Appendix G

The Probability of Correct Selection for PelDLL

In this appendix, an expression for the probability of correct selection given H_β for the PelDLL scheme is obtained. Using (3.18), (3.20), (3.23), (3.24), and (3.25), an expression for each H_β is provided as follows.

G.1) Given H_0

Given H_0, the correct selection occurs if and only if the absolute value of the integrate-and-dump output of branch 0 is the largest. Therefore,

$$\Pr\left(\left|Y_0^{pel}\right| > \left|Y_1^{pel}\right|, \left|Y_2^{pel}\right| \middle| 0,\gamma,q,\delta,n,N\right) =$$

$$\int_{-\infty}^{\infty}\int_{-y_0^{pel}}^{y_0^{pel}}\int_{-y_0^{pel}}^{y_0^{pel}} f\left(y_0^{pel},y_1^{pel},y_2^{pel}\middle|0,\gamma,q,\delta,n,N\right)dy_2^{pel}dy_1^{pel}dy_0^{pel}$$

$$= \int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{z^2}{2}\right)$$

$$\times\left[\Phi\left(\frac{\bar{\sigma}_{pel,0}z+\sqrt{\text{SNR}}\left(s_{0|0}^{pel}-s_{1|0}^{pel}\right)}{\bar{\sigma}_{pel,1}}\right)-\Phi\left(\frac{-\bar{\sigma}_{pel,0}z-\sqrt{\text{SNR}}\left(s_{0|0}^{pel}+s_{1|0}^{pel}\right)}{\bar{\sigma}_{pel,1}}\right)\right]$$

$$\times\left[\Phi\left(\frac{\bar{\sigma}_{pel,0}z+\sqrt{\text{SNR}}\left(s_{0|0}^{pel}-s_{2|0}^{pel}\right)}{\bar{\sigma}_{pel,2}}\right)-\Phi\left(\frac{-\bar{\sigma}_{pel,0}z-\sqrt{\text{SNR}}\left(s_{0|0}^{pel}+s_{2|0}^{pel}\right)}{\bar{\sigma}_{pel,2}}\right)\right]dz, \quad \text{(G.1)}$$

where SNR$=2PT_c/N_0$, which is signal per chip to noise ratio, $\bar{\sigma}_{pel,m}=\sqrt{\sigma_{pel,m}^2/N_0T_c}$ is a normalized version of $\sigma_{pel,m}$ which is defined in (3.20), $s_{m|\beta}^{pel}$ is the signal term, and $\Phi(t)$ is the cumulative distribution function (cdf) of the standard Gaussian random variable, $\Phi(t)=\int_{-\infty}^{t}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{x^2}{2}\right)dx$.

G.2) Given H_1

Given H_1, the correct selection occurs if and only if the absolute value of the integrate-and-dump output of branch 1 is the largest. Therefore,

$$\Pr\left(\left|Y_1^{pel}\right| > \left|Y_0^{pel}\right|, \left|Y_2^{pel}\right| \middle| 1,\gamma,q,\delta,n,N\right) =$$

$$\int_{-\infty}^{\infty}\int_{-y_1^{pel}}^{y_1^{pel}}\int_{-y_1^{pel}}^{y_1^{pel}} f\left(y_0^{pel},y_1^{pel},y_2^{pel}\middle|1,\gamma,q,\delta,n,N\right)dy_2^{pel}dy_0^{pel}dy_1^{pel}$$

$$= \int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{z^2}{2}\right)$$

$$\times\left[\Phi\left(\frac{\bar{\sigma}_{pel,1}z+\sqrt{\text{SNR}}\left(s_{1|1}^{pel}-s_{0|1}^{pel}\right)}{\bar{\sigma}_{pel,0}}\right)-\Phi\left(\frac{-\bar{\sigma}_{pel,1}z-\sqrt{\text{SNR}}\left(s_{1|1}^{pel}+s_{0|1}^{pel}\right)}{\bar{\sigma}_{pel,0}}\right)\right]$$

$$\times\left[\Phi\left(\frac{\bar{\sigma}_{pel,1}z+\sqrt{\text{SNR}}\left(s_{1|1}^{pel}-s_{2|1}^{pel}\right)}{\bar{\sigma}_{pel,2}}\right)-\Phi\left(\frac{-\bar{\sigma}_{pel,1}z-\sqrt{\text{SNR}}\left(s_{1|1}^{pel}+s_{2|1}^{pel}\right)}{\bar{\sigma}_{pel,2}}\right)\right]dz\,. \quad \text{(G.2)}$$

G.3) Given H_2

Given H_2, the correct selection occurs if and only if the absolute value of the integrate-and-dump output of branch 1 is the largest. Hence,

$$\Pr\left(\left|Y_1^{pel}\right|>\left|Y_0^{pel}\right|,\left|Y_2^{pel}\right|\Big|2,\gamma,q,\delta,n,N\right)=$$

$$\int_{-\infty}^{\infty}\int_{-y_1^{pel}}^{y_1^{pel}}\int_{-y_1^{pel}}^{y_1^{pel}} f\left(y_0^{pel},y_1^{pel},y_2^{pel}\big|2,\gamma,q,\delta,n,N\right)dy_2^{pel}dy_0^{pel}dy_1^{pel}$$

$$=\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{z^2}{2}\right)$$

$$\times\left[\Phi\left(\frac{\bar{\sigma}_{pel,1}z+\sqrt{\text{SNR}}\left(s_{1|2}^{pel}-s_{0|2}^{pel}\right)}{\bar{\sigma}_{pel,0}}\right)-\Phi\left(\frac{-\bar{\sigma}_{pel,1}z-\sqrt{\text{SNR}}\left(s_{1|2}^{pel}+s_{0|2}^{pel}\right)}{\bar{\sigma}_{pel,0}}\right)\right]$$

$$\times\left[\Phi\left(\frac{\bar{\sigma}_{pel,1}z+\sqrt{\text{SNR}}\left(s_{1|2}^{pel}-s_{2|2}^{pel}\right)}{\bar{\sigma}_{pel,2}}\right)-\Phi\left(\frac{-\bar{\sigma}_{pel,1}z-\sqrt{\text{SNR}}\left(s_{1|2}^{pel}+s_{2|2}^{pel}\right)}{\bar{\sigma}_{pel,2}}\right)\right]dz\,. \quad \text{(G.3)}$$

G.4) Given H_3

Given H_3, the correct selection occurs if and only if the absolute value of the integrate-and-dump output of branch 2 is the largest. Hence,

$$\Pr\left(\left|Y_2^{pel}\right|>\left|Y_0^{pel}\right|,\left|Y_1^{pel}\right|\Big|3,\gamma,q,\delta,n,N\right)=$$

$$\int_{-\infty}^{\infty}\int_{-y_2^{pel}}^{y_2^{pel}}\int_{-y_2^{pel}}^{y_2^{pel}} f\left(y_0^{pel},y_1^{pel},y_2^{pel}\big|3,\gamma,q,\delta,n,N\right)dy_0^{pel}dy_1^{pel}dy_2^{pel}$$

$$=\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{z^2}{2}\right)$$

$$\times\left[\Phi\left(\frac{\bar{\sigma}_{pel,2}z+\sqrt{\text{SNR}}\left(s_{2|3}^{pel}-s_{0|3}^{pel}\right)}{\bar{\sigma}_{pel,0}}\right)-\Phi\left(\frac{-\bar{\sigma}_{pel,2}z-\sqrt{\text{SNR}}\left(s_{2|3}^{pel}+s_{0|3}^{pel}\right)}{\bar{\sigma}_{pel,0}}\right)\right]$$

$$\times\left[\Phi\left(\frac{\bar{\sigma}_{pel,2}z+\sqrt{\text{SNR}}\left(s_{2|3}^{pel}-s_{1|3}^{pel}\right)}{\bar{\sigma}_{pel,1}}\right)-\Phi\left(\frac{-\bar{\sigma}_{pel,2}z-\sqrt{\text{SNR}}\left(s_{2|3}^{pel}+s_{1|3}^{pel}\right)}{\bar{\sigma}_{pel,1}}\right)\right]dz\,. \quad \text{(G.4)}$$