Appendix E

The Signal Term for PelDLL

In this appendix, an expression for the signal term s_m^{pel} for the PelDLL scheme, which is defined in (3.18), is obtained. This is derived as follows.

$$\begin{aligned} s_m^{pel} &= \frac{1}{T_c}\int_{(q-n)T_c}^{qT_c} c(t-\tau)c_m^{pel}(t)\,dt \\ &= \frac{1}{T_c}\int_{(q-n)T_c}^{qT_c} c(t-\beta T_c-\gamma T_c)\left[c(t-2mT_c-\delta T_c)-c(t-2m+\delta T_c)\right]dt \\ &= s_m^{pel,+}-s_m^{pel,-} \end{aligned} \tag{E.1}$$

where $s_m^{pel,+}$ is the chip normalized partial correlation between the received PN sequence and the delayed version of local PN sequence in the m^{th} branch given by

$$s_m^{pel,+} = \frac{1}{T_c}\int_{(q-n)T_c}^{qT_c} c(t-\beta T_c-\gamma T_c)c(t-2mT_c-\delta T_c)\,dt, \tag{E.2}$$

and $s_m^{pel,-}$ is the chip normalized partial correlation between the received PN sequence and the advanced version of local PN sequence in the m^{th} branch, given by

$$s_m^{pel,-} = \frac{1}{T_c}\int_{(q-n)T_c}^{qT_c} c(t-\beta T_c-\gamma T_c)c(t-2mT_c+\delta T_c)\,dt. \tag{E.3}$$

The signals $s_m^{pel,+}$ and $s_m^{pel,-}$ can be computed as follows.

E.1) Computing the signal $s_m^{pel,+}$. This term can be simplified as

$$\begin{aligned} s_m^{pel,+} &= \frac{1}{T_c}\sum_{i=q-n}^{q-1}\int_{iT_c}^{(i+1)T_c}\sum_{j=-\infty}^{\infty} c_j P_{T_c}(t-\beta T_c-\gamma T_c-jT_c)\sum_{k=-\infty}^{\infty} c_k P_{T_c}(t-2mT_c-\delta T_c-kT_c)\,dt \\ &= \frac{1}{T_c}\sum_{i=q-n}^{q-1} s_{m,i}^{pel,+} \end{aligned} \tag{E.4}$$

where $s_{m,i}^{pel,+}$ is the correlation over one chips duration. Figure E.1 shows the timing relations for the local and received PN code, with $\delta > \gamma$ and $\delta < \gamma$. As shown in the figure, the integration during one period can be broken down into three terms.

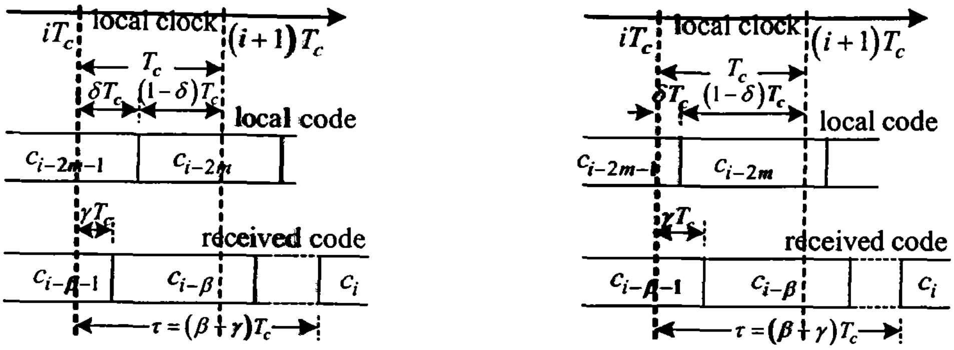
E.1.1) For $\delta > \gamma$. The signal $s_{m,i}^{pel,+}$ can be written as

$$s_{m,i}^{pel,+} = \gamma T_c c_{i-2m-1}c_{i-\beta-1} + (\delta-\gamma)T_c c_{i-2m-1}c_{i-\beta} + (1-\delta)T_c c_{i-2m}c_{i-\beta}. \tag{E.5}$$

E.1.2) For $\delta < \gamma$. The signal $s_{m,i}^{pel,+}$ can be written as

$$s_{m,i}^{pel,+} = \delta T_c c_{i-2m-1}c_{i-\beta-1} + (\gamma-\delta)T_c c_{i-2m}c_{i-\beta-1} + (1-\gamma)T_c c_{i-2m}c_{i-\beta}. \tag{E.6}$$

With some mathematical manipulation, we can write (E.5) and (E.6) as one expression:



(a) when $\delta > \gamma$ (b) when $\delta < \gamma$

Figure E.1. The relation between the delayed version of local code and the received signal at the m^{th} branch during $iT_c \le t \le (i+1)T_c$.

$$s_{m,i}^{pel,+} = \left(\frac{\delta+\gamma}{2} - \frac{|\delta-\gamma|}{2}\right) T_c c_{i-2m-1} c_{i-\beta-1} + \left(\frac{\delta-\gamma}{2} + \frac{|\delta-\gamma|}{2}\right) T_c c_{i-2m-1} c_{i-\beta}$$

$$+\left(\frac{\gamma-\delta}{2} + \frac{|\gamma-\delta|}{2}\right) T_c c_{i-2m} c_{i-\beta-1} + \left(\frac{2-\gamma-\delta}{2} - \frac{|\delta-\gamma|}{2}\right) T_c c_{i-2m} c_{i-\beta}. \quad \text{(E.7)}$$

E.2) Computing the signal $s_{m,i}^{pel,-}$. This term can be written as

$$s_m^{pel,-} = \frac{1}{T_c} \sum_{i=q-n}^{q-1} \int_{iT_c}^{(i+1)T_c} \sum_{j=-\infty}^{\infty} c_j P_{T_c}(t - \beta T_c - \gamma T_c - jT_c) \sum_{k=-\infty}^{\infty} c_k P_{T_c}(t - 2mT_c + \delta T_c - kT_c) dt$$

$$= \frac{1}{T_c} \sum_{i=q-n}^{q-1} s_{m,i}^{pel,-} \quad \text{(E.8)}$$

where $s_{m,i}^{pel,-}$ is the correlation over one chip duration. Figure E.2 shows the timing for the local and received PN codes with $\delta > \gamma$ and $\delta < \gamma$. Similarly, the integration can be broken into three subsections as follows.

E.2.1) For $\delta < \gamma$ or $(1-\delta) > \gamma$. The signal $s_{m,i}^{pel,-}$ can be written as

$$s_{m,i}^{pel,-} = \gamma T_c c_{i-2m} c_{i-\beta-1} + (1-\delta-\gamma) T_c c_{i-2m} c_{i-\beta} + \delta T_c c_{i-2m+1} c_{i-\beta}. \quad \text{(E.9)}$$

E.2.2) For $\delta > \gamma$ or $(1-\delta) < \gamma$. The signal $s_{m,i}^{pel,-}$ can be written as

$$s_{m,i}^{pel,-} = (1-\delta) T_c c_{i-2m} c_{i-\beta-1} + (\gamma+\delta-1) T_c c_{i-2m-1} c_{i-\beta-1} + (1-\gamma) T_c c_{i-2m+1} c_{i-\beta}. \quad \text{(E.10)}$$

Using some mathematical manipulation, a unified expression can be obtained:

$$s_{m,i}^{pel,-} = \left(\frac{(1-\delta)+\gamma}{2} - \frac{|(1-\delta)-\gamma|}{2}\right) T_c c_{i-2m} c_{i-\beta-1} + \left(\frac{(1-\delta)-\gamma}{2} + \frac{|(1-\delta)-\gamma|}{2}\right) T_c c_{i-2m} c_{i-\beta}$$

$$+\left(\frac{\gamma-(1-\delta)}{2} + \frac{|\gamma-(1-\delta)|}{2}\right) T_c c_{i-2m+1} c_{i-\beta-1} + \left(\frac{1+\delta-\gamma}{2} - \frac{|(1-\delta)-\gamma|}{2}\right) T_c c_{i-2m+1} c_{i-\beta}. \quad \text{(E.11)}$$

Finally, from (E.1), (E.4), (E.7), (E.8), and (E.11), a complete expression for the signal s_m^{pel} can be written as

$$s_m^{pel} = \sum_{i=0}^{n-1} \left[\left(\frac{\delta+\gamma}{2} - \frac{|\delta-\gamma|}{2}\right) c_{q-n+i-2m-1} c_{q-n+i-\beta-1} + \left(\frac{\delta-\gamma}{2} + \frac{|\delta-\gamma|}{2}\right) c_{q-n+i-2m-1} c_{q-n+i-\beta} \right.$$

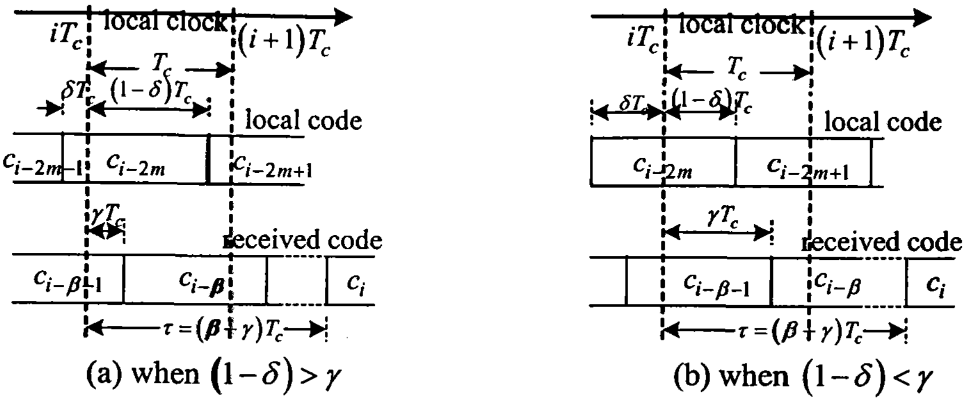


Figure E.2. The relation between the advanced version of local code and the received signal at the m^{th} branch during $iT_c \le t \le (i+1)T_c$.

$$
\begin{aligned}
&+\left(\frac{|(1-\delta)-\gamma|}{2}+\frac{|\gamma-\delta|}{2}-\frac{1}{2}\right)c_{q-n+i-2m}c_{q-n+i-\beta-1}+\left(\frac{1}{2}-\frac{|\delta-\gamma|}{2}-\frac{|(1-\delta)-\gamma|}{2}\right)c_{q-n+i-2m}c_{q-n+i-\beta} \\
&-\left(\frac{\gamma-(1-\delta)}{2}+\frac{|\gamma-(1-\delta)|}{2}\right)c_{q-n+i-2m+1}c_{q-n+i-\beta-1} \\
&-\left(\frac{1+\delta-\gamma}{2}-\frac{|(1-\delta)-\gamma|}{2}\right)c_{q-n+i-2m+1}c_{q-n+i-\beta}\Bigg]. \qquad \text{(E.12)}
\end{aligned}
$$