CHAPTER 1

INTRODUCTION

1.1 General

Fracture mechanics is the subject that deals with the study of behavior of cracks in materials. By definition, fracture mechanics is a method of characterizing the fracture behavior by structural parameters, which can be used directly by engineers. In the early age, the knowledge of fracture mechanics was concentrated and limited to steel. Later on, a number of researchers attempted to take advantage of fracture mechanics in the cases of concrete and other quasi-brittle materials. However, the behavior of such materials is quite complicated and there are many barriers to overcome.

The early fracture mechanics for steel (*Linear Elastic Fracture Mechanics*, LEFM) cannot be applied to matrix-aggregate composite materials such as concrete because of the presence of nonlinear behavior located in a narrow band in front of the crack tip. By using current technology, components and mechanisms of cracking in concrete can be clarified. The formation of the nonlinear narrow band ahead of the crack tip in concrete, called the *fracture process zone* (FPZ), is illustrated in Fig. 1.1. From the figure, it can be seen that in front of the tip of the macroscopic crack, there are a lot of microcracks. This zone of microcracks is called the microcracking zone. Along the tip portion of the macroscopic crack, forces can still be transferred across the crack surfaces. This part of macroscopic crack is called the bridging zone. The bridging zone is a result of the weak interface between the aggregates and the matrix. In this zone, spurious stresses are transmitted by aggregate interlocking and fiber reinforcement, if any. The microcracking and bridging zones comprise the fracture process zone. The fracture process zone can be modeled as a region of strain softening which means that the material will be softener in bigger crack opening. In such region, the stored strain energy is gradually dissipated.

In order to model the fracture behavior of concrete, it is necessary to take the fracture process zone into account. There are a number of models proposed. However, only two of them are widely used. The first model is the *Fictitious Crack Model*

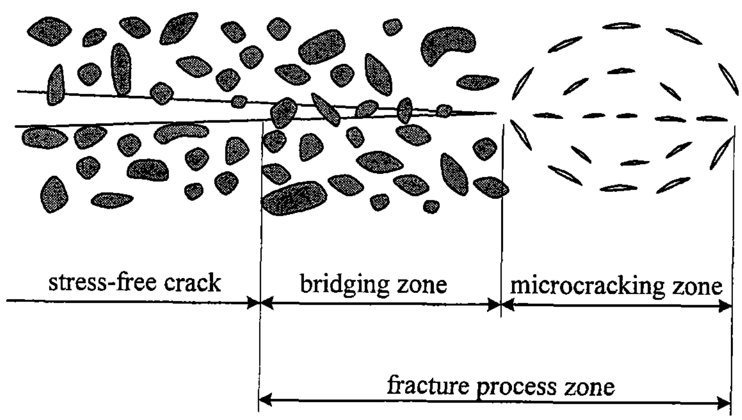


Fig. 1.1 Fracture process zone in concrete

(FCM) proposed by Hillerborg et al. (1976). This model assumes that the material, albeit damaged, can still transfer stress. However, the involved stress decreases as a function of *Crack Opening Displacement* (COD). The other model, the *Crack Band Model* (CBM) (Bazant and Oh 1983), is the fundamental of the *Smeared Crack Model.* It replaces the cracked solid by a continuum and permits a description in terms of stress-strain relationship. This concept fits the nature of the finite element method and its computational algorithm is convenient.

Although these two models give reasonably accurate results for various types of problem, they cannot judge whether the obtained crack patterns are stable or not. Since the two models generally employ a stress criterion for the initiation of cracks, there can be many cracks occurring in the domain. Some of these cracks will, in reality, stop opening and start to unload after some time. Finally, only a few major cracks will prevail. This behavior is called the cracking localization. To illustrate the crack localization in concrete, consider the uniaxial tension test of plain concrete and its typical tensile response shown in Fig. 1.2. After the small degree of nonlinearity caused by microcracking, the material reaches its tensile strength f_t, and then strain softens. Once f_t is reached, subsequent damage is concentrated in a local fracture zone which is the cracking localization. Note that Fig. 1.2 shows a schematic diagram of stress-displacement curve rather than a stress-strain curve. The detail of cracking localization can be described as follows. At the beginning, when concrete resists loading, negligible initiation of internal cracks occurs. Between 30% and 80% of peak load, the internal cracks start to propagate randomly and distribute over the specimen. Before the peak load is reached, those internal cracks start to localize into one or a few major cracks. Finally, after the peak load, the major cracks continuously propagate even after the load decreases. The tensile strain of the material within the localized damage band increases while the other parts of the specimen outside the damage band unload. It can be seen that cracking localization plays a very important role in the prediction of behavior of concrete and other kinds of quasi-brittle material. The insufficient consideration of cracking localization leads to inaccurate prediction of peak and post-peak loads.

To demonstrate the problem that occurs in the fracture analysis of quasi-brittle materials when cracking localization is not considered, a numerical analysis of the uniaxial test shown in Fig. 1.3 is considered. Under the strength criterion in analysis, many cracks will distribute over the specimen as shown in Fig. 1.3a (no localization).

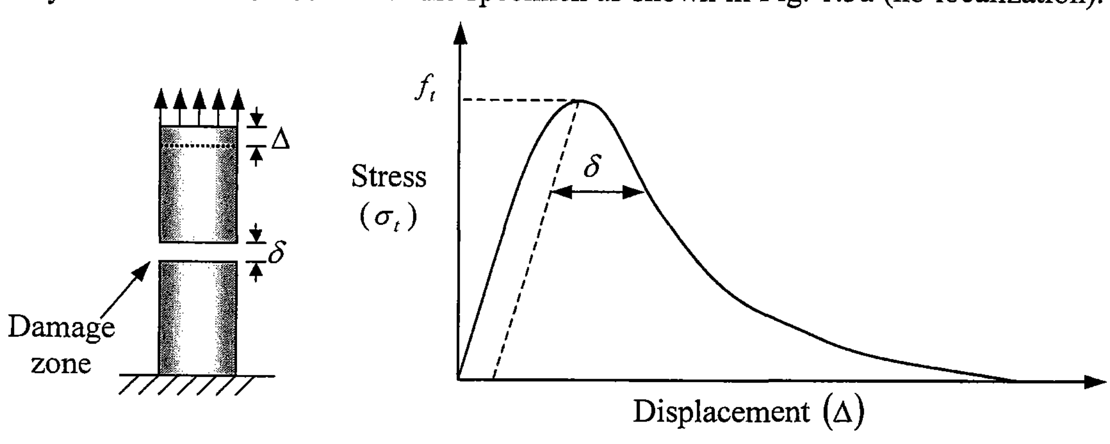
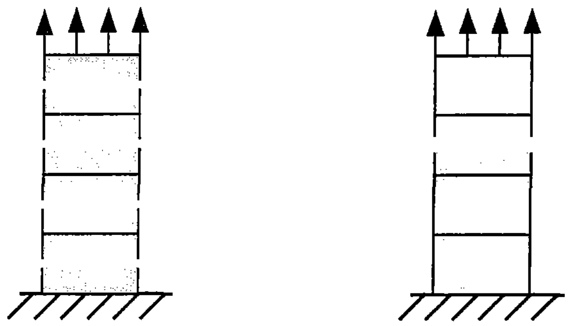


Fig. 1.2 Typical tensile response of concrete



(a) no localization (b) localization

Fig. 1.3 Uniaxial tension test of a concrete specimen

However, this is in contrast to the natural behavior that only one crack will finally open while the others will close as shown in Fig. 1.3b (localization). Since energy is dissipated along the cracks, solutions with and without localization result in different energy dissipation from the domain. Therefore, neglecting the cracking localization in such a problem will obviously result into significant error.

Consideration of cracking localization cannot generally be neglected. However, the consideration of cracking localization needs very expensive computation because solution methods for solving localization problems involve checking stability and bifurcation of many different equilibrium paths. Consequently, many researchers unfortunately avoid the consideration of cracking localization by either allowing many cracks to grow without the consideration of localization or by assuming positions for localized cracks. The first approach is not realistic and can generally lead to inaccurate results. Having many cracks without localization allows incorrect amount of energy to dissipate from the domain. Thus, obtained results will also be inaccurate. However, in some cases where the stress gradients of the problems are very large, and the stress criteria for crack initiation is used, a localized solution may be obtained by this approach (Rots and de Borst 1987, Rots 1989, and Jirasek and Zimmermann 1998). When the stress gradient is very high, it is possible that the major cracks will finally prevail and other cracks will undergo elastic unloading even when cracking localization is not considered. This is due to big difference in the magnitudes of stresses between different locations (Zienkiewicz, Huang and Pastor 1995). The second approach, which assumes positions of localized cracks prior to analysis, may yield reasonable results in some cases. These include cases where assumed positions of localized cracks are reasonably or undoubtedly correct and cases where solutions are not sensitive to locations of localized cracks (Carpinteri 1989, and Shirai 1994). Nevertheless, the approach is not appropriate for general cases since locations of localized cracks may not be easily predicted or solutions may be sensitive to locations of cracks.

Consideration of stability and bifurcation of equilibrium states is one of the major tasks to be done in the analysis of cracking localization. Many researchers have considered stability and bifurcation of equilibrium states by investigating definiteness of stiffness matrices (Riks 1979, de Borst 1987, and Valente 1992). When a stiffness matrix is positive-definite, an equilibrium state is considered stable. The same theory can be applied to the analysis of cracking localization. However, cracking is an irreversible process. In this case, stability and bifurcation of equilibrium states can be determined by investigating definiteness of stiffness matrices (Hessian matrices) constructed with respect to irreversible parameters (Nguyen 1987). These irreversible parameters can be the crack opening displacements (COD) in the discrete crack approach or the crack strains in the smeared crack approach. For the discrete crack approach, the crack opening displacements are usually discretized along crack paths and treated as the degrees of freedom in the analysis. The energy of the system can be expressed in terms of these degrees of freedom. Computing the first and second variations of the energy with respect to these discrete crack opening displacements can be done easily by employing just the ordinary calculus (Brocca 1997). On the contrary, if the smeared crack approach is employed, the energy of the system will be expressed in terms of the irreversible crack strain variables, which are not discretized variables. These crack strain variables are functions of position. To compute the first and the second variations of energy with respect to these crack strain functions, complex mathematics involving calculus of variations must be employed.

Investigating definiteness of Hessian matrices will provide information on the stability of equilibrium paths. Consequently, bifurcation points can be located. Nevertheless, tracing the actual equilibrium path needs more efforts. Employing Gibbs' statement of the second law of thermodynamics, Nemat-Nasser (1979) pointed out that the equilibrium path that makes the total potential energy an absolute minimum would also render the elastic energy an absolute minimum. In addition, this path will also be the actual equilibrium path (Bazant and Cedolin 1991). Employing the same concept, Brocca (1997) used crack opening displacements in the discrete crack finite element analysis as irreversible parameters in the analysis of cracking localization. In his work, Hessian matrices constructed with respect to irreversible crack opening displacements are used to investigate stability and bifurcation of crack patterns. In addition, the equilibrium path is also traced by using the Simplex method to find the path with the minimum total potential energy. From his work, it is clear that Hessian matrices constructed with respect to irreversible parameters can easily be obtained when the discrete crack approach is employed because irreversible parameters are discrete. Nevertheless, the discrete crack approach is not suitable for problems with many cracks in the domain. Usually, in the cracking localization analysis, there will be many cracks occurring in the domain. As the number of cracks increases, the mesh topology may have to be changed to cope with new crack patterns and this leads to more degrees of freedom. On the other hand, the smeared crack approach, which is more suitable for problems with many cracks, does not provide any discrete irreversible parameters for construction of Hessian matrices. Another disadvantage of the smeared crack approach is that, with this approach, it is necessary to define the crack band width or the characteristic length. For fairly regular meshes, the characteristic length is frequently determined in an intuitive way which is difficult to generalize in a formal manner for irregular meshes and arbitrary crack directions. However, for two-dimensional domains, this problem can be overcome. Oliver (1989) proposed a general approach for calculation of the characteristic length. In his study, a crack is modeled as a limiting case of two singular lines that coincide with the boundary of elements covering the crack path. The expression for the characteristic length is obtained by analyzing the dissipated energy in the band bounded by these two singular lines.

To allow the consideration of cracking localization in the smeared crack model, discrete irreversible variables are introduced. Petcherdchoo et al. (1999) introduced the nodal crack displacement variables in the smeared crack model to consider the cracking localization via these discrete variables. In their method, the crack displacements are defined in such a way that their derivatives with respect to the coordinates represent the crack strains. Reasonable results are observed from this technique. However, there is something to be further developed in this procedure. For example, the crack displacement variables used in their work do not have a very clear physical meaning. Moreover, some kinds of constraint have to be used to prevent the rigid-body crack displacements in all cracked elements. The implementation of these constraints can be quite troublesome in some cases. In order to avoid this drawback, Nanakorn and Soparat (2000) proposed an analysis method using the smeared crack finite element model with a mixed formulation. In their work, the discretization is performed not only on the displacement field but also on the crack strain field. The newly introduced discrete nodal crack strain variables serve as discrete irreversible variables needed for the localization analysis. However, their work is limited to stability analysis of crack patterns, and there is no attempt to trace the complete equilibrium path.

In this study, an analysis method for cracking localization is proposed. In the proposed method, stability of crack patterns is investigated by employing the analysis method proposed by Nanakorn and Soparat (2000). When the current crack pattern becomes unstable, the stable crack pattern with the minimum total potential energy is searched for and selected as the solution path (Nemat-Nasser 1979, and Valente 1992). In the search for the stable crack pattern with the minimum total potential energy, exhaustive search and genetic algorithms are used. The proposed analysis method is used to solve cracking localization problems in plain concrete and steel-fiber-reinforced concrete and the obtained results are discussed.

1.2 Objectives

1. To establish an analysis method based on the smeared crack approach for obtaining the actual equilibrium path in the cracking localization analysis of quasi-brittle materials.

2. To investigate the importance of the cracking localization analysis of quasi-brittle materials by using the proposed analysis method.

1.3 Scope of Study

1. Only quasi-brittle materials are considered.

2. Only Two-dimensional problems are considered.

3. Nonlinear material behavior in compression is not considered.