Appendix A

Noise Expected in the VCC Loop

This appendix evaluates the expectation of noise signals in the VCC loop of the proposed noncoherent scheme in Chapter 3. The last 2 terms of (3.28), ζ_R and ζ_I, are the expectation of noise signals where

$$\zeta_R = E\left\{\left(\frac{1}{M}\sum_{k=m-M}^{m-1}\eta_{R1,k+1}\right)\left(\frac{1}{M}\sum_{k=m-M}^{m-1}\eta_{R2,k+1}\right)\right\} \tag{A.1}$$

and

$$\zeta_I = E\left\{\left(\frac{1}{M}\sum_{k=m-M}^{m-1}\eta_{I1,k+1}\right)\left(\frac{1}{M}\sum_{k=m-M}^{m-1}\eta_{I2,k+1}\right)\right\}. \tag{A.2}$$

We can rearrange (A.1)

$$\zeta_R = \frac{1}{N}\sum_{m=1}^{N}E_m\left\{\left(\frac{1}{M}\sum_{k=m-M}^{m-1}\eta_{R1,k+1}\right)\left(\frac{1}{M}\sum_{k=m-M}^{m-1}\eta_{R2,k+1}\right)\right\}. \tag{A.3}$$

Let $\zeta_{R,m} = E_m\left\{\left(\frac{1}{M}\sum_{k=m-M}^{m-1}\eta_{R1,k+1}\right)\left(\frac{1}{M}\sum_{k=m-M}^{m-1}\eta_{R2,k+1}\right)\right\}$ be the statistical expectation with respect to noise at m. We can express $\zeta_{R,m}$ as

$$\zeta_{R,m} = \frac{1}{2P(T_cM)^2}\left\{\sum_{k=m-M}^{m-1}\int_{kT_c}^{(k+1)T_c}\int_{kT_c}^{(k+1)T}R_{z_Rz_R}(t-\lambda)x(t-\hat{\tau}_k)x(\lambda-\hat{\tau}_k-\tfrac{NT_c}{4})d\lambda dt\right.$$
$$\left.+\sum_{k=m-M}^{m-1}\sum_{\substack{j=m-M\\ j\neq k}}^{m}\int_{kT_c}^{(k+1)T_c}\int_{jT_c}^{(j+1)T_c}R_{z_Rz_R}(t-\lambda)x(t-\hat{\tau}_k)x(\lambda-\hat{\tau}_k-\tfrac{NT_c}{4})d\lambda dt\right\} \tag{A.4}$$

where $R_{z_Rz_R}(\gamma)$ is the auto-correlation function of $z_R(t)$. Because $j \neq k$, so $R_{z_Rz_R}(t-\lambda)$ in second term is zero.

$$\begin{aligned}\zeta_{R,m} &= \frac{1}{2P(MT_c)^2}\sum_{k=m-M}^{m-1}\int_{kT_c}^{(k+1)T_c}\int_{kT_c}^{(k+1)T_c}R_{z_Rz_R}(t-\lambda)x(t-\hat{\tau}_k)x(\lambda-\hat{\tau}_k-\tfrac{NT_c}{4})d\lambda dt\\ &= \frac{N_0}{2P(MT_c)^2}\sum_{k=m-M}^{m-1}\int_{kT_c}^{(k+1)T_c}x(t-\hat{\tau}_k)x(t-\hat{\tau}_k-\tfrac{NT_c}{4})dt.\end{aligned} \tag{A.5}$$

From (3.5) and (3.8), we can reduce $x(t)$ as

$$x(t-\tfrac{T_c}{2}) = \alpha(t-T_c) - \alpha(t)$$

$$x(t-\tfrac{T_c}{2}) = \sum_{i=-\frac{N-3}{2}}^{\frac{N-3}{2}} \left(\tfrac{N-1}{2} - |i|\right) c(t - iT_c - T_c) - \sum_{i=-\frac{N-3}{2}}^{\frac{N-3}{2}} \left(\tfrac{N-1}{2} - |i|\right) c(t - iT_c)$$

$$x(t-\tfrac{T_c}{2}) = \sum_{j=-\frac{N-5}{2}}^{\frac{N-1}{2}} \left(\tfrac{N-1}{2} - |j-1|\right) c(t - jT_c) - \sum_{i=-\frac{N-3}{2}}^{\frac{N-3}{2}} \left(\tfrac{N-1}{2} - |i|\right) c(t - iT_c)$$

$$x(t-\tfrac{T_c}{2}) = \sum_{i=1}^{\frac{N-1}{2}} c(t - iT_c) - \sum_{i=-\frac{N-3}{2}}^{0} c(t - iT_c)$$

$$x(t) = \sum_{i=1}^{\frac{N-1}{2}} \left(c(t - iT_c + \tfrac{T_c}{2}) - c(t + iT_c - \tfrac{T_c}{2}) \right). \tag{A.6}$$

From (A.5) and (A.6), (A.3) becomes

$$\begin{aligned}
\zeta_R &= \frac{N_0}{2P(MT_c)^2} E \sum_{k=m-M}^{m-1} \int_{kT_c}^{(k+1)T_c} \left\{ \sum_{i=1}^{\frac{N-1}{2}} \left(c(t - iT_c + \tfrac{T_c}{2} - \hat{\tau}_k) - c(t + iT_c - \tfrac{T_c}{2} - \hat{\tau}_k) \right) \right\} x(t - \hat{\tau}_k - \tfrac{NT_c}{4}) dt \\
&= \frac{N_0}{2PNMT_c^2} \int_0^{NT_c} \left\{ \sum_{i=1}^{\frac{N-1}{2}} c(t - iT_c + \tfrac{T_c}{2} - \hat{\tau}) - c(t + iT_c - \tfrac{T_c}{2} - \hat{\tau}) \right\} x(t - \hat{\tau} - \tfrac{NT_c}{4}) dt \\
&= \frac{N_0}{2PMT_c^2} \sum_{i=1}^{\frac{N-1}{2}} \left(R_{cx}\left((\tfrac{2+N}{4} - i)T \right)_c - R_{cx}\left((\tfrac{N-2}{4} + i)T_c \right) \right) \\
&= \frac{N_0}{2PMT_c^2} \left(\frac{N+1}{N} \right) T_c \\
&= \frac{N+1}{NM(SNR)}.
\end{aligned} \tag{A.7}$$

Similarly, using (A.5) and (A.6), we obtained

$$\zeta_I = \frac{N+1}{NM(SNR)}. \tag{A.8}$$