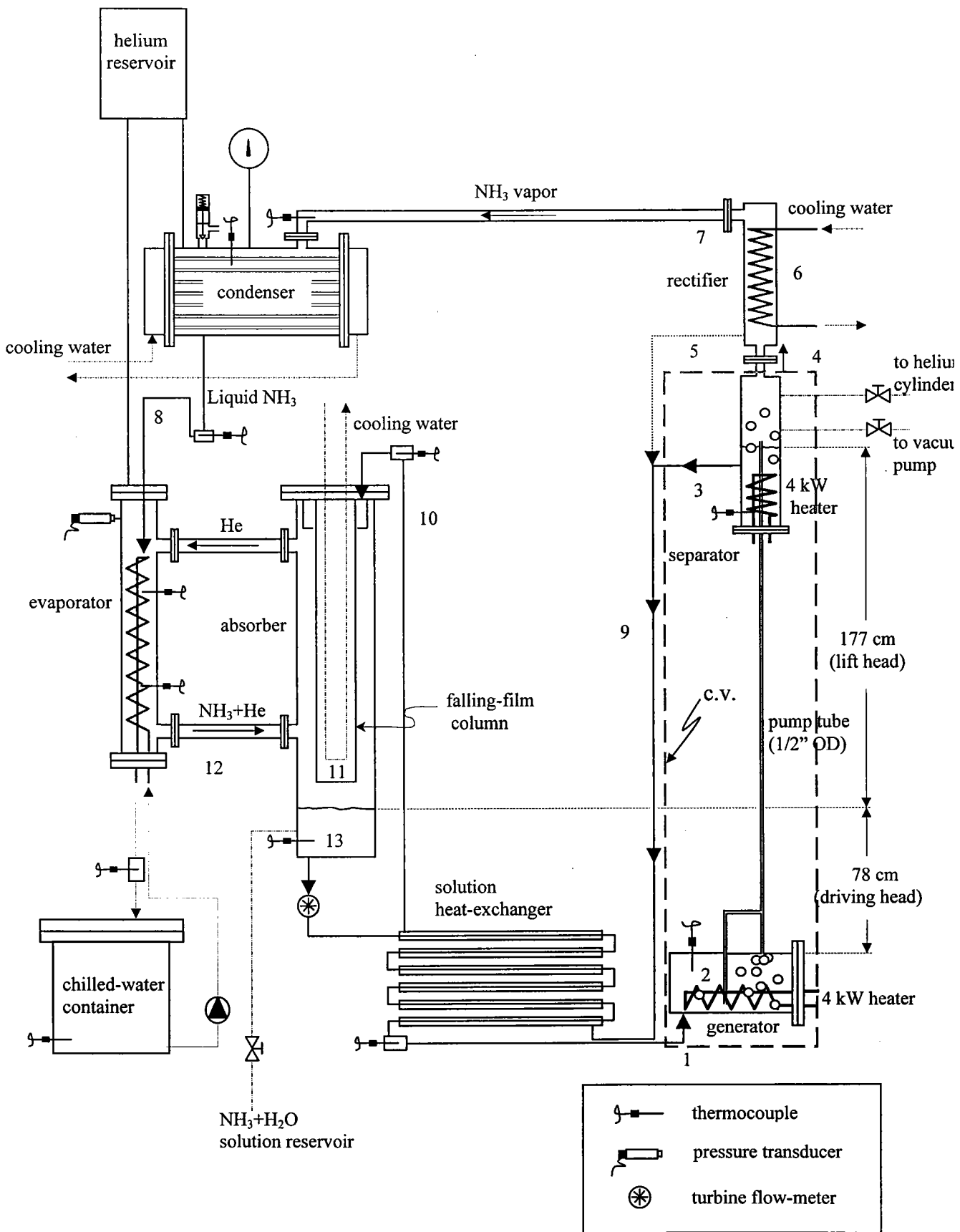


Figure 5.1 Schematic view of the experimental refrigerator.

- For each system component, energy and mass transfer were calculated based on the first law of thermodynamics.
- Based on the assumptions used, the model would provide maximum COP and cooling capacity for given operating conditions.
- The bubble-pump characteristic was obtained as a curve-fitted equation based on the experiment described in chapter 4.
- Properties of aqueous ammonia solution were obtained from correlations provided by Patex and Klomfar [1995] and ASHRAE [1993].

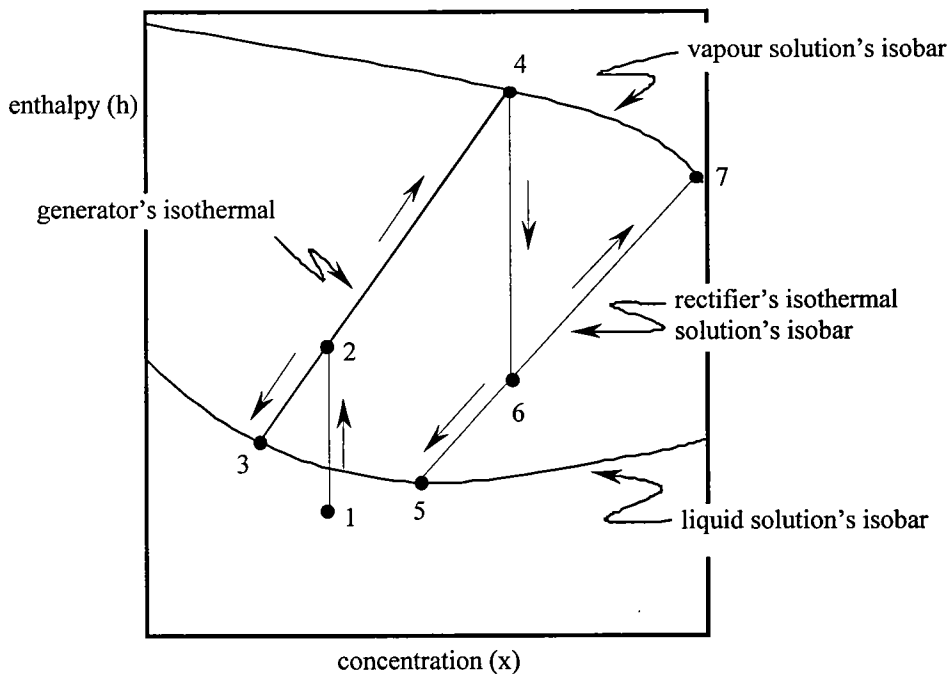


Figure 5.2 P-T-X-h diagram of aqueous ammonia solution for the processes related to the model.

5.1 Generator, Bubble pump, and Separator

According to Figures 5.1 and 5.2, heat input at the generator, \dot{Q}_{in} is used for two purposes. Firstly, it is used for vaporization of weak solution. The solution entering the generator is heated up and vaporized to form bubbles. They agglomerate to become a

larger gas volume pushing columns of liquid above itself in the pump tube. A slug flow occurring in the pump tube is known as a bubble pump. The saturated liquid solution is lifted up to the separator by the vaporized solution flowing upward due to density difference between liquid and vapor. A correlation of vaporized solution and liquid solution flow rate was obtained from the experiment of a simple bubble pump set-up as described in chapter 4. To determine the vaporized solution and the liquid solution flow rate and amount of heat input, an experimental correlation of a bubble pump, and the working solution properties must be considered simultaneously.

Refer to Figure 5.1, an energy balance is applied to a control volume, containing generator, pump tube, and separator,

$$\dot{Q}_{in} = \dot{m}_1 h_1 - \dot{m}_3 h_3 - \dot{m}_4 h_4 \quad (5.1)$$

Then, a mass continuity relation is applied to the total mass,

$$\dot{m}_1 = \dot{m}_3 + \dot{m}_4 \quad (5.2)$$

The experimental correlation of a simple bubble pump which was obtained from chapter 4,

$$\dot{V}_3 = -0.00014\dot{V}_4^4 + 0.00625\dot{V}_4^3 - 0.09706\dot{V}_4^2 + 0.63772\dot{V}_4 - 0.46802 \quad (5.3)$$

As the bubble pump correlation is presented in terms of volumetric flow rate, mass flow rate of both vaporized and liquid solution must be transformed from the volumetric flow rate.

$$\dot{m}_3 = \frac{\dot{V}_3}{v_3} \quad (5.4)$$

$$\dot{m}_4 = \frac{\dot{V}_4}{v_4} \quad (5.5)$$

Specific volume of the liquid solution, v_3 , and the vapor part, v_4 , can be calculated from [Threlkeld, 1970]:

$$v_3 = (1 - X_3) v_{\text{water-liq}} + 0.85 X_3 v_{\text{amm-liq}} \quad (5.6)$$

$$v_4 = (1 - X_4) v_{\text{water-vap}} + X_4 v_{\text{amm-vap}} \quad (5.7)$$

In the calculation procedure, equation (5.1) to (5.7) are simultaneously solved. To determine mass flow rate of fluid leaving and entering the separator and the generator, the following are required:

1. Concentration of the liquid solution at generator inlet, 1, must be initially assumed. Then, the solution enthalpy (1) needed for calculation in equation 5.1 can be calculated from the correlations [Patek and Klomfar 1998].
2. It is reasonable to assume that liquid and vapor leaving the separator are in equilibrium at their temperatures and pressures. Then their concentration, enthalpy and specific volume could be determined.
3. Mass flows (\dot{m}_1 , \dot{m}_3 and \dot{m}_4) can be determined by solving equations 5.1, 5.2 and 5.3 simultaneously.
4. Concentration of weak solution at the generator inlet (1) must be verified. Conservation of mass of ammonia is recalled for the verification,

$$\dot{m}_1 X_1 = \dot{m}_3 X_3 + \dot{m}_4 X_4 \quad (5.8)$$

Unless it is valid, it is used as a corrected concentration for iteration of the enthalpy used in equation 5.1. All calculation steps are repeated until the obtained value is valid by equation 5.8. Then, the obtained results could be used for further calculations.

These calculation steps are shown as a flow chart in Figure 5.3.

5.2 Rectifier

Vaporized solution is separated from the liquid solution in the separator. The vapor flows upward to be purified in the rectifier while the liquid flows downward through the solution heat exchanger (SHX) to the absorber.

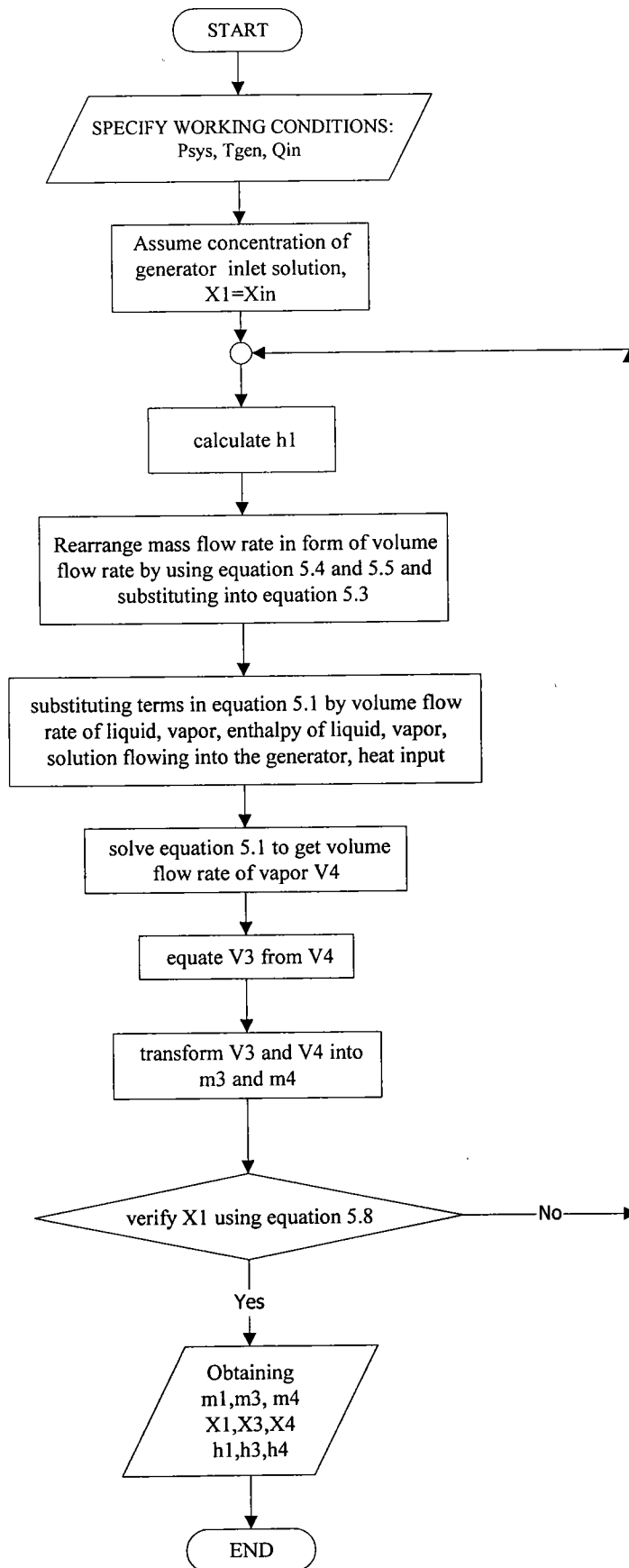


Figure 5.3 A calculation flow chart of the mathematical model.

Water vapor in the vaporized solution is partially condensed in the rectifier. The rectification process is graphically explained as shown in Figure 5.2.

Saturated vapor (4), flowing upward from the separator, is cooled down as two-phase mixture (6) in the rectifier. Condensate obtained from partial condensation of water vapor in the rectifier (5) flows back into the separator. The rectified vapor (7) enters the condenser. Mass flow rate of the purified vapor can be determined from,

$$\dot{m}_7 = \frac{(X_6 - X_5)}{(X_7 - X_5)} \dot{m}_4 \quad (5.9)$$

This amount of ammonia vapor with concentration of X_7 is liquefied in the condenser. There is an amount of heat rejected by partial condensation of water vapor in the rectification process. It is determined from,

$$\dot{Q}_{\text{rec}} = \dot{m}_7 h_7 + \dot{m}_5 h_5 - \dot{m}_4 h_4 \quad (5.10)$$

Condensate (5) and liquid solution (3) are aggregated as the strong solution (9). It flows via the SHX to the absorber. Therefore, mass flow rate of the strong solution can be determined from continuity of mass,

$$\dot{m}_9 = \dot{m}_3 + \dot{m}_5 \quad (5.11),$$

with concentration of,

$$X_9 = \frac{\dot{m}_3 X_3 + \dot{m}_5 X_5}{\dot{m}_9} \quad (5.12)$$

5.3 Condenser

The rectified ammonia vapor from the rectifier (7) enters the condenser. It is liquefied by rejecting heat to the cooling water. The amount of rejected heat is calculated from,

$$\dot{Q}_{\text{con}} = \dot{m}_7 h_7 - \dot{m}_8 h_8 \quad (5.13)$$

5.4 Evaporator and absorber

Liquid ammonia from the condenser (8) enters the evaporator to cause refrigerating effect. Liquid ammonia is evaporated under low partial pressure in an atmosphere of an auxiliary gas, helium. The evaporated ammonia vapor is absorbed by the strong solution. It is initially assumed that liquid ammonia could be totally evaporated and absorbed. Then, the obtained refrigerating capacity could be determined from

$$\dot{Q}_{\text{evap}} = \dot{m}_8 X_8 (h_{12\text{-vap}} - h_8) \quad (5.14)$$

The state 12-vap is considered as the evaporated ammonia after producing refrigerating effect. It is considered that only ammonia is evaporated. Therefore, the amount of water in the evaporator remains constant as that flowing from the condenser. It could be calculated as,

$$\dot{m}_{12\text{-liq}} = \dot{m}_8 (1 - X_8) \quad (5.15)$$

At state 12-liquid, the liquid solution is considered as pure water since all ammonia is already evaporated. Therefore, the obtained refrigerating capacity calculated from equation (5.14) can be considered as the maximum.

After absorption process, the strong solution (10) becomes weak solution (11). Concentration difference between the weak solution and the strong solution was considered as a result of absorption process in the absorber. Concentration of the weak solution is calculated from,

$$X_{11} = \frac{\dot{m}_{10} X_{10} + \dot{m}_8 X_8}{\dot{m}_{10} + \dot{m}_8 X_8} \quad (5.16),$$

where $\dot{m}_9 = \dot{m}_{10}$ and $X_9 = X_{10}$.

Mass of solution after absorption process (11) is the summation of the strong solution mass and the mass of evaporated ammonia.

$$\dot{m}_{11} = \dot{m}_{10} + \dot{m}_8 X_8 \quad (5.17)$$

In the evaporator of a DAR, evaporation of ammonia occurs in the atmosphere of an auxiliary gas. The process is similar to the evaporation of water in a tray exposed to the open-air. The evaporation rate is not dependent only on the ammonia mass flow rate. It requires surface area for the evaporation process as well. The evaporated ammonia vapor causes increments in partial pressure of ammonia in the evaporator, which causes increments in evaporation temperature. To maintain the low evaporation temperature, all evaporated ammonia vapor must be absorbed by the strong solution in the absorber.

5.5 Combined evaporator-absorber effectiveness

With greater heat input, pumping effect should be increased as the relationship of the bubble pump as shown in the experimental correlation. Then, the solution circulation rate should be increased. However, absorption capability of the DAR might be restricted due to performance limitation of the evaporator and the absorber. Therefore, absorbability of the strong solution had to be included into the calculation.

The strong solution entering the absorber can absorb evaporated ammonia only once in a circulation cycle. The ammonia vapor is absorbed while the strong solution flowing downward as falling film over the absorption surface area. No matter what the concentration of the weak solution is, absorption process will stop when the solution reaches the bottom of the absorber (13). Moreover, if the period of solution flow on the absorption surface area was short i.e. fast flow, it might not enough to have time for the strong solution to absorb all of the evaporated ammonia. Then, ammonia vapor will be left in the evaporator causing partial pressure of ammonia to increase. Then, the refrigeration temperature is raised resulting in less refrigerating effect due to small temperature difference of evaporated ammonia and chilled water in the evaporator.

An index named a combined evaporator-absorber effectiveness (ϵ) was introduced. Normally, the effectiveness is defined as a ratio of the actual concentration difference to that of the maximum available. The maximum concentration is normally referred as that when weak solution is in equilibrium with temperature and partial pressure in the absorber. However, in this study, the weak solution is not saturated after finishing the absorption process. Then, the combined evaporator-absorber effectiveness definition is altered as the ratio of evaporated ammonia mass rate to the total ammonia available from the condenser.

$$\epsilon = \frac{\dot{m}_{11}X_{11} - \dot{m}_{10}X_{10}}{\dot{m}_8X_8} \quad (5.18)$$

Mass of evaporated ammonia is calculated as,

$$\dot{m}_{12\text{-vap}} = \dot{m}_{11}X_{11} - \dot{m}_{10}X_{10} \quad (5.19)$$

The refrigerating capacity when including effectiveness into consideration can be determined as,

$$\dot{Q}_{\text{evap}} = \dot{m}_{12\text{-vap}}(h_{12\text{-vap}} - h_8) \quad (5.20)$$

Liquid refrigerant left at bottom of the evaporator flows back to bottom of the absorber having mass flow rate and concentration of,

$$\dot{m}_{12\text{-liq}} = \dot{m}_8 - \dot{m}_{12\text{-vap}} \quad (5.21)$$

$$X_{12\text{-liq}} = \frac{\dot{m}_8X_8 - (\dot{m}_{11}X_{11} - \dot{m}_{10}X_{10})}{\dot{m}_{12\text{-liq}}} \quad (5.22)$$

The calculated results must be verified by employing mass balance of both ammonia and water in the solution as,

$$X_{13} = \frac{\dot{m}_{11}X_{11} + \dot{m}_{12\text{-liq}}X_{12\text{-liq}}}{\dot{m}_{11} + \dot{m}_{12\text{-liq}}} \quad (5.23)$$

This figure, X_{13} , must correspond to the concentration of compressed liquid solution entering the generator (X_1). Heat released due to absorption process from the absorber could be calculated from

$$\dot{Q}_{\text{abs}} = \dot{m}_{10}h_{10} + \dot{m}_{12\text{-vap}}h_{12\text{-vap}} + \dot{m}_{12\text{-liq}}h_{12\text{-liq}} - \dot{m}_{13}h_{13} \quad (5.24)$$

In the absorber, the absorption rate depends on the solution mass flow rate, concentration of strong solution, surface area of absorption, period of absorption etc. Mass flow rate of the strong solution is affected by the bubble pump performance. Thus, various factors are found to affect the evaporation rate in the evaporator. It seems to be that ammonia cannot completely evaporate in the evaporator. The liquid ammonia that is not evaporated will return to the absorber without producing any cooling effect.

5.6 Solution heat exchanger (SHX)

The weak solution flowing out from the absorber into the solution heat exchanger, SHX, will be preheated. It is assumed that there is no heat loss to the surroundings from the SHX. Therefore, the amount of heat transfer internally of the SHX could be balanced as,

$$\dot{m}_{13}(h_1 - h_{13}) = \dot{m}_9(h_9 - h_{10}) \quad (5.25)$$

5.7 Coefficient of Performance

Since DAR is a refrigeration system that uses only heat as input power to drive the system, the coefficient of performance (COP) of the DAR is defined as,

$$\text{COP} = \frac{\dot{Q}_{\text{evap}}}{\dot{Q}_{\text{in}}} \quad (5.26)$$

5.8 Conclusion

A mathematical model was developed to analyze thermodynamic performance of the experimental DAR. The model was developed in order to be used for calculation of the maximum performance under given operating conditions. When calculated results are compared with actual values obtained from the experimental refrigerator, it will show that how losses at different devices occurred and how the system can be modified to improve its performance.