Appendix D

The Probability of Correct Selection of SelDLL

In this appendix, the probability of correct selection of the SelDLL scheme with given H_β, which is defined in (3.14), is explained and an expression is obtained.

The correct selection is defined as the event that the correlated outputs y_β and $y_{\beta+1}$ are larger than others if H_β is true. Equivalently, the values of y_j, y_k, and y_l are not more than $\min(y_\beta, y_{\beta+1})$, the minimum value of y_β and $y_{\beta+1}$. The probability of this event can be expressed as

$$\Pr\left(Y_\beta \,\&\, Y_{\beta+1} > Y_j, Y_k, Y_l \middle| \beta, \gamma, q, n, N\right) =$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\min(y_\beta, y_{\beta+1})}\int_{-\infty}^{\min(y_\beta, y_{\beta+1})}\int_{-\infty}^{\min(y_\beta, y_{\beta+1})} f\left(y_\beta, y_{\beta+1}, y_j, y_k, y_l \middle| \beta, \gamma, q, n, N\right) dy_l dy_k dy_j dy_{\beta+1} dy_\beta \text{ .(D.1)}$$

Eq. (D.1) can be simplified to the sum of two terms corresponding to the regions $Y_\beta > Y_{\beta+1} > Y_j, Y_k, Y_l$ and $Y_{\beta+1} > Y_\beta > Y_j, Y_k, Y_l$, respectively. We can express this in two ways, depending on whether we integrate with respect to y_β first or $y_{\beta+1}$ first. They are

$$\Pr\left(Y_\beta \,\&\, Y_{\beta+1} > Y_j, Y_k, Y_l \middle| \beta, \gamma, q, n, N\right) =$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{y_\beta}\int_{-\infty}^{y_{\beta+1}}\int_{-\infty}^{y_{\beta+1}}\int_{-\infty}^{y_{\beta+1}} f\left(y_\beta, y_{\beta+1}, y_j, y_k, y_l \middle| \beta, \gamma, q, n, N\right) dy_l dy_k dy_j dy_{\beta+1} dy_\beta$$

$$+\int_{-\infty}^{\infty}\int_{y_\beta}^{\infty}\int_{-\infty}^{y_\beta}\int_{-\infty}^{y_\beta}\int_{-\infty}^{y_\beta} f\left(y_\beta, y_{\beta+1}, y_j, y_k, y_l \middle| \beta, \gamma, q, n, N\right) dy_l dy_k dy_j dy_{\beta+1} dy_\beta \,. \qquad \text{(D.2)}$$

and

$$\Pr\left(Y_\beta \,\&\, Y_{\beta+1} > Y_j, Y_k, Y_l \middle| \beta, \gamma, q, n, N\right) =$$

$$\int_{-\infty}^{\infty}\int_{y_{\beta+1}}^{\infty}\int_{-\infty}^{y_{\beta+1}}\int_{-\infty}^{y_{\beta+1}}\int_{-\infty}^{y_{\beta+1}} f\left(y_\beta, y_{\beta+1}, y_j, y_k, y_l \middle| \beta, \gamma, q, n, N\right) dy_l dy_k dy_j dy_\beta dy_{\beta+1}$$

$$+\int_{-\infty}^{\infty}\int_{-\infty}^{y_{\beta+1}}\int_{-\infty}^{y_\beta}\int_{-\infty}^{y_\beta}\int_{-\infty}^{y_\beta} f\left(y_\beta, y_{\beta+1}, y_j, y_k, y_l \middle| \beta, \gamma, q, n, N\right) dy_l dy_k dy_j dy_\beta dy_{\beta+1} \,. \qquad \text{(D.3)}$$

Next, we obtain expression for this probability given H_0, H_1, H_2, and H_3.

D.1) Given H_0

Given H_0, the correct selection is obtained if and only if the correlation outputs of branch 0 and 1 are the two largest values. Therefore, we substitute (3.10) and (3.13) into (D.2) (or, equivalently, (D.3)). After some mathematical manipulation, the probability of correct selection given H_0 can be expressed as

$$\Pr\left(Y_0 \,\&\, Y_1 > Y_2, Y_3, Y_4 \middle| 0, \gamma, q, n, N\right) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right) \int_{-\infty}^{z+\sqrt{\frac{\mathrm{SNR}}{n}}\left(s_{0|0}-s_{1|0}\right)} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)$$

$$\times \Phi\left(x + \sqrt{\frac{\mathrm{SNR}}{n}}\left(s_{1|0} - s_{2|0}\right)\right) \Phi\left(x + \sqrt{\frac{\mathrm{SNR}}{n}}\left(s_{1|0} - s_{3|0}\right)\right) \Phi\left(x + \sqrt{\frac{\mathrm{SNR}}{n}}\left(s_{1|0} - s_{4|0}\right)\right) dxdz$$

$$+ \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{w^2}{2}\right) \left[1 - \Phi\left(w + \sqrt{\frac{\mathrm{SNR}}{n}}\left(s_{0|0} - s_{1|0}\right)\right)\right] \Phi\left(w + \sqrt{\frac{\mathrm{SNR}}{n}}\left(s_{0|0} - s_{2|0}\right)\right)$$

$$\times \Phi\left(w + \sqrt{\frac{\mathrm{SNR}}{n}}\left(s_{0|0} - s_{3|0}\right)\right) \Phi\left(w + \sqrt{\frac{\mathrm{SNR}}{n}}\left(s_{0|0} - s_{4|0}\right)\right) dw, \quad \text{(D.4)}$$

where SNR$=2PT_c/N_0$, which is per-chip signal-to-noise ratio, $\Phi(t)$ is the cumulative distribution function (cdf) of the standard Gaussian random variable, $\Phi(t)= \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx$, and $s_{m|\beta}$ represents the signal term defined in (3.7).

D.2) Given H_1

Given H_1, the correct selection is obtained if and only if the correlation outputs of branch 1 and 2 are the two largest values. Therefore, we substitute (3.10) and (3.13) into (D.2). After some mathematical manipulation, the probability of correct selection given H_1 can be expressed as

$$\Pr\left(Y_1 \,\&\, Y_2 > Y_0, Y_3, Y_4 \middle| 1, \gamma, q, n, N\right) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right) \int_{-\infty}^{z+\sqrt{\frac{\mathrm{SNR}}{n}}\left(s_{1|1}-s_{2|1}\right)} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)$$

$$\times \Phi\left(x + \sqrt{\frac{\mathrm{SNR}}{n}}\left(s_{2|1} - s_{0|1}\right)\right) \Phi\left(x + \sqrt{\frac{\mathrm{SNR}}{n}}\left(s_{2|1} - s_{3|1}\right)\right) \Phi\left(x + \sqrt{\frac{\mathrm{SNR}}{n}}\left(s_{2|1} - s_{4|1}\right)\right) dxdz$$

$$+ \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{w^2}{2}\right) \left[1 - \Phi\left(w + \sqrt{\frac{\mathrm{SNR}}{n}}\left(s_{1|1} - s_{2|1}\right)\right)\right] \Phi\left(w + \sqrt{\frac{\mathrm{SNR}}{n}}\left(s_{1|1} - s_{0|1}\right)\right)$$

$$\times \Phi\left(w + \sqrt{\frac{\mathrm{SNR}}{n}}\left(s_{1|1} - s_{3|1}\right)\right) \Phi\left(w + \sqrt{\frac{\mathrm{SNR}}{n}}\left(s_{1|1} - s_{4|1}\right)\right) dw. \quad \text{(D.5)}$$

D.3) Given H_2

Given H_2, the correct selection is obtained if and only if the correlation outputs of branch 2 and 3 are the two largest values. Therefore, we substitute (3.10) and (3.13) into (D.2). After some mathematical manipulation, the probability of correct selection given H_2 can be expressed as

$$\Pr\left(Y_2 \,\&\, Y_3 > Y_0, Y_1, Y_4 \middle| 2, \gamma, q, n, N\right) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right) \int_{-\infty}^{z+\sqrt{\frac{\mathrm{SNR}}{n}}\left(s_{2|2}-s_{3|2}\right)} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)$$

$$\times \Phi\left(x + \sqrt{\frac{\mathrm{SNR}}{n}}\left(s_{3|2} - s_{0|2}\right)\right) \Phi\left(x + \sqrt{\frac{\mathrm{SNR}}{n}}\left(s_{3|2} - s_{1|2}\right)\right) \Phi\left(x + \sqrt{\frac{\mathrm{SNR}}{n}}\left(s_{3|2} - s_{4|2}\right)\right) dxdz$$

$$+\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{w^2}{2}\right)\left[1-\Phi\left(w+\sqrt{\frac{\text{SNR}}{n}}\left(s_{2|2}-s_{3|2}\right)\right)\right]\Phi\left(w+\sqrt{\frac{\text{SNR}}{n}}\left(s_{2|2}-s_{0|2}\right)\right)$$

$$\times\Phi\left(w+\sqrt{\frac{\text{SNR}}{n}}\left(s_{2|2}-s_{1|2}\right)\right)\Phi\left(w+\sqrt{\frac{\text{SNR}}{n}}\left(s_{2|2}-s_{4|2}\right)\right)dw. \quad \text{(D.6)}$$

D.4) Given H_3

Given H_3, the correct selection is obtained if and only if the correlation outputs of branch 3 and 4 are the two largest values. Therefore, we substitute (3.10) and (3.13) into (D.2). After some mathematical manipulation, the probability of correct selection given H_3 can be expressed as

$$\Pr\left(Y_3\,\&\,Y_4>Y_0,Y_1,Y_2\middle|3,\gamma,q,n,N\right)=\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{z^2}{2}\right)\int_{-\infty}^{z+\sqrt{\frac{\text{SNR}}{n}}\left(s_{3|3}-s_{4|3}\right)}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{x^2}{2}\right)$$

$$\times\Phi\left(x+\sqrt{\frac{\text{SNR}}{n}}\left(s_{4|3}-s_{0|3}\right)\right)\Phi\left(x+\sqrt{\frac{\text{SNR}}{n}}\left(s_{4|3}-s_{1|3}\right)\right)\Phi\left(x+\sqrt{\frac{\text{SNR}}{n}}\left(s_{4|3}-s_{2|3}\right)\right)dxdz$$

$$+\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{w^2}{2}\right)\left[1-\Phi\left(w+\sqrt{\frac{\text{SNR}}{n}}\left(s_{3|3}-s_{4|3}\right)\right)\right]\Phi\left(w+\sqrt{\frac{\text{SNR}}{n}}\left(s_{3|3}-s_{0|3}\right)\right)$$

$$\times\Phi\left(w+\sqrt{\frac{\text{SNR}}{n}}\left(s_{3|3}-s_{1|3}\right)\right)\Phi\left(w+\sqrt{\frac{\text{SNR}}{n}}\left(s_{3|3}-s_{2|3}\right)\right)dw. \quad \text{(D.7)}$$