CHAPTER 6

AGGREGATE STIFFNESS

6.1 General

In simulating the shrinkage of concrete by a two-phase material model, stiffness of aggregate phase has to be obtained as one of the phase properties. The deformational behavior of aggregate particle system can be predicted using the idea from the two-dimensional constitutive model for solid particles under compression. The model, which is applicable to coarse aggregate and fine aggregate individually as single materials, is firstly proposed based on the contact density concept. Then the concept for deriving stiffness of combined coarse and fine aggregates as binary mixtures was introduced later.

6.2 Aggregate Stiffness of Single Material

6.2.1 Concept of the aggregate stiffness model for single material

Aggregate is considered to be composed of particles which are contacting one another. Each contact has its own contact angle (θ) and the density distribution of the contact angle is assumed to be $\Omega(\theta)$. The $\Omega(\theta)$ can be simply explained as to represent the ratio of the numbers of contact which have angles θ to the total numbers of contact. The force system contains normal force which is caused by the deformation normal to the contact plane and friction force which is due to the deformation parallel to the contact plane. Stress-strain relation that is applied for relating the deformation normal to the contact plane to the corresponding stress is assumed. Friction is treated as dry Coulomb's friction. Contact area increases as the deformation progresses and is affected by particle shape, size and grading of the aggregate. Re-arrangement of particles is also a significant factor especially for low friction particles.

6.2.2 Probability density function for contact angle

Particles are considered to have a density function for contact angle as in Fig. 6.1. Li and Maekawa (1987) proposed a different function for effective contact area to model the shear transfer across crack. Here it is reasonable to assume that there are negligible contact angles which are normal and parallel to the principle strain direction (θ equals 0 and $\pi/2$), but most contact angles are nearly or just $\pi/4$. Then the function for contact angle is assumed as

$$\Omega(\theta) = \sin 2\theta \tag{6.1}$$

$$\int_0^{\pi/2} \Omega(\theta) d\theta = 1.0 \tag{6.2}$$

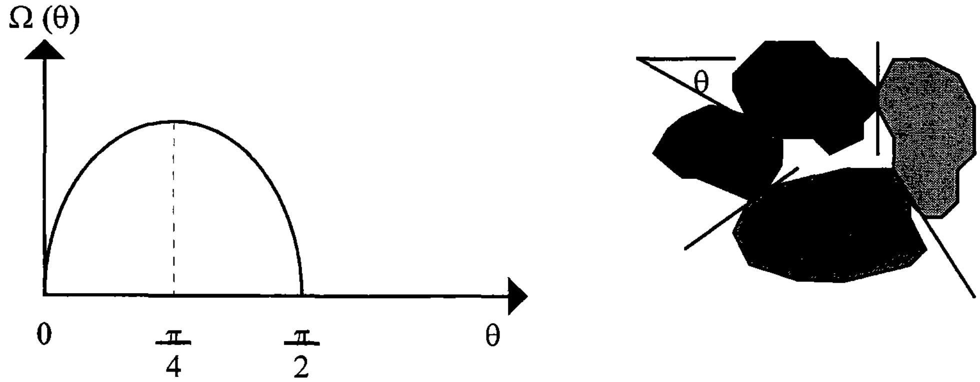

Fig. 6.1 Density function of contact angle

6.2.3 Deformation at a contact

From the geometry in Fig. 6.2, considering a unit volume, the deformation, ω_θ and δ_θ, can be related to strains (ε_y and ε_z) by coordinate transformation. Then

$$\omega_\theta = \varepsilon_z \cdot \cos\theta + \varepsilon_y \cdot \sin\theta \tag{6.3}$$

$$\delta_\theta = \varepsilon_z \cdot \sin\theta - \varepsilon_y \cdot \cos\theta \tag{6.4}$$

As the co-ordinate axis coincides with the principal strain directions, the shear strain, ε_{xy}, equals to zero.

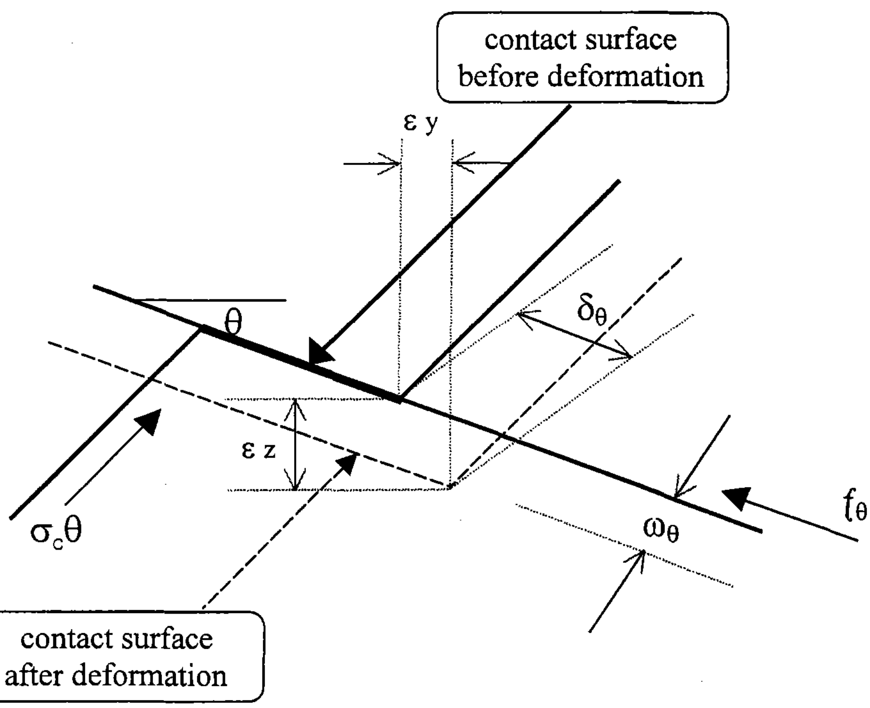

Fig. 6.2 The 2-dimension displacement compatibility of a contact at contact angle θ showing initial as well as deformed configurations of a contact

6.2.4 Constitutive relation for normal direction

In this study, the stress-strain relationship for relating the normal stress ($\sigma_{c\theta}$) to its corresponding deformation (ω_θ) is considered to be linear. The monotonic local stress-strain relation of a contact is assumed to be

$$\sigma_{c\theta} = E_c' \cdot \omega_\theta \tag{6.5}$$

where
$\sigma_{c\theta}$ = contact stress in the normal direction to contact plane θ (kgf/cm^2)
E_c' = stiffness of stress-strain relationship of the contact displacement in the direction normal to the contact plane (kgf/cm^2)
ω_θ = strain normal to contact plane

E_c' was reasonably assumed to have a value of 2.5×10^5 kgf/cm^2 in this study.

6.2.5 Stress in direction parallel to contact plane

To simplify the problem, the frictional stress (f_θ) is assumed constant independent on slip (δ) as in the following expression

$$f_\theta = \mu \cdot \sigma_{c\theta} \tag{6.6}$$

where
f_θ = friction stress of the contact plane (kgf/cm^2)
μ = coefficient of physical friction between grain of particles
$\sigma_{c\theta}$ = contact stress in the normal direction to contact plane θ (kgf/cm^2)

6.2.6 Coefficients of contact friction

Increased amount of water has an effect to lubricate the aggregate particles and then reduce the friction at the contacts of aggregates. As a result, μ is smaller in the mixtures that have greater water to cement ratio. The coefficient of friction depends on the thickness of surface water around aggregate particles and amount of water. Larger thickness of surface water around aggregate particles reduces the friction at the contacts of aggregates. The ratio of volume of water to surface area of aggregates is considered to have relation with the average thickness of surface water around aggregate particles. The surface area of aggregate can be obtained from Appendix C. The relationships between coefficients of friction and the ratio of volume of water to surface area of coarse and fine aggregates are shown in Eq. (6.7) and Eq. (6.8) as well as Fig. 6.3 and Fig. 6.4, respectively.

For coarse aggregate

$$\mu_g = \mu_{g,SSD} - 0.05 \cdot \left(\frac{V_w}{W_s \cdot S_s + W_g \cdot S_g} \right)^{0.24} \tag{6.7}$$

For fine aggregate

$$\mu_s = \mu_{s,SSD} - 0.15 \cdot \left(\frac{V_w}{W_s \cdot S_s + W_g \cdot S_g} \right)^{0.24} \tag{6.8}$$

where μ_g = coefficient of contact friction of coarse aggregate in wet condition
μ_s = coefficient of contact friction of fine aggregate in wet condition
$\mu_{g,SSD}$ = coefficient of contact friction of coarse aggregate in SSD condition
$\mu_{s,SSD}$ = coefficient of contact friction of fine aggregate in SSD condition
V_w = volume of water in a cubic meter of concrete (cm^3/m^3)
W_g = weight of coarse aggregate in a cubic meter of concrete (kg/m^3)
W_s = weight of fine aggregate in a cubic meter of concrete (kg/m^3)
S_g = specific surface area of coarse aggregate (cm^2/m^3)
S_s = specific surface area of fine aggergate (cm^2/m^3)

In this study, the coefficient of friction for crushed limestone coarse aggregate and river sand in SSD condition were assumed to have the values of 0.36 and 0.31, respectively.

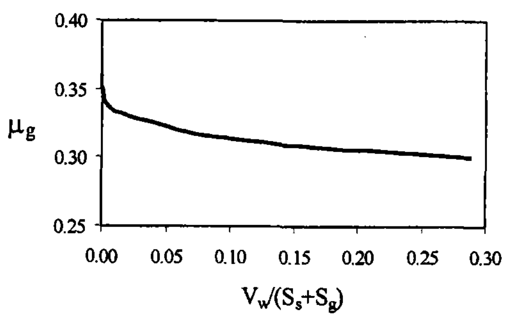

Fig. 6.3 The relationship between the coefficient of friction and the ratio of volume of water to surface area of crushed limestone coarse aggregate

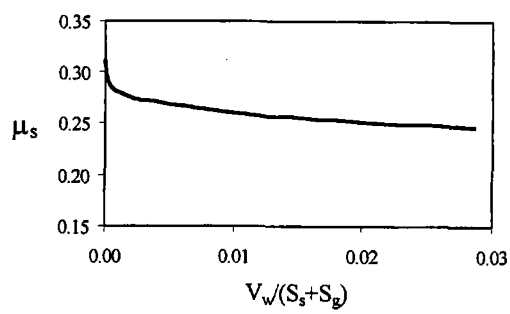

Fig. 6.4 The relationship between the coefficient of friction and the ratio of volume of water to surface area river sand

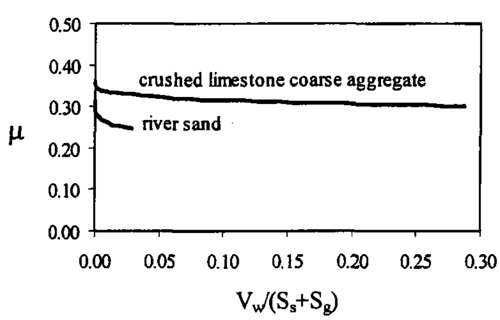

Fig. 6.5 The comparison of coefficient friction between crushed limestone coarse aggregate and river sand

When comparing coefficients of friction (μ_s and μ_g) for coarse aggregate and fine aggregate in Fig. 6.5, it can be seen that coefficient of friction of fine aggregate drops faster than that of the coarse aggregate as the ratio of volume of water to surface area of aggregates increase. It can be explained that small particles like fine aggregate are easier to be dispersed by water film at their surface but the dispersion is difficult for larger particles like coarse aggregate. The increase in amount of surface moisture beyond the saturated and surface-dry (SSD) condition has an effect to decrease the aggregate friction.

6.2.7 Equilibrium equations

The local force system performing on the contact at angle θ can be transformed to be the forces ($F_{z\theta}$, $F_{y\theta}$) in the global coordinate system as

$$F_{z\theta} = (\sigma_{c\theta} \cdot \cos\theta + f_\theta \cdot \sin\theta) \cdot A_{c\theta} \quad (6.9)$$

$$F_{y\theta} = (\sigma_{c\theta} \cdot \sin\theta - f_\theta \cdot \cos\theta) \cdot A_{c\theta} \quad (6.10)$$

where $A_{c\theta}$ is contact area.

Equilibrium is satisfied by integrating the multiplication product of forces with the density function of the contact angle in the global coordinate over contact angles from $\theta = 0$ to $\pi/2$ and equate the integral to the external forces as

$$\sigma_z A_z = \int_0^{\pi/2} \Omega(\theta) \cdot F_{z\theta} \cdot d\theta \quad (6.11)$$

$$\sigma_y A_y = \int_0^{\pi/2} \Omega(\theta) \cdot F_{y\theta} \cdot d\theta \quad (6.12)$$

where σ_y and σ_z are the stresses in y and z directions in global coordinate system, respectively; and A_y and A_z are the area normal to y and z directions in global coordinate system, respectively.

Substituting Eq. (6.9) and Eq. (6.10) into Eq. (6.11) and Eq. (6.12), since $A_y = A_z = 1$, the principal stresses can be obtained as

$$\sigma_z = \int_0^{\pi/2} \Omega(\theta) \cdot (\sigma_{c\theta} \cdot \cos\theta + f_\theta \cdot \sin\theta) \cdot A_{c\theta} \cdot d\theta \quad (6.13)$$

$$\sigma_y = \int_0^{\pi/2} \Omega(\theta) \cdot (\sigma_{c\theta} \cdot \sin\theta - f_\theta \cdot \cos\theta) \cdot A_{c\theta} \cdot d\theta \quad (6.14)$$

By introducing the function for contact area ($A_{c\theta}$), Eq. (6.3), Eq. (6.4), Eq. (6.5), Eq. (6.6), Eq. (6.13), and Eq. (6.14) can be solved simultaneously. Subsequently, the two-dimensional stress-strain relationship of the single materials can be obtained as

$$E_{az} = \frac{\sigma_z}{\varepsilon_z} = \frac{\int_0^{\pi/2} \Omega(\theta) \cdot (\sigma_{c\theta} \cdot \cos\theta + f_\theta \cdot \sin\theta) \cdot A_{c\theta} \cdot d\theta}{\varepsilon_z} \quad (6.15)$$

$$E_{ay} = \frac{\sigma_y}{\varepsilon_y} = \frac{\int_0^{\pi/2} \Omega(\theta) \cdot (\sigma_{c\theta} \cdot \sin\theta - f_\theta \cdot \cos\theta) \cdot A_{c\theta} \cdot d\theta}{\varepsilon_y} \quad (6.16)$$

where E_{az} = aggregate stiffness of stress-strain relation in the z direction of the global coordinate (kgf/cm^2)

E_{az} = aggregate stiffness of stress-strain relation in the y direction of the global coordinate (kgf/cm^2)

σ_z = stress in the z direction in global coordinate system (kgf/cm^2)

σ_y = stress in the y direction in global coordinate system (kgf/cm^2)

ε_z = normal strain in the z direction in global coordinate system

ε_y = normal strain in the y direction in global coordinate system

6.2.8 Contact area

The factors affecting contact area of the particles are size, shape, gradation and re-arrangement of the particles. An important phenomenon is the increase of the contact area as the deformation progresses. By assuming that the contact area of a contact angle θ increases along with the amount of slip in that contact (δ_θ). The contact area at a contact angle θ can be expressed as

$$A_{c\theta} = A_{co} + \int dA_{c\theta} \cdot \phi \quad (6.17)$$

where $A_{c\theta}$ = contact area (cm^2/m^3)

A_{co} = initial contact area (cm^2/m^3)

ϕ = function to govern effect of particle size, shape, grading and re-arrangement on contact area.

For crushed limestone coarse aggregate and river sand, ϕ equals to 1.4 and 1.1, respectively.

1. Initial contact area

The initial contact area of aggregate (A_{co}) can be assumed to be expressed by a non-linear function of the total surface area of aggregate (consider a unit volume of concrete). For coarse aggregate and fine aggregate, the equation for $A_{co(g)}$ and $A_{co(s)}$ were found from the back analysis to be as in Eq. (6.18) and Eq. (6.19), respectively.

$$A_{co(g)} = 16.48 \cdot \left(\frac{\zeta_g}{\zeta_{g,max}} \right) - 19.47 \cdot \left(\frac{\zeta_g}{\zeta_{g,max}} \right)^2 + 8 \cdot \left(\frac{\zeta_g}{\zeta_{g,max}} \right)^3 \quad (6.18)$$

$$A_{co(s)} = 2.23 \cdot \left(\frac{\zeta_s}{\zeta_{s,max}} \right)^{0.77} \quad (6.19)$$

where $A_{co(g)}$ = initial contact area of coarse aggregate in a cubic meter of concrete (cm^2/m^3)

$A_{co(s)}$ = initial contact area of fine aggregate in a cubic meter of concrete(cm^2/m^3)

ζ_G = total surface area of coarse aggregate in a cubic meter of concrete (cm^2/m^3)

ζ_S = total surface area of fine aggregate in a cubic meter of concrete. (cm^2/m^3)

$\zeta_{G,max}$ = total surface area of the densely compacted coarse aggregate in a cubic meter of bulk volume (cm^2/m^3)

$\zeta_{S,max}$ = total surface area of the densely compacted fine aggregate in a cubic meter of bulk volume (cm^2/m^3)

The surface area ratio ($\zeta_g/\zeta_{g,max}$, $\zeta_s/\zeta_{s,max}$) is equal to aggregate volume concentration ratio ($n_g/n_{g,max}$, $n_s/n_{s,max}$) that is defined as

$$\frac{\zeta_g}{\zeta_{g,max}} = \frac{n_g \cdot \rho_g \cdot S_g}{n_{g,max} \cdot \rho_g \cdot S_g} = \frac{n_g}{n_{g,max}} \quad (6.20)$$

$$\frac{\zeta_s}{\zeta_{s,max}} = \frac{n_s \cdot \rho_s \cdot S_s}{n_{s,max} \cdot \rho_s \cdot S_s} = \frac{n_s}{n_{s,max}} \quad (6.21)$$

where n_g = coarse aggregate volume concentration

n_s = fine aggregate volume concentration

$n_{g,max}$ = aggregate volume concentration of the densely compacted coarse aggregate in a cubic meter of bulk volume

$n_{s,max}$ = aggregate volume concentration of the densely compacted fine aggregate in a cubic meter of bulk volume

ρ_g = specific gravity of coarse aggregate

ρ_s = specific gravity of fine aggregate

S_g = specific surface area of coarse aggregate (cm^2/kg)

S_s = specific surface area of fine aggregate (cm^2/kg)

Then, the initial contact area in (A_{co}) in Eq. (6.18) and Eq. (6.19) can be modified as

$$A_{co(g)} = 16.48 \cdot \left(\frac{n_g}{n_{g,max}} \right) - 19.47 \cdot \left(\frac{n_g}{n_{g,max}} \right)^2 + 8 \cdot \left(\frac{n_g}{n_{g,max}} \right)^3 \qquad (6.22)$$

$$A_{co(s)} = 2.23 \cdot \left(\frac{n_s}{n_{s,max}} \right)^{0.77} \qquad (6.23)$$

2. Effect of water content on aggregates contact area reduction

Increasing water content has an effect to increase the thickness of surface water around particles of aggregates. As a result, the contact area among aggregates will be reduced, and the aggregates slip easily when concrete shrinks. Hence, for the same aggregate contents, volume concentration of aggregates, and type of aggregates, but different water to binder ratio, stiffness of aggregates should be changed. The effect of water content on the contact area reduction of aggregates (α) is derived as a function of total volume concentration ratio ($n_a/n_{a,max}$) and water to binder ratio (w/b) as shown in Fig. 6.6. The effect is regarded small at very small and very large volume concentration of aggregates. This is because at very small volume concentration of aggregates, there are not many contacts, while at very large volume concentration of aggregates, contact is too dense and difficult for water to disperse the particles. The effect of water content on contact area is assumed to be 1.0 for water to binder ratio equal to 0.30, less than 1.0 when water to binder ratio is larger than 0.30, and higher than 1.0 when water to binder ratio is smaller than 0.30. Then the equation of the coefficient to govern the effect of water content on the contact area of aggregates is defined as

$$\alpha = 1 - \left[\sqrt{1 - \left(\left(\frac{n_a}{n_{a,max}} - 0.5 \right) \Big/ 0.5 \right)^2} \cdot \left(0.497 \cdot \ln\left(\frac{w}{b} \right) + 0.602 \right) \right] \qquad (6.24)$$

and contact area of aggregates is reduced to be

$$A_{c\theta} = (A_{co} + \int dA_\theta \cdot \phi) \cdot \alpha \qquad (6.25)$$

From the two-dimensional contact configuration in Fig. 6.2, it can be assumed that the contact area of a constant angle θ increases along with the amount of slip (δ_θ) in that contact in a unit volume (m^3) so that a unit width (1 m.) can be applied. As a unit of contact area is cm^2/m^3, a unit width in one meter is transformed to a hundred centimeters. A summation of increase of contact area can be derived from

$$\int dA_{c\theta} = 100 \cdot \delta_\theta \qquad (6.26)$$

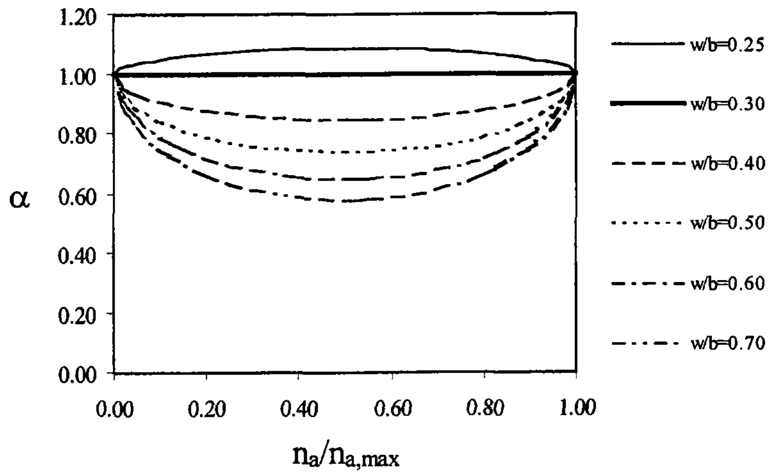

Fig. 6.6 Relationship between the coefficient to govern the effect of water content on the contact area reduction of aggregates (α), volume concentration ratio of aggregates ($n_a/n_{a,max}$), and water to cement ratio (w/c)

3. Effect of maximum size of coarse aggregate (G_{max}) on initial contact area

Low maximum size of coarse aggregate is easier to disperse in the concrete when compared with high maximum size of coarse aggregate at the same volume concentration ratio. There are not many contacts when the maximum size of coarse aggregate is decreased. As a result, initial contact of coarse aggregate will be reduced. Then, the equation of the coefficient to govern the effect of maximum size of coarse aggregate on the initial contact area is defined as

$$\delta = \frac{1}{5.5 - 0.255 \cdot G_{max}} \tag{6.27}$$

and initial contact area of aggregates in Eq. (6.22) is modified to be

$$A_{co(g)} = \left[16.48 \cdot \left(\frac{n_g}{n_{g,max}} \right) - 19.47 \cdot \left(\frac{n_g}{n_{g,max}} \right)^2 + 8 \cdot \left(\frac{n_g}{n_{g,max}} \right)^3 \right] \cdot \delta \tag{6.28}$$

The relation between the coefficient to govern the effect of maximum size of coarse aggregate (δ) and maximum size of coarse aggregate (G_{max}) are shown in Fig. 6.7.

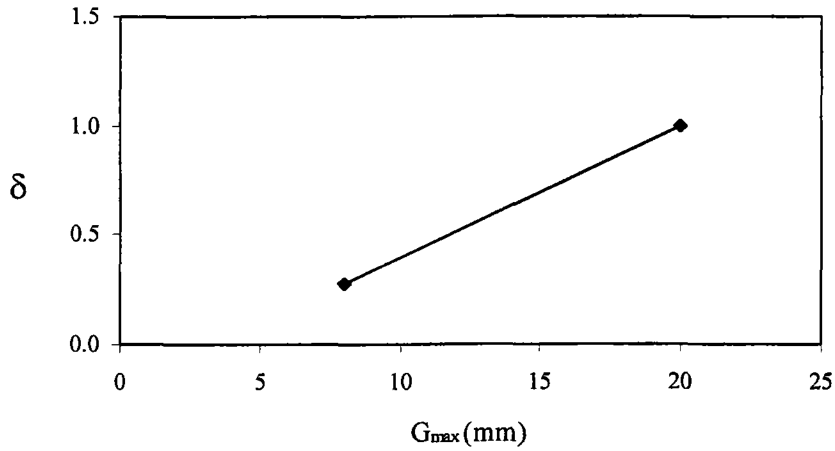

Fig. 6.7 Relationship between the coefficient to govern the effect of maximum size of coarse aggregate on the initial contact area reduction (δ) and maximum size of coarse aggregate (G_{max})

6.3 Stiffness of Binary Mixtures of Aggregates

The above stiffness model was proposed for single materials (coarse and fine aggregates individually). However, aggregates in concrete are usually mixtures of coarse and fine aggregates. The stress of the mixture of aggregates is considered to be the combined results of stresses contributed by each single material, namely stress produced by coarse aggregates (σ_g) and stress produced by fine aggregates (σ_s). The stress contributed by coarse aggregates can be obtained from the summation of stresses from coarse aggregate–coarse aggregate interaction (σ_{g-g}) and fine aggregate–coarse aggregate interaction. The fine aggregate–coarse aggregate interaction stress can be obtained from the fine aggregate–fine aggregate interaction (σ_{s-s}) and the coarse aggregate volumetric ratio as

$$\sigma_g = \sigma_{g-g} + n_g \cdot \sigma_{s-s} \tag{6.29}$$

where
- σ_g = stress produced by coarse aggregate (kgf/cm^2)
- σ_{g-g} = stress from coarse aggregate–coarse aggregate interaction (kgf/cm^2)
- σ_{s-s} = stress from sand aggregate–sand aggregate interaction (kgf/cm^2)
- n_g = coarse aggregate volume concentration

In the same way, the stress contributed by fine aggregates can be obtained from the fine aggregate-fine aggregate interaction as

$$\sigma_s = (1-n_g) \cdot \sigma_{s-s} \tag{6.30}$$

Then the total stress of the binary aggregate phase (σ_t) is calculated from

$$\sigma_t = \sigma_g + \sigma_s = \sigma_{g-g} + \sigma_{s-s} \tag{6.31}$$

where σ_s = stress produced by fine aggregate (kgf/cm^2)
σ_t = total stress produced by coarse aggregate and fine aggregate (kgf/cm^2)

$\sigma_{g\text{-}g}$ and $\sigma_{s\text{-}s}$ are obtained from the stiffness model of single material, so

$$\sigma_{t(z)} = \int_0^{\pi/2} \Omega(\theta)\cdot(\sigma_{c\theta(g)}\cdot\cos\theta + f_{\theta(g)}\cdot\sin\theta)\cdot A_{c\theta(g)}\cdot d\theta + \int_0^{\pi/2} \Omega(\theta)\cdot(\sigma_{c\theta(s)}\cdot\cos\theta + f_{\theta(s)}\cdot\sin\theta)\cdot A_{c\theta(s)}\cdot d\theta \qquad (6.32)$$

$$\sigma_{t(y)} = \int_0^{\pi/2} \Omega(\theta)\cdot(\sigma_{c\theta(g)}\cdot\sin\theta - f_{\theta(g)}\cdot\cos\theta)\cdot A_{c\theta(g)}\cdot d\theta + \int_0^{\pi/2} \Omega(\theta)\cdot(\sigma_{c\theta(s)}\cdot\sin\theta - f_{\theta(s)}\cdot\cos\theta)\cdot A_{c\theta(s)}\cdot d\theta \qquad (6.33)$$

where $A_{c\theta(s)}$ can be obtained in the same manner as single material in Eq. (6.25) as

$$A_{c\theta(s)} = (A_{co(s)} + \int dA_{c\theta(s)}\cdot\phi)\cdot\alpha \qquad (6.34)$$

In binary mixture of coarse and fine aggregates, a part of concrete volume is occupied by coarse aggregate with volume concentration n_g. Then, $(1\text{-}n_g)$ represent the space that can be filled by fine aggregate. So, initial contact of fine aggregate for single material in Eq. (6.23) can be modified to be

$$A_{co(s)} = 2.23\cdot\left(\frac{n_s}{n_{s,max}\cdot(1-n_g)}\right)^{0.77} \qquad (6.35)$$

The fine aggregate particles usually fill the space among the coarse aggregates. When the mixture contains small amount of fine aggregates, they can fill the voids without disturbing the contacts among coarse aggregate. However, all fine aggregate cannot get inside the voids among coarse aggregate when there is large amount of fine aggregate in the mixture. When the amount of fine aggregate is increased so that the particles of coarse aggregates are far apart from each other, the contact area of coarse aggregate will be reduced from this particle interference by fine aggregate. The effect of particle interference by fine aggregate will be smaller when the coarse aggregate volume concentration is increased. Then, the contact area of coarse aggregate in the binary mixture can be computed from

$$A_{c\theta(g)} = (A_{co(g)} + \int dA_{\theta(g)}\cdot\phi)\cdot\alpha\cdot(1-\varphi) \qquad (6.36)$$

where $A_{co(g)}$ was defined in Eq. (6.28), and φ is the coefficient to govern the effect of particle interference of fine aggregate on the contact area reduction of coarse aggregate which was derived to be

$$\varphi = \left[\tan^{-1}\left(15\cdot\frac{n_s}{n_{s,max}}\right)\right]^{\left(1.024\cdot\left(\frac{n_g}{n_{g,max}}\right)^{0.016}\right)}\cdot\left[0.429+\exp\left(-0.561-8.538\cdot\frac{n_g}{n_{g,max}}\right)\right]$$

$$\cdot\left[1-\left(\frac{n_g}{n_{g,max}}\right)^{5.402}\right] \tag{6.37}$$

where φ = coefficient to govern the effect of particle interference of fine aggregate on the contact area reduction of coarse aggregate

n_g = coarse aggregate volume concentration

n_s = fine aggregate volume concentration

$n_{g,max}$ = aggregate volume concentration of the densely compacted coarse aggregate in a cubic meter of bulk volume

$n_{s,max}$ = aggregate volume concentration of the densely compacted fine aggregate in a cubic meter of bulk volume

Fig. 6.8 shows the relationship between the coefficient to govern the effect of particle interference of fine aggregate on the contact area reduction of coarse aggregate (φ), volume concentration ratio of fine aggregate ($n_s/n_{s,max}$), and volume concentration ratio of coarse aggregate ($n_g/n_{g,max}$). It can be seen that for the same volume concentration ratio of coarse aggregate, the coefficient φ is larger when volume concentration ratio of fine aggregate is increased. On the other hand, for the same volume concentration ratio of fine aggregate, the coefficient φ is larger when volume concentration ratio of coarse aggregate is decreased. The coefficient φ is 1.0 when there is no coarse aggregate in the mixture, and is 0 when there is no fine aggregate in the mixture.

Then the stiffness of binary mixture of aggregate can be obtained from

$$E_{az} = \frac{\sigma_{t(z)}}{\varepsilon_z} \tag{6.38}$$

$$E_{ay} = \frac{\sigma_{t(y)}}{\varepsilon_y} \tag{6.39}$$

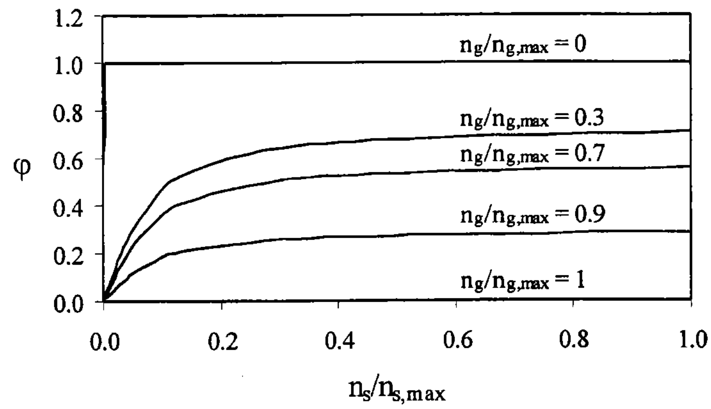

Fig. 6.8 Relationship between the coefficient to govern the effect of particle interference of fine aggregate on the contact area reduction of coarse aggregate, volume concentration ratio of fine aggregate ($n_s/n_{s,max}$), and volume concentration ratio of coarse aggregate ($n_g/n_{g,max}$)