# CHAPTER 3

# MODELING OF UNKNOWN RESTORING FORCE BY ANNS

## 3.1 General

The identification and modeling of linear and nonlinear dynamical systems through the use of measured experimental data is a problem of considerable importance in engineering. For engineering structures, the main contributions come from complexity and uncertainty of structures in real environment. Some researchers tried to solve this problem by using some existing methods i.e., parametric, and nonparametric identification. Anyway these methods still have limitation, e.g. limited amount of informative data, or complexity of the system. The capability of ANNs to learn, the ability to generalize to new inputs, the robustness to noise, the inherent ability to handle nonlinear systems, and the ability to perform fast computations are the major advantages in the application of ANNs to solving these difficulties. In this chapter the method of using ANNs for identification of dynamical behavior of engineering structures are clearly explained using the simple mechanical system as an example through out this study.

## 3.2 Model Characteristics of Dynamical Systems

Consider a Single-Degree-of-Freedom (SDOF) system whose Free-Body-Diagram (FBD) is shown in Fig. 3.1. This mass, $m$, is subjected to external force, $P_{ext}$, that can be any type of loading (e.g. wind, seismic loading). The equation of motion of this system is

$$m\ddot{u} + f_c + f_s = P_{ext} \tag{3.1}$$

where $f_c$ and $f_s$ are damping force and spring force. $u$, $\dot{u}$, $\ddot{u}$ are displacement , velocity, and acceleration of the system respectively. The restoring force, $f_R$, is the summation of $f_c$ and $f_s$. Eq. (3.1) is rewritten as:

$$m\ddot{u} + f_R = P_{ext}\,. \tag{3.2}$$

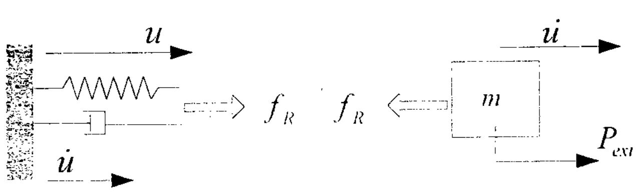


Fig. 3.1 FBD of SDOF system with unknown restoring force

In general the characteristic of restoring force-displacement, $f_R$, can be linear or nonlinear. The nonlinearity can be classified in to conservative types (path-independent) or non-conservative types (path-dependent).

### 3.3 Identification of Restoring Force by ANNs

The identification of restoring force by ANNs consists of two procedures: the network training (or calibration) procedure and the network testing (or validation) procedure. In the training procedure, the network is presented with the sequence of input vectors and the corresponding target values, which is restoring forces. The weights between each layer are initially random as explained in section 2.7.4. Every input vector is fed into the network and the resulting output value is compared with the target value. The error between these two values is then propagated back by using generalized delta rule to adjust weight in such a way that this error is reduced. This procedure is continued until maximum number of training data set is obtained and the error criterion will be computed. The performance index in this study is sum square relative error (*SREE*),

$$SREE = \sqrt{\frac{\sum_{i=1}^{l}\left(\frac{t-o}{t}\right)^2}{l}}, \tag{3.3}$$

in which $t$ is the target value and $o$ is the output that is computed by ANN. $l$ is a number of training data set. The training procedure is terminated when *SREE* value lies within an acceptable range, e.g. $SREE \leq 0.1$, or when maximum number of training epoch reaches the preset number.

In the testing procedure, the well-trained network is presented with training and testing data sets to check the network ability in generalization. When the network shows poor performance, new data sets are required for further training.

### 3.4 Architecture of Emulating ANNs

#### 3.4.1 Selection of network inputs

Before deciding on the architecture of Neural Network, it is important to select appropriate input parameters for modeling the output of network. As described in section 2.7.1, a priori knowledge about the relevant inputs can enhance the network performance. In the following ANNs as a functional mapping from the mathematical viewpoint is explicitly shown. The criterion for the input selection is then described in detail.

From the Fig. 2.7, the mathematical relation of an ANN with two hidden layers and one output neuron is

$$o = f\left[ w_0 + \sum_{j=1}^{p} w_j f\left[ v_{0j} + \sum_{h=1}^{q} v_{hj} f\left[ u_{0h} + \sum_{i=1}^{n} u_{ih} y_i \right]\right]\right] \tag{3.4}$$

in which $y_1, y_2, y_3, \ldots, y_n$ are the inputs, and $f(\cdot)$ denotes an activation function. $[u_{ih}, v_{hj}, w_j]^T$ is the vector of weights and biases, which are adjustable network parameters. The intricate relation between the inputs and output as emulated by the trained network, can be written in a much simpler functional form as:

$$o = ANN\left(\mathbf{y}; u_{ih}^{*}, v_{hj}^{*}, w_j^{*}\right) \tag{3.5a}$$

or shortly,

$$o = ANN\left(\mathbf{y}\right) \tag{3.5b}$$

in which *ANN* is the functional form representing the designed ANN-architecture with the optimal weights $u^{*}_{ih}$, $v^{*}_{hj}$, and $w^{*}_{j}$ respectively. $\boldsymbol{y}$ and $o$ are the input and its resulting output. The expression Eq. (3.5) is illustrated in Fig. 3.2.

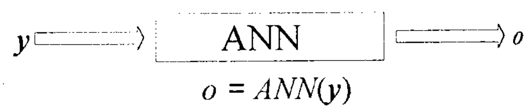


Fig. 3.2 Functional form of ANN

From Fig. 3.2, relation between input and output strongly influences the appropriate ANN architecture and its efficiency.

1) Nonlinear conservative (path-independent) system

In this study, the duffing of hardening type system is employed as a nonlinear conservative system. The corresponding equation of motion is

$$m\ddot{u} + c\dot{u} + k_1 u + k_2 u^3 = -mA_g \tag{3.6}$$

where $c$, $k_1$, $k_2$ are the damping, the linear stiffness, and nonlinear stiffness respectively. $A_g$ is seismic record. The nonlinear spring force is described by

$$f_s = k_1 u + k_2 u^3 . \tag{3.7}$$

In the equation of motion for the verification procedure, the spring force will be assumed unknown and replaced by network an *ANN*. Eq. (3.6) is changed to be:

$$m\ddot{u} + c\dot{u} + f_{ANN} = -mA_g . \quad (3.8)$$

From Eq. (3.7) and Eq. (3.8), $f_{ANN}$ is the functional form of *ANN*. Correspondingly,

$$f_{ANN,t} = ANN(u_t) . \quad (3.9)$$

When the damping is also unknown, $f_{ANN}$, can include the damping. Thus, the functional form of Eq. (3.9) is replaced by

$$f_{ANN,t} = ANN(u_t, \dot{u}_t) . \quad (3.10)$$

From Eq. (3.9) and Eq. (3.10), the inputs to the network for predicting the restoring force are shown in Fig. 3.3 and Fig. 3.4 respectively.

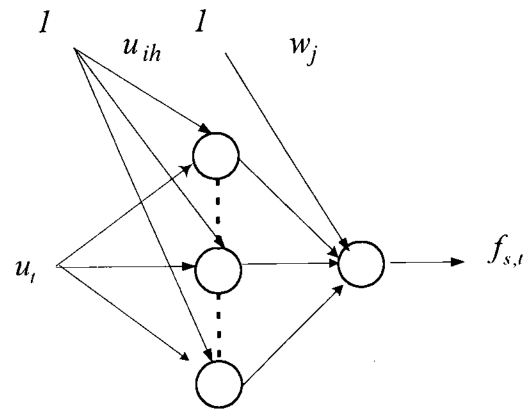


Fig. 3.3 Architecture of ANNs for modeling of nonlinear conservative spring force

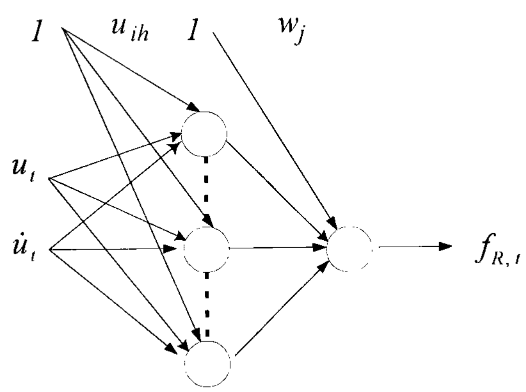


Fig. 3.4 Architecture of ANNs for modeling of nonlinear conservative restoring force

2) Nonlinear non-conservative (path-dependent) system

The nonlinear non-conservative system to be considered herein is Clough-Johnston (CJ) Hysteretic and Extended-Bouc-Wen-Baber-Noori (EBWBN) Hysteretic system. The equation of motion for these systems is:

$$m\ddot{u} + c\dot{u} + \alpha k_l u + (1-\alpha) k_l Z = -mA_g \quad (3.11)$$

in which $\alpha$ determine the elastic fraction in the total restoring force. $Z$ is the hysteretic displacement. The detailing of these two models, CJ and EBWBN hysteretic, are shown in section 4.5.1 and 4.5.2 respectively. The spring force is combined with linear, $\alpha k_l u$, and nonlinear part, $(1-\alpha) k_l Z$.

Since the restoring force, $f_R$, of a hysteretic system depends not only on the instantaneous displacement, but also on its response history, and past restoring force. The input vector, $y$, in the current study can be written as:

$$y = \left[ u_t \dots u_{t-n_u} \ \dot{u}_t \dots \dot{u}_{t-n_v} \ f_{R,t-1} \cdots f_{R,t-n_f} \right] \quad (3.12)$$

where $n_u, n_v, n_f$ are numbers of time steps of displacement, velocity, and past restoring force respectively. This set of inputs will be the initial condition for locating the next position in the hyper-space of the force-kinematic variables when the dynamic of the systems evolves. The reason of using the past restoring force as input is that the restoring force intrinsically contains the information about the other state variables which control but are not easily correlated to the hysteretic component, e.g. propagating crack length or reduced material strength. From the ANNs architecture and input-output variables, the proposed ANNs belong to the class of dynamic or recurrent neural networks. This type of input is used for the least of this CJ and EBWBN hysteretic system. Fig. 3.5 shows the network that use to solve nonlinear non-conservative system.

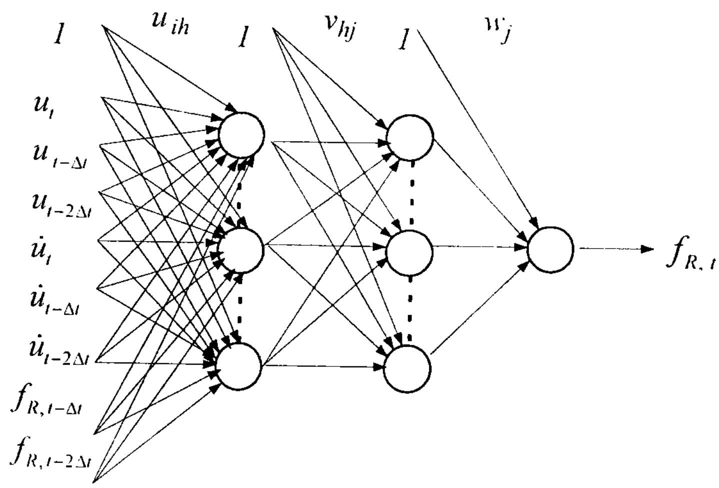


Fig. 3.5 Architecture of ANNs for modeling of hysteretic restoring force

The application of the trained artificial neural networks to the prediction of the responses due to new excitations ($A_{g2}$ for this study) or disturbances has to be done prudently. The nature of the path- or history-dependent behavior in hysteretic systems must indispensably appear in the application. For such purpose, the values of the ANNs-input state-variables at the tail of the immediately previous response histories, $A_{g1}$, need to be used as the delayed inputs to the trained artificial neural networks. In other words, the path-dependent behavior is conserved by means of the sequential evolution of the state variables that characterize the hysteretic system.

### 3.4.2 Selection of network topology

There is no general rule for selecting the number of neurons in a hidden layer. The choice of hidden layer size is another mainly problem specific and to some extent depends on the number and the quality of training patterns. To be able to design a stable MLP network, it would be more appropriate to carry out a parametric study by changing the number of neurons in the hidden layer in order to test the stability of the network. In this study, the parametric study (varied number of hidden layer, neuron in hidden layer) had been done to obtain the appropriate parameters for each numerical example cases i.e., duffing (hardening type), hysteretic dynamical systems.

## 3.5 Numerical Time-Domain Analysis of Discrete-ANNs Systems

By a discrete-ANNs system it is meant that the system is represented by a hybrid discrete and ANNs-based model. The discrete models are built up using, for examples, the finite element method. To demonstrate the computational procedure. the Newmark method is introduced as the numerical integration scheme. This is because the Newmark method is widely recognized and utilized in engineering mechanics. The equation of motion for a discrete-ANNs system with the application of the Newmark method can be written as, based on the notation used in (Bathe and Wilson 1976),

$$m\ddot{u}_{t+\Delta t} + f_{ANN,t+\Delta t} = P_{ext,t+\Delta t}, \tag{3.13}$$

where $m$ = mass, $f_{ANN}$ = restoring force of unknown subsystems, and $P_{ext}$ = external excitation. $f_{\mathrm{ANN}}$ is corresponding to an ANNs-based model of each unknown subsystem.

To compute the responses in the next time step, the following assumptions are imposed on the Newmark method

$$\dot{u}_{t+\Delta t} = \dot{u}_t + 0.5\left(\ddot{u}_t + \ddot{u}_{t+\Delta t}\right)\Delta t \tag{3.14a}$$

$$u_{t+\Delta t} = u_t + \dot{u}_t\Delta t + 0.25\left(\ddot{u}_t + \ddot{u}_{t+\Delta t}\right)\Delta t^2. \tag{3.14b}$$

Since the Newmark method is an implicit integration scheme, it requires that iteration be performed at each time instant for solving non-linear systems. Using the modified Newton-Raphson iteration. Eq. (3.13) becomes

$$m\ddot{u}^{(k)}_{t+\Delta t} + f^{(k-1)}_{ANN,t+\Delta t} = P_{ext,t+\Delta t} \tag{3.15a}$$

with

$$u^{(k)}_{t+\Delta t} = u^{(k-1)}_{t+\Delta t} + \Delta u^{(k)} \tag{3.15b}$$

where $k$ denotes the $k$-th iteration. From the assumption Eq. (3.14a) and Eq. (3.14b),

$$\ddot{u}_{t+\Delta t}^{(k)} = \frac{4}{\Delta t^2}\left(u_{t+\Delta t}^{(k)} - u_t\right) - \frac{4}{\Delta t}\dot{u}_t - \ddot{u}_t \tag{3.16a}$$

$$\dot{u}_{t+\Delta t}^{(k)} = 0.5\Delta t\left(\ddot{u}_{t+\Delta t}^{(k)} + \ddot{u}_t\right) + \dot{u}_t\,. \tag{3.16b}$$

Substituting Eq. (3.16a) into Eq. (3.15a) with the updating scheme Eq. (3.15b) yields

$$m\Delta u^{(k)} = \hat{P}_{ext,t+\Delta t} - f_{ANN,t+\Delta t}^{(k-1)} - \frac{4}{\Delta t^2} m u_{t+\Delta t}^{(k-1)} \tag{3.17}$$

in which

$$\hat{P}_{ext,t+\Delta t} = P_{ext,t+\Delta t} + m\left(\frac{4}{\Delta t^2}u_t + \frac{4}{\Delta t}\dot{u}_t + \ddot{u}_t\right). \tag{3.18}$$

At each time instance $t+\Delta t$, a trial $u$ is assumed first with $k$ equal to 1 and the first updating value $\Delta u^{(1)}$ is solved according to Eq. (3.17). Based on Eq. (3.15b), a new solution is obtained with the use of $\Delta u^{(1)}$. The new solution is then inserted into Eq. (3.17). The procedure is then repeated with the increase in the iteration index $k$. It should be noted that the velocity and acceleration terms have to be updated as well during the iterative procedure, i.e. following Eq. (3.16a) and Eq. (3.16b). The convergence of the solution is reached when the following conditions are satisfied:

$$\frac{\left\| P_{ext,t+\Delta t} - f_{ANN,t+\Delta t}^{(k-1)} - m\ddot{u}_{t+\Delta t}^{(k-1)} \right\|_2}{\left\| P_{ext,t+\Delta t} \right\|_2} \le RTOL \tag{3.19a}$$

$$\frac{\Delta u^{(k)T}\left(P_{ext,t+\Delta t} - f_{ANN,t+\Delta t}^{(k-1)} - m\ddot{u}_{t+\Delta t}^{(k-1)}\right)}{\Delta u^{(k)T}\left(P_{ext,t+\Delta t} - f_{ANN,t} - m\ddot{u}_t\right)} \le ETOL \tag{3.19b}$$

in which $RTOL$ is a force tolerance and $ETOL$ is an energy tolerance, respectively. The obtained solution is then treated as the initial condition for the solution at the following time instance.