Appendix K

Correlation Coefficient Between $w_{r,L}$ and $w_{s,L'}$

In this appendix, the correlation coefficient $\rho_{r,s}\left(L,L'|\tau\right)$ between $w_{r,L}$ and $w_{s,L'}$ is derived for $r,s\in\left\{1,2,3,\ldots,2^{M}\right\}$ and L, L' are positive integers.

$$\rho_{r,s}\left(L,L'|\tau\right)=\rho_{s,r}\left(L',L|\tau\right)=\frac{Cov\left(w_{r,L},w_{s,L'}|\tau\right)}{\sigma_{r,L}\sigma_{s,L'}}$$
$$=\frac{E\left\{w_{r,L}w_{s,L'}|\tau\right\}-E\left\{w_{r,L'}|\tau\right\}E\left\{w_{s,L'}|\tau\right\}}{\sigma_{r,L}\sigma_{s,L'}} \qquad \text{(K.1)}$$

where $Cov\left(w_{r,L},w_{s,L'}|\tau\right)$ is the covariance between $w_{r,L}$ and $w_{s,L'}$ given τ and $E\{\cdot\}$ is the expectation operation. The standard deviations of $w_{m,L}$ is obtained from (J.11),

$$\sigma_{m,L}=\sqrt{\left(N^{2}+N-1\right)LN_{0}T_{c}}\,. \qquad \text{(K.2)}$$

The mean of $w_{m,L}$ given τ can be directly obtained from (4.15) as

$$E\left\{w_{m,L}|\tau\right\}=\sqrt{2P}LT_{c}R_{m}\left(\tau\right). \qquad \text{(K.3)}$$

From (K.1), $E\left\{w_{r,L}w_{s,L'}|\tau\right\}$ is obtained as

$$E\left\{w_{r,L}w_{s,L'}|\tau\right\}=E\left\{\left[\sqrt{2P}LT_{c}R_{r}\left(\tau\right)+\eta_{r,L}\right]\left[\sqrt{2P}L'T_{c}R_{s}\left(\tau\right)+\eta_{s,L'}\right]\right\}$$
$$=2PLL'T_{c}^{2}R_{r}\left(\tau\right)R_{s}\left(\tau\right)+\sqrt{2P}LT_{c}R_{r}\left(\tau\right)E\left\{\eta_{s,L'}\right\}$$
$$+\sqrt{2P}L'T_{c}R_{s}\left(\tau\right)E\left\{\eta_{r,L}\right\}+E\left\{\eta_{r,L}\eta_{s,L'}\right\}, \qquad \text{(K.4)}$$

where $E\left\{\eta_{r,L}\right\}=0$ is the mean of Gaussian noise and $E\left\{\eta_{r,L}\eta_{s,L'}\right\}$ is the cross-correlation of $\eta_{r,L}$ and $\eta_{s,L'}$, which can be computed as follows.

$$E\left\{\eta_{r,L}\eta_{s,L'}\right\}=E\left\{\left[\sum_{i=0}^{L-1}\int_{iNT_c}^{(i+1)NT_c}\mathrm{Re}\left\{\eta(t)\right\}a_{r}(t)\,dt\right]\left[\sum_{j=0}^{L'-1}\int_{jNT_c}^{(j+1)NT_c}\mathrm{Re}\left\{\eta(\lambda)\right\}a_{s}(\lambda)\,d\lambda\right]\right\}$$
$$=\sum_{i=0}^{L-1}\sum_{j=0}^{L'-1}\int_{iNT_c}^{(i+1)NT_c}\int_{jNT_c}^{(j+1)NT_c}E\left\{\mathrm{Re}\left\{\eta(t)\right\}\mathrm{Re}\left\{\eta(\lambda)\right\}\right\}a_{r}(t)a_{s}(\lambda)\,d\lambda\,dt. \qquad \text{(K.5)}$$

The autocorrelation of the real part of the noise is

$$E\left\{\mathrm{Re}\left\{\eta(t)\right\}\mathrm{Re}\left\{\eta(\lambda)\right\}\right\}=N_{0}\delta\left(t-\lambda\right), \qquad \text{(K.6)}$$

where $\delta(t)$ is a delta function. Without loss of generality, suppose that $L\le L'$. Substituting (K.6) into (K.5) and recalling that $a_{m}(t)=\sum_{i=0}^{N-1}a_{m,i}P_{T_c}\left(t-iT_{c}\right)$, we obtain

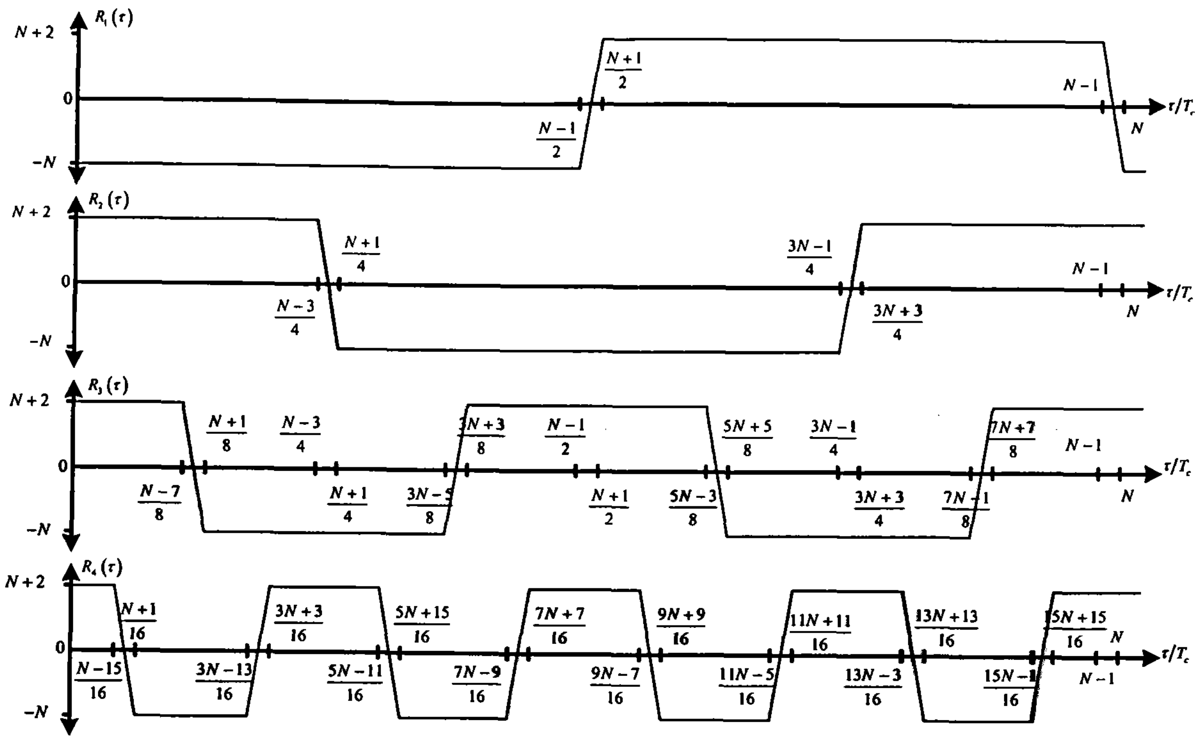

Figure K.1. The correlation function $R_m(\tau)$ for m=1, 2, 3, 4.

$$E\{\eta_{r,L}\eta_{s,L'}\} = N_0 \sum_{i=0}^{L-1} \int_{iNT_c}^{(i+1)NT_c} a_r(t)a_s(t)dt$$
$$= N_0 L T_c \sum_{i=0}^{N-1} a_{r,i}a_{s,i} \,. \tag{K.7}$$

Therefore, the correlation coefficient can be expressed as

$$\rho_{r,s}(L,L'|\tau) = \rho_{r,s}(L,L') = \sqrt{\frac{L}{L'}} \frac{\sum_{i=0}^{N-1} a_{r,i}a_{s,i}}{N^2+N-1}, \qquad \text{for } L \le L', \tag{K.8}$$

which is no longer a function of τ. It remains to find the correlation term in the numerator.

If r=s, from Appendix J, the autocorrelation term $\sum_{i=0}^{N-1} a_{r,i}a_{s,i}$ is

$$\sum_{i=0}^{N-1} a_{r,i}a_{r,i} = N^2 + N - 1 \,. \tag{K.9}$$

However, from Figure K.1, if $r \neq s$, the cross-correlation of $a_{r,i}$ and $a_{s,i}$, $CC(r,s) = \sum_{i=0}^{N-1} a_{r,i}a_{s,i}$, is the summation of $2^{s-1}+1$ cross-correlations between $a_{r,i}$ and a summation of PN signals at different code phases. For example, $CC(1,2)$ is the summation among $\sum_{i=0}^{N-1} a_{r,i}\left(\sum_{q=0}^{\frac{N-3}{4}} c_{i+q}\right)$, $\sum_{i=0}^{N-1} a_{r,i}\left((-1)\sum_{q=\frac{N+1}{4}}^{\frac{3N-1}{4}} c_{i+q}\right)$, and $\sum_{i=0}^{N-1} a_{r,i}\left(\sum_{q=\frac{3N+3}{4}}^{N-1} c_{i+q}\right)$.

These are obtained below.

Without loss of generality, we assume that $s>r$. Recall that $a_{m,i}$ comprises $(N-1)/2$ PN signals of different code phases with positively weighted coefficients and $(N+1)/2$ PN signals of different code phases with negatively weighted coefficients. There are eight correlations to be derived for use in computing $\mathrm{CC}(r,s)$. These correlations are obtained as follows.

(i) $\gamma_{+}^{\mathrm{beg}}(r,s)$, for $r<s$. Let $\gamma_{+}^{\mathrm{beg}}(r,s)$ denote the summation of the cross-correlations between $a_{r,i}$ and the PN signals with the code phases locating in the first flat region of $R_s(\tau)$, i.e., in $\left[0,\ \frac{N-(2^s-1)}{2^s}\right]T_c$. The superscript "beg" indicates that the code phases of the PN signals are located around the beginning region of the uncertainty region. The positive subscript indicates that the coefficients of such PN signals are positive. For example, see Figure K.1, $\gamma_{+}^{\mathrm{beg}}(1,2)$ is the summation of the cross-correlation between $a_{1,i}$ and the PN signals with code phases locating in $[0,\ (N-3)/4]T_c$. $\gamma_{+}^{\mathrm{beg}}(r,s)$ can be categorized into two cases.

For $r=1$,

$$\gamma_{+}^{\mathrm{beg}}(1,s)=\sum_{i=0}^{N-1}\left\{\left[\sum_{k=0}^{\frac{N-1}{2}}[-c_{i+k}]+\sum_{j=0}^{\frac{N-3}{2}}c_{i+j+\frac{N+1}{2}}\right]\left[\sum_{q=0}^{\frac{N-(2^s-1)}{2^s}}c_{i+q}\right]\right\}$$

$$=(-1)(N)\left(\frac{N+1}{2^s}\right)+(-1)(-1)\left[\left(\frac{N+1}{2}\right)\left(\frac{N+1}{2^s}\right)-\left(\frac{N+1}{2^s}\right)\right]$$

$$+(-1)\left(\frac{N-1}{2}\right)\left(\frac{N+1}{2^s}\right) \tag{K.10}$$

$$=\frac{-N^2-N}{2^s}, \tag{K.11}$$

where the first square bracket at the first line of (K.10) is $a_{1,i}$ defined in (4.9). In (K.10), $\gamma_{+}^{\mathrm{beg}}(1,s)$ is the summation of three terms. The first and second terms are obtained from the correlations between negatively weighted $(N+1)/2$ PN signals with code phases from zero to $\left(\frac{N-1}{2}\right)T_c$ due to $a_{1,i}$, i.e., the first term in the first square brackets of the equation above (K.10), and positively weighted $(N+1)/2^s$ PN signals with code phases from zero to $\left[\frac{N-(2^s-1)}{2^s}\right]T_c$, i.e., the term in the second square brackets. The first term is obtained because there are $(N+1)/2^s$ autocorrelations with value of N. The remaining $\left(\frac{N+1}{2}\right)\left(\frac{N+1}{2^s}\right)-\left(\frac{N+1}{2^s}\right)$ cross-correlations have a value of -1, which become the second term. The factor -1 in both terms is due to the product of coefficients with opposite polarities. The third term is obtained from the cross-correlation between positively weighted, $(N-1)/2$ PN signals with code phases from $(N+1)/2$ to $N-1$, due to $a_{1,i}$, i.e., the second term in the first square brackets of the equation above (K.10), and positively weighted, $(N+1)/2^s$ PN signals with code phases from zero to $\frac{N-(2^s-1)}{2^s}$, i.e., the term in the second square brackets. These result in $\left(\frac{N-1}{2}\right)\left(\frac{N+1}{2^s}\right)$ cross-correlations with value of -1.

For $r>1$,

$$\gamma_+^{\text{beg}}(r,s)=\sum_{i=0}^{N-1}\left\{\left[a_{r,i}\right]\left[\sum_{q=0}^{\frac{N-(2^s-1)}{2^s}} c_{i+q}\right]\right\}$$

where $a_{r,i}$ is defined in (4.22). Similar to the case for r=1, $\gamma_+^{\text{beg}}(r,s)$ can be obtained as

$$\begin{aligned}\gamma_+^{\text{beg}}(r,s)&=(N)\left(\frac{N+1}{2^s}\right)+(-1)\left[\left(\frac{N-1}{2}\right)\left(\frac{N+1}{2^s}\right)-\left(\frac{N+1}{2^s}\right)\right]\\&\quad+(-1)(-1)\left(\frac{N+1}{2}\right)\left(\frac{N+1}{2^s}\right)\\&=\frac{N^2+3N+2}{2^s}\end{aligned} \qquad \text{(K.12)}$$

Finally, from (K.11) and (K.12), we conclude that

$$\gamma_+^{\text{beg}}(r,s)=\begin{cases}\frac{-N^2-N}{2^s}, & \text{for } r=1\\ \frac{N^2+3N+2}{2^s}, & \text{for } r>1\end{cases} \qquad \text{(K.13)}$$

(ii) $\gamma_+^{\text{fin}}(r,s)$, for r<s. Let $\gamma_+^{\text{fin}}(r,s)$ denote the summation of the cross-correlations between $a_{r,i}$ and the PN signals with the code phases locating in the last flat region of $R_s(\tau)$, i.e., in $\left[\frac{2^s-1}{2^s}(N+1),\ N-1\right]T_c$. The superscript "fin" indicates that the code phases of the PN signals are located in the last part of the uncertainty region. The positive subscript indicates that the coefficients are positive. For example, see Figure K.1, $\gamma_+^{\text{fin}}(1,2)$ is the summation of the cross-correlation between $a_{1,i}$ and the PN signals with code phases locating in $\left[(3N+3)/4,\ N-1\right]T_c$. As explained in (i), $\gamma_+^{\text{fin}}(r,s)$ can be similarly computed as follows.

$$\begin{aligned}\gamma_+^{fin}(r,s)&=\sum_{i=0}^{N-1}\left\{a_{r,i}\left[\sum_{k=0}^{\frac{N-(2^{s+1}-1)}{2^s}} c_{i+k+\frac{2^s-1}{2^s}(N+1)}\right]\right\}\\&=(N)\left(\frac{N-(2^s-1)}{2^s}\right)+(-1)\left[\left(\frac{N-1}{2}\right)\left(\frac{N-(2^s-1)}{2^s}\right)-\left(\frac{N-(2^s-1)}{2^s}\right)\right]\end{aligned}$$

$$+(-1)(-1)\left(\frac{N+1}{2}\right)\left(\frac{N-(2^s-1)}{2^s}\right) \tag{K.14}$$

$$=\frac{N^2+(3-2^s)N+(2-2^{s+1})}{2^s}. \tag{K.15}$$

In (K.14), the first two terms are obtained from the correlations between $(N-1)/2$ PN signals with positive coefficients from $a_{r,i}$, and $\left[N-(2^s-1)\right]/2^s$ PN signals of consecutive phases from $a_{s,i}$, and they are the results of autocorrelations and cross-correlations, respectively. The third term is the cross-correlation result between $(N+1)/2$ PN signals with negative coefficients, due to $a_{r,i}$, and $\left[N-(2^s-1)\right]/2^s$ PN signals of consecutive phases with positive coefficients.

(iii) $\gamma_-^-(r,s)$, for $r<s$. Let $\gamma_-^-(r,s)$ denote the summation of the cross-correlations between $a_{r,i}$ and $(N+1)/2^{s-1}$ PN signals with negative coefficients and consecutive code phases resulting in both the negative values of $R_s(\tau)$ and $R_r(\tau)$. The negative superscript indicates that the code phases of the PN signals cause the negative values of $R_r(\tau)$. The negative subscript indicates that the coefficients of such PN signals are negative. For example, see Figure K.1, $\gamma_-^-(1,3)$ is the correlation between $a_{1,i}$ and $(N+1)/4$ PN signals with negative coefficients and having code phases locating in $\left[(N+1)/8,\ (3N-5)/8\right]T_c$. Notice that these code phases result in both the negative values of $R_1(\tau)$ and $R_3(\tau)$. Similar to (i), $\gamma_-^-(r,s)$ can be computed as follows.

$$\gamma_-^-(r,s)=\sum_{i=0}^{N-1}\left\{a_{r,i}\left[\sum_{k=0}^{\frac{N-(2^{s-1}-1)}{2^{s-1}}}(-1)c_{i+k+\theta_-^-(r,s)}\right]\right\}$$

$$=(-1)(-1)(N)\left(\frac{N+1}{2^{s-1}}\right)+(-1)(-1)(-1)\left[\left(\frac{N+1}{2}\right)\left(\frac{N+1}{2^{s-1}}\right)-\left(\frac{N+1}{2^{s-1}}\right)\right]$$

$$+(-1)(-1)\left(\frac{N-1}{2}\right)\left(\frac{N+1}{2^{s-1}}\right) \tag{K.16}$$

$$=\frac{2N^2+2N}{2^s}, \tag{K.17}$$

where $\theta_-^-(r,s)$ is the first code phase that makes the PN signals follow the above definition of $\gamma_-^-(r,s)$. For example, see Figure K.1, $\theta_-^-(1,3)$ is $(N+1)/8$.

(iv) $\gamma_+^-(r,s)$, for $r<s$. Let $\gamma_+^-(r,s)$ denote the summation of the cross-correlations between $a_{r,i}$ and $(N+1)/2^{s-1}$ PN signals with positive coefficients and consecutive code phases causing the positive values of $R_s(\tau)$ and the negative values of $R_r(\tau)$. The negative superscript indicates that the code phases of the PN signals are located in the negative flat region of $R_r(\tau)$. The positive subscript indicates that the coefficients of such

PN signals are positive. For example, see Figure K.1, $\gamma_+^-(3,4)$ is the correlation between $a_{3,i}$ and $(N+1)/8$ PN signals with positive coefficients and having the code phases locating in $\left[\frac{3N+3}{16}, \frac{5N-11}{16}\right]T_c$ or, equivalently, $\left[\frac{11N+11}{16}, \frac{13N-3}{16}\right]T_c$. Notice that these code phases result in the negative values of $R_3(\tau)$ and the positive values of $R_4(\tau)$. Similar to (i), $\gamma_+^-(r,s)$ can be computed as follows.

$$\gamma_+^-(r,s)=\sum_{i=0}^{N-1}\left\{a_{r,i}\left[\sum_{k=0}^{\frac{N-(2^{s-1}-1)}{2^{s-1}}} c_{i+k+\theta_+^-(r,s)}\right]\right\}$$

$$=(-1)(N)\left(\frac{N+1}{2^{s-1}}\right)+(-1)(-1)\left[\left(\frac{N+1}{2}\right)\left(\frac{N+1}{2^{s-1}}\right)-\left(\frac{N+1}{2^{s-1}}\right)\right]$$

$$+(-1)\left(\frac{N-1}{2}\right)\left(\frac{N+1}{2^{s-1}}\right) \tag{K.18}$$

$$=\frac{-2N^2-2N}{2^s}, \tag{K.19}$$

where $\theta_+^-(r,s)$ is the first code phase that makes the PN signals follow the above definition of $\gamma_+^-(r,s)$. For example, $\theta_+^-(3,4)$ is $(3N+3)/16$ or, $(11N+11)/16$.

(v) $\gamma_-^+(r,s)$, for $r<s$. Let $\gamma_-^+(r,s)$ denote the summation of the cross-correlations between $a_{r,i}$ and $(N+1)/2^{s-1}$ PN signals with negative coefficient and consecutive code phases causing the negative values of $R_s(\tau)$ and the positive values of $R_r(\tau)$. The positive superscript indicates that the code phases of the PN signals are located in the positive flat region of $R_r(\tau)$. The negative subscript indicates that the coefficients of such PN signals are negative. For example, see Figure K.1, $\gamma_-^+(2,4)$ is the correlation between $a_{2,i}$ and $(N+1)/16$ PN signals with negative coefficients and having the code phase locating in $\left[\frac{N+1}{16}, \frac{3N-13}{16}\right]T_c$, or, equivalently, $\left[\frac{13N+13}{16}, \frac{15N-1}{16}\right]T_c$. Notice that these code phases result in the positive values of $R_2(\tau)$ and the negative values of $R_4(\tau)$. Similar to (i), $\gamma_-^+(r,s)$ can be computed as follows.

$$\gamma_-^+(r,s)=\sum_{i=0}^{N-1}\left\{a_{r,i}\left[\sum_{k=0}^{\frac{N-(2^{s-1}-1)}{2^{s-1}}} (-1)c_{i+k+\theta_-^+(r,s)}\right]\right\}$$

$$=(-1)(N)\left(\frac{N+1}{2^{s-1}}\right)+(-1)(-1)\left[\left(\frac{N-1}{2}\right)\left(\frac{N+1}{2^{s-1}}\right)-\left(\frac{N+1}{2^{s-1}}\right)\right]$$

$$+(-1)(-1)(-1)\left(\frac{N+1}{2}\right)\left(\frac{N+1}{2^{s-1}}\right) \tag{K.20}$$

$$=\frac{-2N^2-6N-4}{2^s}, \tag{K.21}$$

where $\theta_{-}^{+}(r,s)$ is the first code phase that makes the PN signals follow the above definition of $\gamma_{-}^{+}(r,s)$. For example, $\theta_{-}^{+}(2,4)$ is $(N+1)/16$, or $(13N+13)/16$.

(vi) $\gamma_{+}^{+}(r,s)$, for $r<s$. Let $\gamma_{+}^{+}(r,s)$ denote the summation of the cross-correlations between $a_{r,i}$ and $(N+1)/2^{s-1}$ PN signals with positive coefficients and consecutive code phases causing the positive values of both $R_s(\tau)$ and $R_r(\tau)$. The positive superscript indicates that the code phases of the PN signals are located in the positive flat region of $R_r(\tau)$. The positive subscript indicates that the coefficients of such PN signals are positive. For example, see Figure K.1, $\gamma_{+}^{+}(3,4)$ is the correlation between $a_{3,i}$ and the $(N+1)/8$ PN signals with negative coefficients and having code phases locating in $\left[\frac{7N+7}{16},\frac{9N-7}{16}\right]T_c$. Similar to (i), $\gamma_{+}^{+}(r,s)$ can be computed as follows.

$$\gamma_{+}^{+}(r,s)=\sum_{i=0}^{N-1}\left\{a_{r,i}\left[\sum_{k=0}^{\frac{N-(2^{s-1}-1)}{2^{s-1}}} c_{i+k+\theta_{+}^{+}(r,s)}\right]\right\}$$

$$=(N)\left(\frac{N+1}{2^{s-1}}\right)+(-1)\left[\left(\frac{N-1}{2}\right)\left(\frac{N+1}{2^{s-1}}\right)-\left(\frac{N+1}{2^{s-1}}\right)\right]$$

$$+(-1)(-1)\left(\frac{N+1}{2}\right)\left(\frac{N+1}{2^{s-1}}\right) \tag{K.22}$$

$$=\frac{2N^2+6N+4}{2^s}, \tag{K.23}$$

where $\theta_{+}^{+}(r,s)$ is the first code phase that makes the PN signals follow the above definition of $\gamma_{+}^{+}(r,s)$. For example, $\theta_{+}^{+}(3,4)$ is $(7N+7)/16$.

(vii) $\gamma_{-}^{tr}(r,s)$, for $r<s$. Let $\gamma_{-}^{tr}(r,s)$ denote the summation of the cross-correlations between $a_{r,i}$ and $(N+1)/2^{s-1}$ PN signals with negative coefficients and having consecutive code phases locating in the negative flat region of $R_s(\tau)$ and around the transition region of $R_r(\tau)$. The superscript "tr" indicates that the code phases of the PN signals are located around the transition region of $R_r(\tau)$. The negative subscript indicates that the coefficients of such PN signals are negative. For example, see Figure K.1, $\gamma_{-}^{tr}(2,3)$ is obtained from the correlations between $a_{2,i}$ and $(N+1)/4$ PN signals with negatives coefficients and code phases locating in $\left[\frac{N+1}{8},\frac{3N-5}{8}\right]T_c$ or, equivalently, $\left[\frac{5N+5}{8},\frac{7N-1}{8}\right]T_c$. Similar to (i), $\gamma_{-}^{tr}(r,s)$ can be computed as follows.

$$\gamma_-^{\mathrm{tr}}(r,s)=\sum_{i=0}^{N-1}\left\{a_{r,i}\left[\sum_{k=0}^{\frac{N-(2^{s-1}-1)}{2^{s-1}}}(-1)c_{i+k+\theta_-^{\mathrm{tr}}(r,s)}\right]\right\}$$

$$=(-1)(-1)(N)\left(\frac{N+1}{2^s}\right)+(-1)(-1)(-1)\left[\left(\frac{N+1}{2}\right)\left(\frac{N+1}{2^{s-1}}\right)-\left(\frac{N+1}{2^s}\right)\right]$$

$$+(-1)(N)\left(\frac{N+1}{2^s}\right)+(-1)(-1)\left[\left(\frac{N-1}{2}\right)\left(\frac{N+1}{2^{s-1}}\right)-\left(\frac{N+1}{2^s}\right)\right] \quad \text{(K.24)}$$

$$=\frac{-2N-2}{2^s}, \quad \text{(K.25)}$$

where $\theta_-^{\mathrm{tr}}(r,s)$ is the first code phase that makes the PN signals follow the definition of $\gamma_-^{\mathrm{tr}}(r,s)$. For example, $\theta_-^{\mathrm{tr}}(2,3)$ is $(N+1)/8$ or, equivalently, $(5N+5)/8$. The first two terms in (K.24) are obtained from the correlations between $(N+1)/2$ PN signals with negative coefficients from $a_{r,i}$, and $\frac{N+1}{2^{s-1}}$ PN signals of consecutive phases with negative coefficients. Similarly, the last two terms in (K.24) are obtained from the correlations between $\frac{N-1}{2}$ PN signals with positive coefficients from $a_{r,i}$ and $\frac{N+1}{2^{s-1}}$ PN signals of consecutive phases with negative coefficients.

(viii) $\gamma_+^{\mathrm{tr}}(r,s)$, for $r<s$. Let $\gamma_+^{\mathrm{tr}}(r,s)$ denote the summation of the cross-correlations between $a_{r,i}$ and $(N+1)/2^{s-1}$ PN signals with positive coefficients and having consecutive code phases locating in the positive flat region of $R_s(\tau)$ and around the transition region of $R_r(\tau)$. The superscript "tr" indicates that the code phases of the PN signals are located around the transition region of $R_r(\tau)$. The positive subscript indicates that the coefficients of such PN signals are positive. For example, see Figure K.1, $\gamma_+^{\mathrm{tr}}(2,4)$ is obtained from the correlations between $a_{2,i}$ and $(N+1)/8$ PN signals with positive coefficients and code phases locating in $\left[\frac{3N+3}{16},\ \frac{5N-11}{16}\right]T_c$ or, equivalently, $\left[\frac{11N+11}{16},\ \frac{13N-3}{16}\right]T_c$. Similar to (i), $\gamma_+^{\mathrm{tr}}(r,s)$ can be computed as follows

$$\gamma_+^{\mathrm{tr}}(r,s)=\sum_{i=0}^{N-1}\left\{a_{r,i}\left[\sum_{k=0}^{\frac{N-(2^{s-1}-1)}{2^{s-1}}}c_{i+k+\theta_+^{\mathrm{tr}}(r,s)}\right]\right\}$$

$$=(-1)(N)\left(\frac{N+1}{2^s}\right)+(-1)(-1)\left[\left(\frac{N+1}{2}\right)\left(\frac{N+1}{2^{s-1}}\right)-\left(\frac{N+1}{2^s}\right)\right]$$

$$+(N)\left(\frac{N+1}{2^s}\right)+(-1)\left[\left(\frac{N-1}{2}\right)\left(\frac{N+1}{2^{s-1}}\right)-\left(\frac{N+1}{2^s}\right)\right] \quad \text{(K.26)}$$

$$=\frac{2N+2}{2^s} \quad \text{(K.27)}$$

where $\theta_+^{tr}(r,s)$ is the first code phase that makes the PN signals follow the definition of $\gamma_+^{tr}(r,s)$. For example, see Figure K.1, $\theta_+^{tr}(2,4)$ is $(3N+3)/16$ or, equivalently, $(11N+11)/16$. The explanation for (K.26) is the same as described for (K.24).

Using the above results, we can obtain $\mathrm{CC}(r,s)=\sum_{i=0}^{N-1} a_{r,i}a_{s,i}$ in terms of cross-correlations between $a_{r,i}$ and a summation of different code phase PN signals defined in (K.13)-(K.27) as follows:

(a) For $r=1$ and $s=2$

$$\mathrm{CC}(1,2)=\gamma_+^{beg}(1,2)+\gamma_+^{fin}(1,2)+\gamma_-^{tr}(1,2). \tag{K.28}$$

Substituting (K.13), (K.15), and (K.25) into (K.28), we obtain

$$\begin{aligned}\mathrm{CC}(1,2)&=\left(\frac{-N^2-N}{4}\right)+\left(\frac{N^2-N-6}{4}\right)+\left(\frac{-2N-2}{4}\right)\\&=-N-2.\end{aligned} \tag{K.29}$$

(b) For $r=1$ and $s\geq 3$

$$\begin{aligned}\mathrm{CC}(1,s)=\gamma_+^{beg}(1,s)+\gamma_+^{fin}(1,s)+\gamma_-^{tr}(1,s)+\left(2^{s-3}\right)\gamma_-^{-}(1,s)+\left(2^{s-3}-1\right)\gamma_+^{-}(1,s)\\+\left(2^{s-3}\right)\gamma_-^{+}(1,s)+\left(2^{s-1}-1\right)\gamma_+^{+}(1,s).\end{aligned} \tag{K.30}$$

Since $\gamma_-^{-}(q,s)+\gamma_+^{-}(r,s)=0$, $\gamma_-^{+}(q,s)+\gamma_+^{+}(r,s)=0$, and substituting (K.13), (K.15), (K.17), (K.21), and (K.27) into (K.30), we obtain

$$\begin{aligned}\mathrm{CC}(1,s)&=\gamma_+^{beg}(1,s)+\gamma_+^{fin}(1,s)+\gamma_+^{tr}(1,s)+\gamma_-^{-}(1,s)+\gamma_-^{+}(1,s)\\&=\left(\frac{-N^2-N}{2^s}\right)+\left(\frac{N^2+\left(3-2^s\right)N+\left(2-2^{s+1}\right)}{2^s}\right)+\left(\frac{2N+2}{2^s}\right)\\&\quad+\left(\frac{2N^2+2N}{2^s}\right)+\left(\frac{-2N^2-6N-4}{2^s}\right)\\&=-N-2.\end{aligned} \tag{K.31}$$

(c) For $r=2$ and $s=3$

$$\mathrm{CC}(2,3)=\gamma_+^{beg}(2,3)+\gamma_+^{fin}(2,3)+2\gamma_-^{tr}(2,3)+\gamma_+^{-}(2,3). \tag{K.32}$$

Substituting (K.13), (K.15), (K.19), and (K.25) into (K.32), we obtain

$$\begin{aligned}\mathrm{CC}(2,3)&=\left(\frac{N^2+3N+2}{8}\right)+\left(\frac{N^2-5N-14}{8}\right)+2\left(\frac{-2N-2}{8}\right)+\left(\frac{-2N^2-2N}{8}\right)\\&=-N-2.\end{aligned} \tag{K.33}$$

(d) For $r=2$ and $s\geq 4$

$$\begin{aligned}\mathrm{CC}(2,s)=\gamma_+^{beg}(2,s)+\gamma_+^{fin}(2,s)+2\gamma_+^{tr}(2,s)+\left(2^{s-3}\right)\gamma_-^{+}(2,s)+\left(2^{s-3}-2\right)\gamma_+^{+}(2,s)\\+\left(2^{s-3}\right)\gamma_-^{-}(2,s)+\left(2^{s-3}-1\right)\gamma_+^{-}(2,s).\end{aligned} \tag{K.34}$$

Since $\gamma_-^{-}(q,s)+\gamma_+^{-}(r,s)=0$, $\gamma_-^{+}(r,s)+\gamma_+^{+}(r,s)=0$, and substituting (K.13), (K.15), (K.17), (K.21), and (K.27) into (K.34), we obtain

$$\begin{aligned}\mathrm{CC}(2,s)&=\gamma_+^{beg}(2,s)+\gamma_+^{fin}(2,s)+2\gamma_+^{tr}(2,s)+2\gamma_-^{+}(2,s)+\gamma_-^{-}(2,s)\\&=\left(\frac{N^2+3N+2}{2^s}\right)+\left(\frac{N^2+\left(3-2^s\right)N+\left(2-2^{s+1}\right)}{2^s}\right)+2\left(\frac{2N+2}{2^s}\right)\end{aligned}$$

$$+2\left(\frac{-2N^2-6N-4}{2^s}\right)+\left(\frac{2N^2+2N}{2^s}\right)$$

$$=-N-2. \qquad \text{(K.35)}$$

(e) For $r=s-1$ and $s\geq 4$

$$\mathrm{CC}(s-1,s)=\gamma_+^{\mathrm{beg}}(s-1,s)+\gamma_+^{\mathrm{fin}}(s-1,s)+\left(2^{s-2}\right)\gamma_-^{\mathrm{tr}}(s-1,s)$$
$$+\left(2^{s-3}\right)\gamma_+^-(s-1,s)+\left(2^{s-3}-1\right)\gamma_+^+(s-1,s). \qquad \text{(K.36)}$$

Substituting (K.13), (K.15), (K.19), (K.23) and (K.25) into (K.36), we obtain

$$\mathrm{CC}(s-1,s)=\left(\frac{N^2+3N+2}{2^s}\right)+\left(\frac{N^2-\left(3-2^s\right)N+\left(2-2^{s+1}\right)}{2^s}\right)+\left(2^{s-2}\right)\left(\frac{-2N-2}{2^s}\right)$$
$$+\left(2^{s-3}\right)\left(\frac{-2N^2-2N}{2^s}\right)+\left(2^{s-3}-1\right)\left(\frac{2N^2+6N+4}{2^s}\right)$$
$$=-N-2. \qquad \text{(K.37)}$$

(f) For $r\geq 3$, $s\geq 5$, and $r<s-1$

$$\mathrm{CC}(r,s)=\gamma_+^{\mathrm{beg}}(r,s)+\gamma_+^{\mathrm{fin}}(r,s)+\left(2^{r-1}\right)\gamma_+^{\mathrm{tr}}(r,s)+\left(2^{s-3}\right)\gamma_-^-(r,s)+\left(2^{s-3}-2^{r-2}\right)\gamma_+^-(r,s)$$
$$+\left(2^{s-3}\right)\gamma_-^+(r,s)+\left(2^{s-3}-2^{r-2}-1\right)\gamma_+^+(r,s). \qquad \text{(K.38)}$$

Since $\gamma_-^-(q,s)+\gamma_+^-(r,s)=0$, $\gamma_-^+(r,s)+\gamma_+^+(r,s)=0$, and substituting (K.13), (K.15), (K.17), (K.21), and (K.27) into (K.38), we obtain

$$\mathrm{CC}(r,s)=\gamma_+^{\mathrm{beg}}(r,s)+\gamma_+^{\mathrm{fin}}(r,s)+\left(2^{r-1}\right)\gamma_+^{\mathrm{tr}}(r,s)+\left(2^{r-2}\right)\gamma_-^-(r,s)+\left(2^{r-2}+1\right)\gamma_-^+(r,s)$$
$$=\left(\frac{N^2+3N+2}{2^s}\right)+\left(\frac{N^2+\left(3-2^s\right)N+\left(2-2^{s+1}\right)}{2^s}\right)+\left(2^{r-1}\right)\left(\frac{2N+2}{2^s}\right)$$
$$+\left(2^{r-2}\right)\left(\frac{2N^2+2N}{2^s}\right)+\left(2^{r-2}+1\right)\left(\frac{-2N^2-6N-4}{2^s}\right)$$
$$=-N-2. \qquad \text{(K.39)}$$

Therefore, from (K.29), (K.31), (K.33), (K.35), (K.37), and (K.39), we can conclude that

$$\mathrm{CC}(r,s)=-N-2, \quad r,s\in\left\{1,2,3,\ldots,2^M\right\} \text{ and } r\neq s. \qquad \text{(K.40)}$$

Finally, from (K.8), (K.9), and (K.40), we obtain

$$\rho_{r,s}(L,L')=\begin{cases}1, & \text{for } r=s \text{ and } L=L'\\ \sqrt{\dfrac{L}{L'}}, & \text{for } r=s \text{ and } L<L'\\ \dfrac{-N-2}{N^2+N-1}, & \text{for } r\neq s \text{ and } L=L'\\ \sqrt{\dfrac{L}{L'}}\left(\dfrac{-N-2}{N^2+N-1}\right), & \text{for } r\neq s \text{ and } L<L'\end{cases} \qquad \text{(K.41)}$$

observe that the fourth line is the product between the second and the third line.