# Appendix H

## Correlation Coefficient, Covariance, and Variance for PerDLL

In this appendix, we derive the expressions for the correlation coefficient $\rho_{m,k}^{per}$, covariance $Cov\left(y_m^{per}, y_k^{per}\right)$ between the $m^{\text{th}}$ and $k^{\text{th}}$ branches, and the variance $\sigma_{per,m}^2$ of the $m^{\text{th}}$ branch for the PerDLL scheme.

The correlation coefficient $\rho_{m,k}^{per}$ is defined as

$$\rho_{m,k}^{per} = \frac{Cov\left(y_m^{per}, y_k^{per}\right)}{\sqrt{\sigma_{per,m}^2 \cdot \sigma_{per,k}^2}} \tag{H.1}$$

while $Cov\left(y_m^{per}, y_k^{per}\right)$ is defined as

$$Cov\left(y_m^{per}, y_k^{per}\right) = E\left\{y_m^{per} y_k^{per}\right\} - E\left\{y_m^{per}\right\} E\left\{y_k^{per}\right\}, \tag{H.2}$$

where $E\{\cdot\}$ is the expectation operator and $E\left\{y_m^{per}\right\}$ is the mean of $y_m^{per}$, which is equal to $\sqrt{2P}T_c s_m^{per} - \sqrt{2\hat{P}}T_c B_m^{per}$. Substituting (3.31) in (H.2) yields

$$\begin{aligned} Cov\left(y_m^{per}, y_k^{per}\right) = {} & E\left\{\left(\sqrt{2P}T_c s_m^{per} - \sqrt{2\hat{P}}T_c B_m^{per}\right)\left(\sqrt{2P}T_c s_k^{per} - \sqrt{2\hat{P}}T_c B_k^{per}\right)\right\} \\ & + E\left\{\left(\sqrt{2P}T_c s_m^{per} - \sqrt{2\hat{P}}T_c B_m^{per}\right)\eta_k^{per}\right\} + E\left\{\left(\sqrt{2P}T_c s_k^{per} - \sqrt{2\hat{P}}T_c B_m^{per}\right)\eta_m^{per}\right\} \\ & + E\left\{\eta_m^{per}\eta_k^{per}\right\} - \left(\sqrt{2P}T_c s_m^{per} - \sqrt{2\hat{P}}T_c B_m^{per}\right)\left(\sqrt{2P}T_c s_k^{per} - \sqrt{2\hat{P}}T_c B_k^{per}\right), \end{aligned} \tag{H.3}$$

where $\eta_m^{per}$ is defined as (3.38). The first and last terms in (H.3) cancel each other, while the second and third terms are zero since the mean of the noise is zero. Therefore, (H.3) is reduced to

$$Cov\left(y_m^{per}, y_k^{per}\right) = E\left\{\eta_m^{per}\eta_k^{per}\right\}. \tag{H.4}$$

The noise $\eta_m^{per}$ can be written as

$$\begin{aligned} \eta_m^{per} &= \sum_{i=q-n}^{q-1} \int_{iT_c}^{(i+1)T_c} \mathrm{Re}\{z(t)\}\left[\sum_{j=-\infty}^{\infty} c_{m,j}^{per} P_{T_c}\left(t - jT_c\right)\right] dt \\ &= \sum_{i=q-n}^{q-1} c_{m,i}^{per} \int_{iT_c}^{(i+1)T_c} \mathrm{Re}\{z(t)\}\, dt. \end{aligned} \tag{H.5}$$

Note that $c_{m,i}^{per}$ is defined in (3.28)-(3.30) for $m$=0, 1, 2, respectively. Substituting (H.5) into (H.4), $Cov\left(y_m^{per}, y_k^{per}\right)$ can be obtained

$$Cov\left(y_m^{per}, y_k^{per}\right) = E\left\{\left[\sum_{i=q-n}^{q-1} c_{m,i}^{per} \int_{iT_c}^{(i+1)T_c} \mathrm{Re}\{\eta(t)\}\,dt\right]\left[\sum_{j=q-n}^{q-1} c_{k,j}^{per} \int_{jT_c}^{(j+1)T_c} \mathrm{Re}\{\eta(\lambda)\}\,d\lambda\right]\right\}$$

$$= \sum_{i=q-n}^{q-1} \sum_{j=q-n}^{q-1} c_{m,i}^{per} c_{k,j}^{per} \int_{iT_c}^{(i+1)T_c} \int_{jT_c}^{(j+1)T_c} R_{\eta_R}(t-\lambda)\,d\lambda\,dt, \tag{H.6}$$

$R_{\eta_R}(t)$ is the autocorrelation function of $\mathrm{Re}\{\eta(t)\}$, which is $N_0\delta(t)$. The integration duration of the variable $t$ and $\lambda$ overlap if and only if the values of $i$ and $j$ are equal. Consequently, $Cov\left(y_m^{per}, y_k^{per}\right)$ will be expressed as

$$Cov\left(y_m^{per}, y_k^{per}\right) = N_0 T_c \sum_{i=q-n}^{q-1} c_{m,i}^{per} c_{k,i}^{per}. \tag{H.7}$$

The variance of $y_m^{per}$, which is equal to the variance of $\eta_m^{per}$, can be obtained by replacing the variable $k$ with $m$ in (H.7), yielding

$$\sigma_{per,m}^2 = N_0 T_c \sum_{i=q-n}^{q-1} \left(c_{m,i}^{per}\right)^2. \tag{H.8}$$

Finally, substituting (H.7) and (H.8) into (H.1), we obtain an expression for $\rho_{m,k}^{per}$.