Appendix F

Correlation Coefficient, Covariance, and Variance for PelDLL

In this appendix, we derive expressions for the correlation coefficient $\rho_{m,k}^{pel}$, the covariance $Cov\left(y_m^{pel}, y_k^{pel}\right)$ between the m^{th} and k^{th} branch, and the variance $\sigma_{pel,m}^2$ of the m^{th} branch for the PelDLL scheme.

The correlation coefficient $\rho_{m,k}^{pel}$ is defined as

$$\rho_{m,k}^{pel} = \frac{Cov\left(y_m^{pel}, y_k^{pel}\right)}{\sqrt{\sigma_{pel,m}^2 \cdot \sigma_{pel,k}^2}} \tag{F.1}$$

while $Cov\left(y_m^{pel}, y_k^{pel}\right)$ is defined as

$$Cov\left(y_m^{pel}, y_k^{pel}\right) = E\left\{y_m^{pel} y_k^{pel}\right\} - E\left\{y_m^{pel}\right\} E\left\{y_k^{pel}\right\}, \tag{F.2}$$

where $E\{\cdot\}$ is the expectation operator and $E\left\{y_m^{pel}\right\}$ is the mean of y_m^{pel}, which is equal to $\sqrt{2P}T_c s_m^{pel}$. Substituting (3.17) in (F.2), we obtain

$$\begin{aligned} Cov\left(y_m^{pel}, y_k^{pel}\right) = {} & E\left\{\left(\sqrt{2P}T_c s_m^{pel}\right)\left(\sqrt{2P}T_c s_k^{pel}\right)\right\} + E\left\{\sqrt{2P}T_c s_m^{pel} \eta_k^{pel}\right\} + E\left\{\sqrt{2P}T_c s_k^{pel} \eta_m^{pel}\right\} \\ & + E\left\{\eta_m^{pel} \eta_k^{pel}\right\} - \left(\sqrt{2P}T_c s_m^{pel}\right)\left(\sqrt{2P}T_c s_k^{pel}\right), \end{aligned} \tag{F.3}$$

where η_m^{pel} is defined in (3.19). The first and last terms in (F.3) cancel each other, while the second and third terms are zero since the mean of the noise is zero. Therefore, (F.3) is reduced to

$$Cov\left(y_m^{pel}, y_k^{pel}\right) = E\left\{\eta_m^{pel} \eta_k^{pel}\right\}. \tag{F.4}$$

The noise η_m^{pel} can be written as

$$\begin{aligned} \eta_m^{pel} = \sum_{i=q-n}^{q-1} \int_{iT_c}^{(i+1)T_c} \operatorname{Re}\{\eta(t)\} \Bigg[& \sum_{j=-\infty}^{\infty} c_j P_{T_c}\left(t - 2mT_c - \delta T_c - jT_c\right) \\ & - \sum_{k=-\infty}^{\infty} c_k P_{T_c}\left(t - 2mT_c + \delta T_c - kT_c\right) \Bigg] dt. \end{aligned} \tag{F.5}$$

Figure F.1 shows the time relation between the delayed and advanced versions of the local PN code in the m^{th} branch over one-chip duration. The integration can be split into three integration intervals. Therefore, (F.5) can be expressed as

$$\eta_m^{pel} = \sum_{i=q-n}^{q-1} \Bigg[\left(c_{i-2m-1} - c_{i-2m}\right) \int_{iT_c}^{(i+\delta)T_c} \operatorname{Re}\{\eta(t)\} dt + \left(c_{i-2m} - c_{i-2m}\right) \int_{(i+\delta)T_c}^{(i+1-\delta)T_c} \operatorname{Re}\{\eta(t)\} dt$$

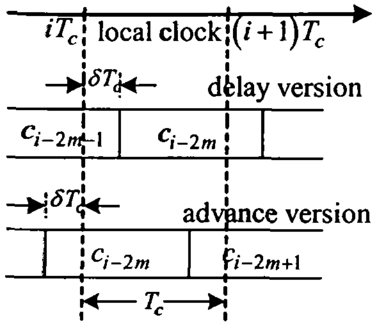

Figure F.1. The relation between the delay and advance version of local code at the m^{th} branch during $iT_c \le t \le (i+1)T_c$.

$$+\left(c_{i-2m}-c_{i-2m+1}\right)\int_{(i+1-\delta)T_c}^{(i+1)T_c}\mathrm{Re}\{\eta(t)\}\,dt\Bigg]$$

$$=\sum_{i=q-n}^{q-1}\left[\left(c_{i-2m-1}-c_{i-2m}\right)\int_{iT_c}^{(i+\delta)T_c}\mathrm{Re}\{\eta(t)\}\,dt+\left(c_{i-2m}-c_{i-2m+1}\right)\int_{(i+1-\delta)T_c}^{(i+1)T_c}\mathrm{Re}\{\eta(t)\}\,dt\right]. \quad \text{(F.6)}$$

Substituting (F.6) into (F.4), we have

$$Cov\left(y_m^{pel},y_k^{pel}\right)=$$

$$E\left\{\sum_{i=q-n}^{q-1}\left[\left(c_{i-2m-1}-c_{i-2m}\right)\int_{iT_c}^{(i+\delta)T_c}\mathrm{Re}\{\eta(t)\}\,dt+\left(c_{i-2m}-c_{i-2m+1}\right)\int_{(i+1-\delta)T_c}^{(i+1)T_c}\mathrm{Re}\{\eta(t)\}\,dt\right]\right.$$

$$\left.\times\sum_{j=q-n}^{q-1}\left[\left(c_{j-2k-1}-c_{j-2k}\right)\int_{jT_c}^{(j+\delta)T_c}\mathrm{Re}\{\eta(\lambda)\}\,d\lambda+\left(c_{j-2k}-c_{j-2k+1}\right)\int_{(j+1-\delta)T_c}^{(j+1)T_c}\mathrm{Re}\{\eta(\lambda)\}\,d\lambda\right]\right\}$$

$$=\sum_{i=q-n}^{q-1}\sum_{j=q-n}^{q-1}\Bigg[\left(c_{i-2m-1}-c_{i-2m}\right)\left(c_{j-2k-1}-c_{j-2k}\right)\int_{iT_c}^{(i+\delta)T_c}\int_{jT_c}^{(j+\delta)T_c}R_{\eta_R}(t-\lambda)\,dt\,d\lambda$$

$$+\left(c_{i-2m}-c_{i-2m+1}\right)\left(c_{j-2k-1}-c_{j-2k}\right)\int_{(i+1-\delta)T_c}^{(i+1)T_c}\int_{jT_c}^{(j+\delta)T_c}R_{\eta_R}(t-\lambda)\,dt\,d\lambda$$

$$+\left(c_{i-2m-1}-c_{i-2m}\right)\left(c_{j-2k}-c_{j-2k+1}\right)\int_{iT_c}^{(i+\delta)T_c}\int_{(j+1-\delta)T_c}^{(j+1)T_c}R_{\eta_R}(t-\lambda)\,dt\,d\lambda$$

$$+\left(c_{i-2m}-c_{i-2m+1}\right)\left(c_{j-2k}-c_{j-2k+1}\right)\int_{(j+1-\delta)T_c}^{(i+1)T_c}\int_{(j+1-\delta)T_c}^{(j+1)T_c}R_{\eta_R}(t-\lambda)\,dt\,d\lambda\Bigg] \quad \text{(F.7)}$$

where $R_{\eta_R}(t)$ is the autocorrelation of $\mathrm{Re}\{\eta(t)\}$, equal to $N_0\delta(t)$. Since $\delta\in(0,\ 0.5]$, the integration ranges of the second and third terms do not overlap for all possible values of i and j. Therefore, they are zero. Furthermore, the integration ranges of the first and fourth terms overlap if and only if the values of i and j are identical. Therefore, the expression of $Cov\left(y_m^{pel},y_k^{pel}\right)$ become

$$Cov\left(y_m^{pel},y_k^{pel}\right)=\delta N_0 T_c\sum_{i=q-n}^{q-1}\left[\left(c_{i-2m-1}-c_{i-2m}\right)\left(c_{i-2k-1}-c_{i-2k}\right)\right.$$

$$+\left(c_{i-2m}-c_{i-2m+1}\right)\left(c_{i-2k}-c_{i-2k+1}\right)\Big]. \qquad \text{(F.8)}$$

The variance of y_m^{pel}, which is equal to the variance of η_m^{pel}, can be obtained by replacing k with m in (F.8), yielding

$$\sigma_{pel,m}^2 = \delta N_0 T_c \sum_{i=q-n}^{q-1}\left[\left(c_{i-2m-1}-c_{i-2m}\right)^2+\left(c_{i-2m}-c_{i-2m+1}\right)^2\right]$$

$$=\delta N_0 T_c\left[4n-2\sum_{i=q-n}^{q-1}\left(c_{i-2m-1}c_{i-2m}+c_{i-2m}c_{i-2m+1}\right)\right]. \qquad \text{(F.9)}$$

Finally, substituting (F.8) and (F.9) into (F.1), we obtain an expression for $\rho_{m,k}^{pel}$.