

## CHAPTER 5

### AN OVERVIEW OF A CONCRETE SHRINKAGE MODEL TAKING INTO ACCOUNT AGGREGATE RESTRAINT

#### 5.1 Concrete as a 2-Phase Material

In this study, concrete is regarded as a two-phase material comprising of paste phase and aggregate phase. Paste phase being the part to undergo shrinkage, consists of all cementitious and powder materials, water, all kinds of mineral and chemical admixtures and air voids. The aggregate phase, considered much more stable in volume, consists of coarse and fine aggregates. Fig. 5.1 shows the conceptual illustration of concrete regarded as a two-phase material. It was assumed that both phases developed complete interaction, which means that there is full bond between the two-phases. The aggregate phase acts as an uniformly distributed so that strains of both phases were equivalent which was equal to the strain of the concrete.

Since shrinkage occurs only in paste phase and aggregate has been proved to provide restraining effect on shrinkage of cement paste in concrete. So the aggregate restraint model which will be essential for calculation of shrinkage of concrete from shrinkage of the paste phase will be proposed.

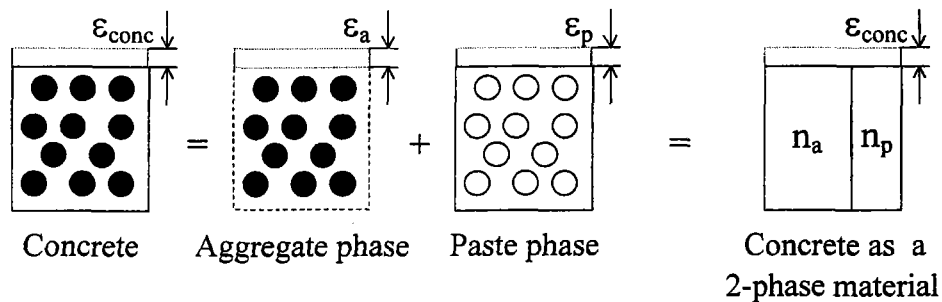


Fig. 5.1 Illustration of concrete as a two-phase

In this study, the model for predicting concrete shrinkage, which was proposed by Sudsangiam (1993), was utilized. The model considers shrinkage restraint by aggregate interaction. It is pointed out that the assumptions, which are used for this model, can be expressed as follow:

- The cement paste does not crack around the aggregate particles.
- Mineral powder, liquid and air voids are considered in paste phase.
- Concrete is considered to consist of two phases, namely aggregate and paste phases.

- The aggregate and paste phases are inelastic. Aggregates are considered as an aggregate assembly having uniformly dispersed aggregate particles in the paste phase.

As a Fig. 5.1, one can defined the volume ratio of paste phase ( $n_p$ ) and aggregate phase ( $n_a$ ) as follows:

$$n_p = \frac{V_p}{V_{\text{conc}}} \quad (5.1)$$

$$n_a = \frac{V_a}{V_{\text{conc}}} \quad (5.2)$$

where  $V_p$  = volume of paste in a cubic meter ( $\text{m}^3/\text{m}^3$ )  
 $V_a$  = volume of aggregate in a cubic meter ( $\text{m}^3/\text{m}^3$ )  
 $V_{\text{conc}}$  = volume of concrete = 1.0 ( $\text{m}^3$ )

Considering a unit volume of concrete, the relation between  $n_p$  and  $n_a$  can be written in the equation as follow:

$$n_p + n_a = 1.0 \quad (5.3)$$

## 5.2 Equilibrium Condition

A general equilibrium equation can be written as

$$\sum \sigma_e = \sum \sigma_i \quad (5.4)$$

where  $\sum \sigma_e$  = total external stress ( $\text{kg}/\text{cm}^2$ )  
 $\sum \sigma_i$  = total internal stress ( $\text{kg}/\text{cm}^2$ )

When concrete undergoes shrinkage, the concrete develops self stress inside the concrete mass (Fig. 5.2) without any external applied stress. Therefore, the equilibrium condition can be written as

$$\sum \sigma_i = \sigma_{(p-a)} + \sigma_{(a-p)} = 0 \quad (5.5)$$

where  $\sigma_{(p-a)}$  = stress acting on aggregate phase from paste phase ( $\text{kg}/\text{cm}^2$ )  
 $\sigma_{(a-p)}$  = stress acting on paste phase from aggregate phase ( $\text{kg}/\text{cm}^2$ )

### 5.2.1 Stress acting on aggregate phase from paste phase

By assuming that the paste phase is uniformly distributed in concrete, the strain of the paste phase is the averaged spatial strain based on a unit volume of the concrete.  $\epsilon_{p0}$  is defined as the strain due to shrinkage of cement paste (concrete with  $n_a = 0$ ). If the aggregate phase does not resist the shrinkage of the paste phase,

shrinkage of the paste phase in concrete with aggregate volume ratio  $n_a$  in case of no aggregate restraint ( $\epsilon_p$ ) will be proportional to the volume ratio of the paste phase as

$$\epsilon_p = \epsilon_{p0} \cdot (1 - n_a) \quad (5.6)$$

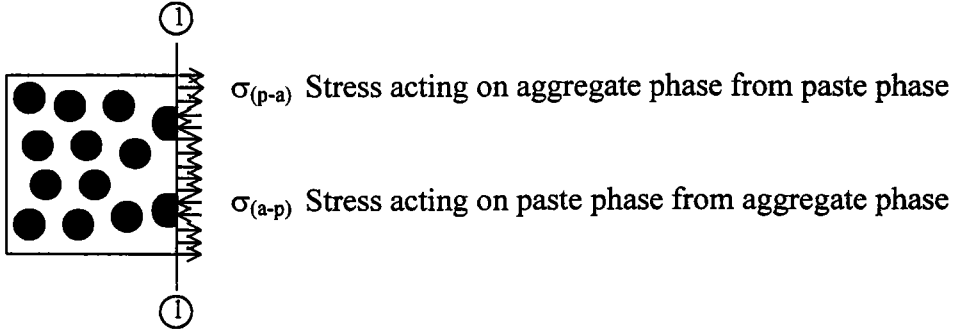


Fig. 5.2 Internal stress between paste and aggregate phases due to shrinkage

However, in actual, the shrinkage can be restrained by particle interaction of the aggregate phase. Assuming that the shrinkage of the paste phase considering aggregate restraint is equal to  $\epsilon_{par}$ , a portion of shrinkage restrained by aggregate phase can be determined as follow:

$$\epsilon_{(p-a)} = \epsilon_p - \epsilon_{par} = \epsilon_{p0} \cdot (1 - n_a) - \epsilon_{par} \quad (5.7)$$

Stress on aggregate phase ( $\sigma_{(p-a)}$ ) is resulted from the restrained shrinkage of the paste phase by the aggregate phase, consequently  $\sigma_{(p-a)}$  can be derived from

$$\sigma_{(p-a)} = E_p \cdot \epsilon_{(p-a)} = E_p \cdot \epsilon_{p0} \cdot (1 - n_a) - E_p \cdot \epsilon_{par} \quad (5.8)$$

where  $E_p$  is the paste stiffness and is considered to be a time dependent material constant.

### 5.2.2 Stress acting on paste phase from aggregate phase

Aggregates are considered as an aggregate assembly, having uniformly dispersed aggregate particles in the paste phase. In this study, the displacement of the aggregate is the summation of the local displacement of all pairs of contact particles. The displacement of particle assembly consists of mainly two types providing that crushing of the particles does not occur. One is the displacement due to particle slip and rearrangement ( $\epsilon_{av}$ ), the other is the displacement due to particle deformation ( $\epsilon_{ad}$ ). The particle slip and rearrangement is easier to occur than the particle deformation. The total strain of the aggregate phase is

$$\epsilon_a = \epsilon_a (\epsilon_{av}, \epsilon_{ad}) \quad (5.9)$$

where

- $\epsilon_a$  = total strain of the aggregate phase
- $\epsilon_{av}$  = strain due to particle slip and rearrangement
- $\epsilon_{ad}$  = strain due to particle deformation

Accordingly, the resisting stress of the aggregate phase ( $\sigma_{(a-p)}$ ) can be calculated from

$$\sigma_{(a-p)} = E_a \cdot \varepsilon_a \quad (5.10)$$

where  $E_a$  is stiffness of the aggregate assembly and can be derived by using the aggregate stiffness model in Chapter 6.

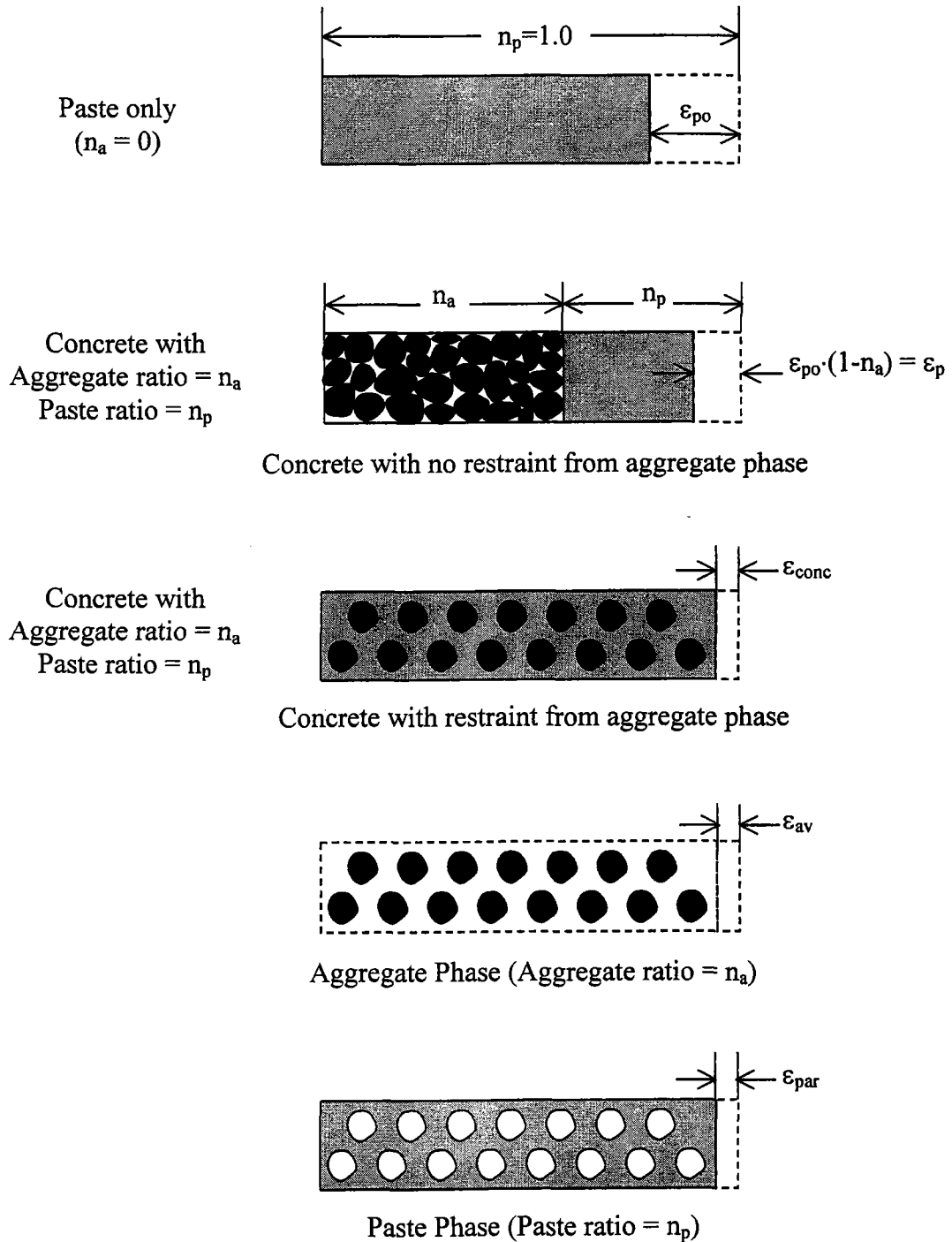


Fig. 5.3 Concept of 2-phase material with perfect paste-aggregate interaction

### 5.3 Strain Compatibility

Since the strain of paste and aggregate phase are not defined based on the volume of their own phase but based on the total volume of concrete and both paste and aggregate phase are assumed to be uniformly distributed in the concrete with the circumferential volume of each phase equal the concrete volume, also the two phases are assumed to be perfectly interacted with no relative average displacement, the following compatibility condition can be obtained from

$$\epsilon_{\text{conc}} = \epsilon_a = \epsilon_{\text{par}} \quad (5.11)$$

where

- $\epsilon_{\text{conc}}$  = strain of the concrete
- $\epsilon_a$  = strain of the aggregate phase
- $\epsilon_{\text{par}}$  = strain of the paste phase with the restrained by aggregate phase

From Eq. (5.8) to Eq. (5.11), we obtain the equation for computing shrinkage strain of concrete as

$$\epsilon_{\text{conc}} = \frac{\epsilon_{\text{po}} \cdot E_p \cdot (1 - n_a)}{E_p + E_a} \quad (5.12)$$

where

- $\epsilon_{\text{po}}$  = free shrinkage of paste in concrete obtained from Chapter 8
- $n_a$  = volume concentration of aggregate
- $E_p$  = paste stiffness ( $\text{kg}/\text{cm}^2$ ) obtained from Chapter 7
- $E_a$  = aggregate stiffness ( $\text{kg}/\text{cm}^2$ ) obtained from Chapter 6