Appendix I

The Probability of Correct Selection of PerDLL

In this appendix, we obtain an expression for the probability of correct selection given H_β of the PerDLL.

I.1) Given H_0

Given H_0, the correct selection occurs if and only if the absolute value of the integrate-and-dump output of branch 0 is the smallest. Therefore,

$$\Pr(\text{correct selection}|0,\gamma,q,n,N) = \Pr\left(\left(\left|Y_0^{per}\right| < \left|Y_1^{per}\right|\right) \cap \left(\left|Y_0^{per}\right| < \left|Y_2^{per}\right|\right) \middle| 0,\gamma,q,n,N\right). \quad \text{(I.1)}$$

We know that

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B) \quad \text{(I.2)}$$

and

$$\Pr(A \cup B) = 1 - \Pr\left(A^C \cap B^C\right), \quad \text{(I.3)}$$

where $A = \left|Y_0^{per}\right| < \left|Y_1^{per}\right|$, $B = \left|Y_0^{per}\right| < \left|Y_2^{per}\right|$, and C is the complement operator. Thereafter, we obtain

$$\Pr\left(\left(\left|Y_0^{per}\right| < \left|Y_1^{per}\right|\right) \cap \left(\left|Y_0^{per}\right| < \left|Y_2^{per}\right|\right) \middle| 0,\gamma,q,n,N\right) = \Pr\left(\left(\left|Y_0^{per}\right| < \left|Y_1^{per}\right|\right) \middle| 0,\gamma,q,n,N\right) +$$
$$\Pr\left(\left(\left|Y_0^{per}\right| < \left|Y_2^{per}\right|\right) \middle| 0,\gamma,q,n,N\right) + \Pr\left(\left(\left|Y_0^{per}\right| > \left|Y_1^{per}\right|\right) \cap \left(\left|Y_0^{per}\right| > \left|Y_2^{per}\right|\right) \middle| 0,\gamma,q,n,N\right) - 1, \quad \text{(I.4)}$$

where

$$\Pr\left(\left(\left|Y_0^{per}\right| > \left|Y_1^{per}\right|\right) \cap \left(\left|Y_0^{per}\right| > \left|Y_2^{per}\right|\right) \middle| 0,\gamma,q,n,N\right)$$
$$= \int_{-\infty}^{\infty} \int_{-y_0^{per}}^{y_0^{per}} \int_{-y_0^{per}}^{y_0^{per}} f(y_0^{per}, y_1^{per}, y_2^{per} | 0,\gamma,q,n,N) dy_2^{per} dy_1^{per} dy_0^{per}$$
$$= \int_{-\infty}^{\infty} \int_{\left[-z_0 - \sqrt{\text{SNR}}\left(s_{0|0}^{per} + s_{1|0}^{per}\right) + \sqrt{\hat{\text{SNR}}}\left(B_0^{per} + B_1^{per}\right)\right]}^{\left[z_0 + \sqrt{\text{SNR}}\left(s_{0|0}^{per} - s_{1|0}^{per}\right) - \sqrt{\hat{\text{SNR}}}\left(B_0^{per} - B_1^{per}\right)\right]} \int_{\left[-z_0 - \sqrt{\text{SNR}}\left(s_{0|0}^{per} + s_{2|0}^{per}\right) + \sqrt{\hat{\text{SNR}}}\left(B_0^{per} + B_2^{per}\right)\right]}^{\left[z_0 + \sqrt{\text{SNR}}\left(s_{0|0}^{per} - s_{2|0}^{per}\right) - \sqrt{\hat{\text{SNR}}}\left(B_0^{per} - B_2^{per}\right)\right]} \frac{\exp\left\{-\frac{1}{2}\mathbf{z}^{\mathrm{T}}\left(\mathbf{K}_{nor}\right)^{-1}\mathbf{z}\right\}}{(2\pi)^{3/2}\left(\left|\mathbf{K}_{nor}\right|\right)^{1/2}} dz_2 dz_1 dz_0, \quad \text{(I.5)}$$

$$\Pr\left(\left(\left|Y_0^{per}\right| < \left|Y_1^{per}\right|\right) \middle| 0,\gamma,q,n,N\right) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-y_1^{per}}^{y_1^{per}} f(y_0^{per}, y_1^{per}, y_2^{per} | 0,\gamma,q,n,N) dy_0^{per} dy_1^{per} dy_2^{per}$$
$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{\left[-z_1 - \sqrt{\text{SNR}}\left(s_{1|0}^{per} + s_{0|0}^{per}\right) + \sqrt{\hat{\text{SNR}}}\left(B_1^{per} + B_0^{per}\right)\right]}^{\left[z_1 + \sqrt{\text{SNR}}\left(s_{1|0}^{per} - s_{0|0}^{per}\right) - \sqrt{\hat{\text{SNR}}}\left(B_1^{per} - B_0^{per}\right)\right]} \frac{\exp\left\{-\frac{1}{2}\mathbf{z}^{\mathrm{T}}\left(\mathbf{K}_{nor}\right)^{-1}\mathbf{z}\right\}}{(2\pi)^{3/2}\left(\left|\mathbf{K}_{nor}\right|\right)^{1/2}} dz_0 dz_1 dz_2, \quad \text{(I.6)}$$

$$\Pr\left(\left(\left|Y_0^{per}\right|<\left|Y_2^{per}\right|\right)\middle|0,\gamma,q,n,N\right)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-y_2^{per}}^{y_2^{per}} f(y_0^{per},y_1^{per},y_2^{per}\,|\,0,\gamma,q,n,N)\,dy_0^{per}dy_2^{per}dy_1^{per}$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{\left[-z_2-\sqrt{\mathrm{SNR}}\left(s_{2|0}^{per}+s_{0|0}^{per}\right)+\sqrt{\hat{\mathrm{SNR}}}\left(B_2^{per}+B_0^{per}\right)\right]}^{\left[z_2+\sqrt{\mathrm{SNR}}\left(s_{2|0}^{per}-s_{0|0}^{per}\right)-\sqrt{\hat{\mathrm{SNR}}}\left(B_2^{per}-B_0^{per}\right)\right]} \frac{\exp\left\{-\frac{1}{2}\mathbf{z}^{\mathrm{T}}\left(\mathbf{K}_{nor}\right)^{-1}\mathbf{z}\right\}}{(2\pi)^{3/2}\left(\left|\mathbf{K}_{nor}\right|\right)^{1/2}}dz_0dz_2dz_1\,. \quad \text{(I.7)}$$

Where z is a column matrix defined as

$$\mathbf{z}=\begin{bmatrix} z_0 \\ z_1 \\ z_2 \end{bmatrix}=\begin{bmatrix} \dfrac{y_0^{per}-\sqrt{2PT_c}s_{0|0}^{per}+\sqrt{2\hat{P}T_c}B_0^{per}}{\sqrt{N_0T_c}} \\ \dfrac{y_1^{per}-\sqrt{2PT_c}s_{1|0}^{per}+\sqrt{2\hat{P}T_c}B_1^{per}}{\sqrt{N_0T_c}} \\ \dfrac{y_2^{per}-\sqrt{2PT_c}s_{2|0}^{per}+\sqrt{2\hat{P}T_c}B_2^{per}}{\sqrt{N_0T_c}} \end{bmatrix}, \quad \text{(I.8)}$$

$\mathrm{SNR}=2PT_c/N_0$, $\hat{\mathrm{SNR}}=2\hat{P}T_c/N_0$, the estimated per-chip signal-to-noise ratio. $\mathbf{K}_{nor}$ is defined in (3.51), $(\cdot)^{-1}$ and $|\cdot|$ are inverse and determinant operators, B_m^{per} is defined in (3.35)-(3.37), and $s_{m|\beta}^{per}$ is the signal term given in (3.28)-(3.30), m=0, 1, 2.

I.2) Given H_1

Given H_1, the correct selection occurs if and only if the absolute value of the integrate-and-dump output of branch 1 is the smallest. Therefore,

$$\Pr(\text{correct selection}|1,\gamma,q,n,N)=\Pr\left(\left(\left|Y_1^{per}\right|<\left|Y_0^{per}\right|\right)\cap\left(\left|Y_1^{per}\right|<\left|Y_2^{per}\right|\right)\middle|1,\gamma,q,n,N\right)$$

$$=\Pr\left(\left(\left|Y_1^{per}\right|<\left|Y_0^{per}\right|\right)\middle|1,\gamma,q,n,N\right)+\Pr\left(\left(\left|Y_1^{per}\right|<\left|Y_2^{per}\right|\right)\middle|1,\gamma,q,n,N\right)$$

$$+\Pr\left(\left(\left|Y_1^{per}\right|>\left|Y_0^{per}\right|\right)\cap\left(\left|Y_1^{per}\right|>\left|Y_2^{per}\right|\right)\middle|1,\gamma,q,n,N\right)-1, \quad \text{(I.9)}$$

where

$$\Pr\left(\left(\left|Y_1^{per}\right|>\left|Y_0^{per}\right|\right)\cap\left(\left|Y_1^{per}\right|>\left|Y_2^{per}\right|\right)\middle|1,\gamma,q,n,N\right)$$

$$=\int_{-\infty}^{\infty}\int_{-y_1^{per}}^{y_1^{per}}\int_{-y_1^{per}}^{y_1^{per}} f(y_0^{per},y_1^{per},y_2^{per}\,|\,1,\gamma,q,n,N)\,dy_0^{per}dy_2^{per}dy_1^{per}$$

$$=\int_{-\infty}^{\infty}\int_{\left[-z_1-\sqrt{\mathrm{SNR}}\left(s_{1|1}^{per}+s_{2|2}^{per}\right)+\sqrt{\hat{\mathrm{SNR}}}\left(B_1^{per}+B_2^{per}\right)\right]}^{\left[z_1+\sqrt{\mathrm{SNR}}\left(s_{1|1}^{per}-s_{2|1}^{per}\right)-\sqrt{\hat{\mathrm{SNR}}}\left(B_1^{per}-B_2^{per}\right)\right]}\int_{\left[-z_1-\sqrt{\mathrm{SNR}}\left(s_{1|1}^{per}+s_{0|1}^{per}\right)+\sqrt{\hat{\mathrm{SNR}}}\left(B_1^{per}+B_0^{per}\right)\right]}^{\left[z_1+\sqrt{\mathrm{SNR}}\left(s_{1|1}^{per}-s_{0|1}^{per}\right)-\sqrt{\hat{\mathrm{SNR}}}\left(B_1^{per}-B_0^{per}\right)\right]}$$

$$\frac{\exp\left\{-\frac{1}{2}\mathbf{z}^{\mathrm{T}}\left(\mathbf{K}_{nor}\right)^{-1}\mathbf{z}\right\}}{(2\pi)^{3/2}\left(\left|\mathbf{K}_{nor}\right|\right)^{1/2}}dz_0dz_2dz_1\,, \quad \text{(I.10)}$$

$$\Pr\left(\left(\left|Y_1^{per}\right|<\left|Y_0^{per}\right|\right)\middle|1,\gamma,q,n,N\right)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-y_0^{per}}^{y_0^{per}} f(y_0^{per},y_1^{per},y_2^{per}\,|\,1,\gamma,q,n,N)\,dy_1^{per}dy_0^{per}dy_2^{per}$$

$$
= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{\left[-z_0-\sqrt{\mathrm{SNR}}\left(s_{0|1}^{per}+s_{1|1}^{per}\right)+\sqrt{\hat{\mathrm{SNR}}}\left(B_0^{per}+B_1^{per}\right)\right]}^{\left[z_0+\sqrt{\mathrm{SNR}}\left(s_{0|1}^{per}-s_{1|1}^{per}\right)-\sqrt{\hat{\mathrm{SNR}}}\left(B_0^{per}-B_1^{per}\right)\right]} \frac{\exp\left\{-\frac{1}{2}\mathbf{z}^{\mathrm{T}}\left(\mathbf{K}_{nor}\right)^{-1}\mathbf{z}\right\}}{(2\pi)^{3/2}\left(\left|\mathbf{K}_{nor}\right|\right)^{1/2}} dz_1 dz_0 dz_2 , \quad \text{(I.11)}
$$

$$
\Pr\left(\left(\left|Y_1^{per}\right|<\left|Y_2^{per}\right|\right)\middle|1,\gamma,q,n,N\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-y_2^{per}}^{y_2^{per}} f(y_0^{per},y_1^{per},y_2^{per}|1,\gamma,q,n,N)\,dy_1^{per}dy_2^{per}dy_0^{per}
$$

$$
= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{\left[-z_2-\sqrt{\mathrm{SNR}}\left(s_{2|1}^{per}+s_{1|1}^{per}\right)+\sqrt{\hat{\mathrm{SNR}}}\left(B_2^{per}+B_1^{per}\right)\right]}^{\left[z_2+\sqrt{\mathrm{SNR}}\left(s_{2|1}^{per}-s_{1|1}^{per}\right)-\sqrt{\hat{\mathrm{SNR}}}\left(B_2^{per}-B_1^{per}\right)\right]} \frac{\exp\left\{-\frac{1}{2}\mathbf{z}^{\mathrm{T}}\left(\mathbf{K}_{nor}\right)^{-1}\mathbf{z}\right\}}{(2\pi)^{3/2}\left(\left|\mathbf{K}_{nor}\right|\right)^{1/2}} dz_1 dz_2 dz_0 , \quad \text{(I.12)}
$$

where z is a column matrix defined as

$$
\mathbf{z} = \begin{bmatrix} z_0 \\ z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} \dfrac{y_0^{per}-\sqrt{2PT_c}s_{0|1}^{per}+\sqrt{2\hat{P}T_c}R_0^{per}}{\sqrt{N_0T_c}} \\ \dfrac{y_1^{per}-\sqrt{2PT_c}s_{1|1}^{per}+\sqrt{2\hat{P}T_c}R_1^{per}}{\sqrt{N_0T_c}} \\ \dfrac{y_2^{per}-\sqrt{2PT_c}s_{2|1}^{per}+\sqrt{2\hat{P}T_c}R_2^{per}}{\sqrt{N_0T_c}} \end{bmatrix} . \quad \text{(I.13)}
$$

I.3) Given H_2

Given H_2, the correct selection occurs if and only if the absolute value of the integrate-and-dump output of branch 1 is the smallest. Therefore,

$$
\Pr(\text{correct selection}|2,\gamma,q,n,N) = \Pr\left(\left(\left|Y_1^{per}\right|<\left|Y_0^{per}\right|\right)\cap\left(\left|Y_1^{per}\right|<\left|Y_2^{per}\right|\right)\middle|2,\gamma,q,n,N\right)
$$

$$
= \Pr\left(\left(\left|Y_1^{per}\right|<\left|Y_0^{per}\right|\right)\middle|2,\gamma,q,n,N\right) + \Pr\left(\left(\left|Y_1^{per}\right|<\left|Y_2^{per}\right|\right)\middle|2,\gamma,q,n,N\right)
$$

$$
+\Pr\left(\left(\left|Y_1^{per}\right|>\left|Y_0^{per}\right|\right)\cap\left(\left|Y_1^{per}\right|>\left|Y_2^{per}\right|\right)\middle|2,\gamma,q,n,N\right)-1, \quad \text{(I.14)}
$$

where

$$
\Pr\left(\left(\left|Y_1^{per}\right|>\left|Y_0^{per}\right|\right)\cap\left(\left|Y_1^{per}\right|>\left|Y_2^{per}\right|\right)\middle|2,\gamma,q,n,N\right)
$$

$$
= \int_{-\infty}^{\infty}\int_{-y_1^{per}}^{y_1^{per}}\int_{-y_1^{per}}^{y_1^{per}} f(y_0^{per},y_1^{per},y_2^{per}|2,\gamma,q,n,N)\,dy_0^{per}dy_2^{per}dy_1^{per}
$$

$$
= \int_{-\infty}^{\infty} \int_{\left[-z_1-\sqrt{\mathrm{SNR}}\left(s_{1|2}^{per}+s_{2|2}^{per}\right)+\sqrt{\hat{\mathrm{SNR}}}\left(B_1^{per}+B_2^{per}\right)\right]}^{\left[z_1+\sqrt{\mathrm{SNR}}\left(s_{1|2}^{per}-s_{2|2}^{per}\right)-\sqrt{\hat{\mathrm{SNR}}}\left(B_1^{per}-B_2^{per}\right)\right]} \int_{\left[-z_1-\sqrt{\mathrm{SNR}}\left(s_{1|2}^{per}+s_{0|2}^{per}\right)+\sqrt{\hat{\mathrm{SNR}}}\left(B_1^{per}+B_0^{per}\right)\right]}^{\left[z_1+\sqrt{\mathrm{SNR}}\left(s_{1|2}^{per}-s_{0|2}^{per}\right)-\sqrt{\hat{\mathrm{SNR}}}\left(B_1^{per}-B_0^{per}\right)\right]}
$$

$$
\frac{\exp\left\{-\frac{1}{2}\mathbf{z}^{\mathrm{T}}\left(\mathbf{K}_{nor}\right)^{-1}\mathbf{z}\right\}}{(2\pi)^{3/2}\left(\left|\mathbf{K}_{nor}\right|\right)^{1/2}} dz_0 dz_2 dz_1 , \quad \text{(I.15)}
$$

$$
\Pr\left(\left(\left|Y_1^{per}\right|<\left|Y_0^{per}\right|\right)\middle|2,\gamma,q,n,N\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-y_0^{per}}^{y_0^{per}} f(y_0^{per},y_1^{per},y_2^{per}|2,\gamma,q,n,N)\,dy_1^{per}dy_0^{per}dy_2^{per}
$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{\left[-z_0-\sqrt{\mathrm{SNR}}\left(s_{0|2}^{per}+s_{1|2}^{per}\right)+\sqrt{\hat{\mathrm{SNR}}}\left(B_0^{per}+B_1^{per}\right)\right]}^{\left[z_0+\sqrt{\mathrm{SNR}}\left(s_{0|2}^{per}-s_{1|2}^{per}\right)-\sqrt{\hat{\mathrm{SNR}}}\left(B_0^{per}-B_1^{per}\right)\right]} \frac{\exp\left\{-\frac{1}{2}\mathbf{z}^{\mathrm{T}}\left(\mathbf{K}_{nor}\right)^{-1}\mathbf{z}\right\}}{(2\pi)^{3/2}\left(\left|\mathbf{K}_{nor}\right|\right)^{1/2}} dz_1 dz_0 dz_2 , \quad \text{(I.16)}$$

$$\Pr\left(\left(\left|Y_1^{per}\right| < \left|Y_2^{per}\right|\right)\middle| 2,\gamma,q,n,N\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-y_2^{per}}^{y_2^{per}} f(y_0^{per}, y_1^{per}, y_2^{per} | 2,\gamma,q,n,N) dy_1^{per} dy_2^{per} dy_0^{per}$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{\left[-z_2-\sqrt{\mathrm{SNR}}\left(s_{2|2}^{per}+s_{1|2}^{per}\right)+\sqrt{\hat{\mathrm{SNR}}}\left(B_2^{per}+B_1^{per}\right)\right]}^{\left[z_2+\sqrt{\mathrm{SNR}}\left(s_{2|2}^{per}-s_{1|2}^{per}\right)-\sqrt{\hat{\mathrm{SNR}}}\left(B_2^{per}-B_1^{per}\right)\right]} \frac{\exp\left\{-\frac{1}{2}\mathbf{z}^{\mathrm{T}}\left(\mathbf{K}_{nor}\right)^{-1}\mathbf{z}\right\}}{(2\pi)^{3/2}\left(\left|\mathbf{K}_{nor}\right|\right)^{1/2}} dz_1 dz_2 dz_0 , \quad \text{(I.17)}$$

where $\mathbf{z}$ is a column matrix defined as

$$\mathbf{z} = \begin{bmatrix} z_0 \\ z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} \dfrac{y_0^{per} - \sqrt{2PT_c}s_{0|2}^{per} + \sqrt{2\hat{P}T_c}B_0^{per}}{\sqrt{N_0 T_c}} \\ \dfrac{y_1^{per} - \sqrt{2PT_c}s_{1|2}^{per} + \sqrt{2\hat{P}T_c}B_1^{per}}{\sqrt{N_0 T_c}} \\ \dfrac{y_2^{per} - \sqrt{2PT_c}s_{2|2}^{per} + \sqrt{2\hat{P}T_c}B_2^{per}}{\sqrt{N_0 T_c}} \end{bmatrix}. \quad \text{(I.18)}$$

I.4) Given H_3

Given H_3, the correct selection occurs if and only if the absolute value of the integrate-and-dump output of branch 2 is the smallest. Therefore,

$$\Pr(\text{correct selection}|3,\gamma,q,n,N) = \Pr\left(\left(\left|Y_2^{per}\right| < \left|Y_0^{per}\right|\right)\cap\left(\left|Y_2^{per}\right| < \left|Y_1^{per}\right|\right)\middle| 3,\gamma,q,n,N\right)$$

$$= \Pr\left(\left(\left|Y_2^{per}\right| < \left|Y_0^{per}\right|\right)\middle| 3,\gamma,q,n,N\right) + \Pr\left(\left(\left|Y_2^{per}\right| < \left|Y_1^{per}\right|\right)\middle| 3,\gamma,q,n,N\right)$$

$$+ \Pr\left(\left(\left|Y_2^{per}\right| > \left|Y_0^{per}\right|\right)\cap\left(\left|Y_2^{per}\right| > \left|Y_1^{per}\right|\right)\middle| 3,\gamma,q,n,N\right) - 1, \quad \text{(I.19)}$$

where

$$\Pr\left(\left(\left|Y_2^{per}\right| > \left|Y_0^{per}\right|\right)\cap\left(\left|Y_2^{per}\right| > \left|Y_1^{per}\right|\right)\middle| 3,\gamma,q,n,N\right)$$

$$= \int_{-\infty}^{\infty}\int_{-y_2^{per}}^{y_2^{per}}\int_{-y_2^{per}}^{y_2^{per}} f(y_0^{per}, y_1^{per}, y_2^{per} | 3,\gamma,q,n,N) dy_0^{per} dy_1^{per} dy_2^{per}$$

$$= \int_{-\infty}^{\infty}\int_{\left[-z_2-\sqrt{\mathrm{SNR}}\left(s_{2|3}^{per}+s_{1|3}^{per}\right)+\sqrt{\hat{\mathrm{SNR}}}\left(B_2^{per}+B_1^{per}\right)\right]}^{\left[z_2+\sqrt{\mathrm{SNR}}\left(s_{2|3}^{per}-s_{1|3}^{per}\right)-\sqrt{\hat{\mathrm{SNR}}}\left(B_2^{per}-B_1^{per}\right)\right]}\int_{\left[-z_2-\sqrt{\mathrm{SNR}}\left(s_{2|3}^{per}+s_{0|3}^{per}\right)+\sqrt{\hat{\mathrm{SNR}}}\left(B_2^{per}+B_0^{per}\right)\right]}^{\left[z_2+\sqrt{\mathrm{SNR}}\left(s_{2|3}^{per}-s_{0|3}^{per}\right)-\sqrt{\hat{\mathrm{SNR}}}\left(B_2^{per}-B_0^{per}\right)\right]}$$

$$\frac{\exp\left\{-\frac{1}{2}\mathbf{z}^{\mathrm{T}}\left(\mathbf{K}_{nor}\right)^{-1}\mathbf{z}\right\}}{(2\pi)^{3/2}\left(\left|\mathbf{K}_{nor}\right|\right)^{1/2}} dz_0 dz_1 dz_2 , \quad \text{(I.20)}$$

$$\Pr\left(\left(\left|Y_2^{per}\right| < \left|Y_0^{per}\right|\right)\middle| 3,\gamma,q,n,N\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-y_0^{per}}^{y_0^{per}} f(y_0^{per}, y_1^{per}, y_2^{per} | 3,\gamma,q,n,N) dy_2^{per} dy_0^{per} dy_1^{per}$$

$$
= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{\left[-z_0-\sqrt{\mathrm{SNR}}\left(s_{0|3}^{per}+s_{2|3}^{per}\right)+\sqrt{\hat{\mathrm{SNR}}}\left(B_0^{per}+B_2^{per}\right)\right]}^{\left[z_0+\sqrt{\mathrm{SNR}}\left(s_{0|3}^{per}-s_{2|3}^{per}\right)-\sqrt{\hat{\mathrm{SNR}}}\left(B_0^{per}-B_2^{per}\right)\right]} \frac{\exp\left\{-\frac{1}{2}\mathbf{z}^{\mathrm{T}}\left(\mathbf{K}_{nor}\right)^{-1}\mathbf{z}\right\}}{\left(2\pi\right)^{3/2}\left(\left|\mathbf{K}_{nor}\right|\right)^{1/2}} dz_2 dz_0 dz_1 , \quad \text{(I.21)}
$$

$$
\Pr\left(\left(\left|Y_2^{per}\right| < \left|Y_1^{per}\right|\right)\middle| 3,\gamma,q,n,N\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-y_1^{per}}^{y_1^{per}} f(y_0^{per},y_1^{per},y_2^{per} \mid 3,\gamma,q,n,N)\, dy_2^{per} dy_1^{per} dy_0^{per}
$$

$$
= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{\left[-z_1-\sqrt{\mathrm{SNR}}\left(s_{1|3}^{per}+s_{1|3}^{per}\right)+\sqrt{\hat{\mathrm{SNR}}}\left(B_1^{per}+B_2^{per}\right)\right]}^{\left[z_1+\sqrt{\mathrm{SNR}}\left(s_{1|3}^{per}-s_{2|3}^{per}\right)-\sqrt{\hat{\mathrm{SNR}}}\left(B_1^{per}-B_2^{per}\right)\right]} \frac{\exp\left\{-\frac{1}{2}\mathbf{z}^{\mathrm{T}}\left(\mathbf{K}_{nor}\right)^{-1}\mathbf{z}\right\}}{\left(2\pi\right)^{3/2}\left(\left|\mathbf{K}_{nor}\right|\right)^{1/2}} dz_2 dz_1 dz_0 , \quad \text{(I.22)}
$$

where $\mathbf{z}$ is a column matrix defined as

$$
\mathbf{z} = \begin{bmatrix} z_0 \\ z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} \dfrac{y_0^{per} - \sqrt{2P}T_c s_{0|3}^{per} + \sqrt{2\hat{P}}T_c B_0^{per}}{\sqrt{N_0 T_c}} \\ \dfrac{y_1^{per} - \sqrt{2P}T_c s_{1|3}^{per} + \sqrt{2\hat{P}}T_c B_1^{per}}{\sqrt{N_0 T_c}} \\ \dfrac{y_2^{per} - \sqrt{2P}T_c s_{2|3}^{per} + \sqrt{2\hat{P}}T_c B_2^{per}}{\sqrt{N_0 T_c}} \end{bmatrix} . \quad \text{(I.23)}
$$