Appendix B

Correlation Coefficient, Covariance, and Variance for SelDLL

In this appendix, expressions for the correlation coefficient $\rho_{m,k}$, covariance $Cov(y_m, y_k)$ between the m^{th} and k^{th} branches, and the variance σ_m^2 of the m^{th} branch for the SelDLL scheme, are derived.

For $\rho_{m,k}$, it is defined as

$$\rho_{m,k} = \frac{Cov(y_m, y_k)}{\sqrt{\sigma_m^2 \cdot \sigma_k^2}} \tag{B.1}$$

and $Cov(y_m, y_k)$ is

$$Cov(y_m, y_k) = E\{y_m y_k\} - E\{y_m\}E\{y_k\}. \tag{B.2}$$

where $E\{\cdot\}$ is the expectation operator and $E\{y_m\}$ is mean of the y_m which is equal to $\sqrt{2P}T_c s_m$. Substituting (3.5) in (B.2) yields

$$\begin{aligned} Cov(y_m, y_k) = {} & E\left\{\left(\sqrt{2P}T_c s_m\right)\left(\sqrt{2P}T_c s_k\right)\right\} + E\left\{\sqrt{2P}T_c s_m \eta_k\right\} + E\left\{\sqrt{2P}T_c s_k \eta_m\right\} \\ & + E\{\eta_m \eta_k\} - \left(\sqrt{2P}T_c s_m\right)\left(\sqrt{2P}T_c s_k\right), \end{aligned} \tag{B.3}$$

where η_m is defined in (3.8). The first and last terms in (B.3) cancel each other, while the second and third terms are zero since the mean of the noise is zero. Therefore, (B.3) is reduced to

$$Cov(y_m, y_k) = E\{\eta_m \eta_k\}, \tag{B.4}$$

.The noise can be written as

$$\begin{aligned} \eta_m &= \sum_{i=q-n}^{q-1} \int_{iT_c}^{(i+1)T_c} \mathrm{Re}\{\eta(t)\} \left[\sum_{j=-\infty}^{\infty} c_j P_{T_c}(t - mT_c - jT_c)\right] dt \\ &= \sum_{i=q-n}^{q-1} c_{i-m} \int_{iT_c}^{(i+1)T_c} \mathrm{Re}\{\eta(t)\} dt, \end{aligned} \tag{B.5}$$

Substituting (B.5) into (B.4), we have

$$\begin{aligned} Cov(y_m, y_k) &= E\left\{\left[\sum_{i=q-n}^{q-1} c_{i-m} \int_{iT_c}^{(i+1)T_c} \mathrm{Re}\{\eta(t)\} dt\right]\left[\sum_{j=q-n}^{q-1} c_{j-k} \int_{jT_c}^{(j+1)T_c} \mathrm{Re}\{\eta(\lambda)\} d\lambda\right]\right\} \\ &= \sum_{i=q-n}^{q-1} \sum_{j=q-n}^{q-1} c_{i-m} c_{j-k} \int_{iT_c}^{(i+1)T_c} \int_{jT_c}^{(j+1)T_c} R_{\eta_R}(t - \lambda) d\lambda dt \end{aligned} \tag{B.6}$$

where $R_{\eta_R}(t)$ is the autocorrelation function of $\mathrm{Re}\{\eta(t)\}$ and its value is $N_0\delta(t)$, where $\delta(t)$ is Kronecker delta function. The integration intervals of the variables t and λ

overlap if and only if the values of i and j are equal. Consequently, $Cov(y_m, y_k)$ can be expressed as

$$Cov(y_m, y_k) = N_0 T_c \sum_{i=q-n}^{q-1} c_{i-m} c_{i-k} \,. \tag{B.7}$$

The variance of y_m, which is equal to the variance of η_m, is obtained by replacing the variable k with m in (B.7), yields

$$\sigma_m^2 = N_0 T_c \sum_{i=q-n}^{q-1} \left(c_{i-m}\right)^2 = nN_0 T_c \,. \tag{B.8}$$

Finally, substituting (B.7) and (B.8) into (B.1), we obtain an expression for $\rho_{m,k}$.