Appendix C

The Approximate Correlation Coefficient for SelDLL

In this appendix, we obtain an approximate expression, as shown in (3.11) for the correlation coefficient $\tilde{\rho}_{m,k}$ between y_m and y_k for the SelDLL scheme.

We can write (B.7) as

$$Cov(y_m, y_k) = N_0 T_c \sum_{i=q-n}^{q-n'-1} c_{i-m} c_{i-k} + N_0 T_c \sum_{i=q-n'}^{q-1} c_{i-m} c_{i-k}, \tag{C.1}$$

where $n' = n - LN$, $L = \lfloor n/N \rfloor$, and $\lfloor x \rfloor$ represents the integer part of x. The first term is the summation over LN chips, so it is L times the periodic autocorrelation at a phase difference of $m-k$. Since $m \neq k$, such first term is reduced to $-LN_0T_c$. Based on 99.9% confidence interval [68], the second term can be approximated as

$$N_0 T_c \sum_{i=q-n'}^{q-1} c_{i-m} c_{i-k} \approx \left\{ 3\left[\sqrt{\left(1 - \frac{n'}{N}\right) n'} \right] - \frac{n'}{N} \right\} N_0 T_c . \tag{C.2}$$

Therefore, $Cov(y_m, y_k)$ can be approximated by

$$Cov(y_m, y_k) \approx -LN_0 T_c + \left\{ 3\left[\sqrt{\left(1 - \frac{n'}{N}\right) n'} \right] - \frac{n'}{N} \right\} N_0 T_c . \tag{C.3}$$

Finally, from (B.1), (B.8), and (C.3), the approximate correlation coefficient $\tilde{\rho}_{m,k}$ is

$$\tilde{\rho}_{m,k} = \frac{-L + 3\left[\sqrt{\left(1 - \frac{n'}{N}\right) n'} \right] - \frac{n'}{N}}{n}, \quad \text{for } m \neq k . \tag{C.4}$$