CHAPTER 2

LITERATURE REVIEW

2.1 Fracture Mechanics of Quasi-Brittle Materials

It is commonly known that the linear elastic fracture mechanics (LEFM) cannot be applied to concrete and other kinds of quasi-brittle material because of some unique fracture characteristics of these materials. One of these unique fracture characteristics is the presence of a narrow nonlinear band in front of the crack tip (Shah and McGarry 1971). This nonlinear band is called the fracture process zone and it is composed of microcracking and bridging zones. The microcracking zone is the area at the tip of the crack which the initiation and the expansion of microcracks are the major mechanism. The bridging zone is a part of crack across which stress can transfer.

Many models of the fracture process zone have been proposed. Based on different energy dissipation mechanisms, these models can be divided into two categories, i.e., effective crack approach and fictitious crack approach. In the effective crack approach, the fracture process zone is modeled by using an equivalent traction-free elastic crack and governed by the criterion from LEFM. The fictitious crack approach can be further categorized into two different models, i.e., fictitious crack model and crack band model. The fictitious crack model is proposed by Hillerborg et al. (1976). In this model, the fracture process zone is considered as a virtual crack across which stress can transfer. The ability of stress transferation will decrease when the crack opens wider and it will finally vanish. At that point, the virtual crack changes into a real crack. The relationship between crack width and transferred stress across the crack can be treated as one of material properties. It is usually called the *Tension-Softening Relationship* (Anderson 1995). The crack band model is proposed by Bazant and Oh (1983). The boundary of the fracture process zone is defined as the boundary of the strain-softening region. In this region, the stress decreases when strain increases and the constitutive law is the relation of crack strain and tensile stress. This model requires one more parameter, i.e., the width of the crack band which is used to define the size the strain-softening region.

2.2 Finite Element Method

The finite element method (FEM) is a well-known numerical technique for solving mathematical engineering problems. According to the basic concept of the finite element method, the interested domain is divided into small elements connected by nodes. Normally, elements are connected to other adjacent elements at nodes which are located at the corners and sometimes also at sides of each element. At these connections, the condition of equilibrium of tractions and compatibility of deformation are satisfied. In the field of fracture mechanics, there are two major approaches of the finite element method that are widely used. These two approaches are the discrete crack approach and the smeared crack approach.

2.2.1 Discrete crack approach

The discrete crack model was proposed by Ngo and Scordelis (1967). This approach models the crack as a real displacement discontinuity in the displacement field in the domain (Fig. 2.1). It can be done by releasing the element connectivity requirement between elements adjacent to the crack or embedding the displacement discontinuity into the element in which the crack appears. The advantage of modeling the real crack by the displacement discontinuity in the discrete crack model is that the displacement discontinuity is what actually happens at the crack. However, there are some disadvantages of the discrete crack model. For example, the arrangement of the crack path is usually assumed to coincide with the boundaries between elements by remeshing and changing the node connectivity which consume long computational time. Embedding the displacement discontinuity into the element also involves creating new shape functions for special displacement interpolations. This can be quite troublesome. As a result, the discrete crack approach may not be very suitable for problems with many simultaneous cracks.

Nanakorn and Horii (1995) developed a new element into which the displacement discontinuity is embedded by adjusting the interpolation functions of the element. In this case, the arrangement of the crack path can cut through the elements. The interpolations function is obtained from the consideration of the relationship between displacement induced by the rigid translations and the rigid rotation between two cracks surface separated by the discontinuity.

2.2.2 Smeared crack approach

In this approach, cracks in an element are replaced by a continuous medium with degraded material properties. The displacement discontinuity in the discrete crack approach is distributed. As a result, the displacement is continuous over the

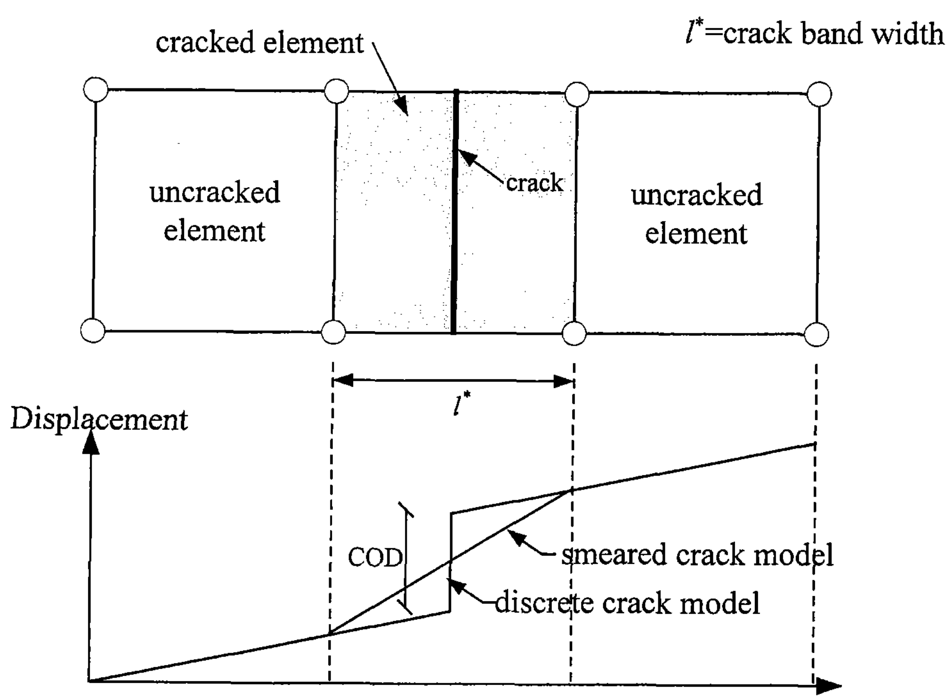

Fig. 2.1 Smeared and discrete crack finite element concepts

degraded medium or damaged area in the element as shown in Fig. 2.1. The weak point of the smeared crack approach is that the size of damaged area affects the accuracy of result. However, there are many methods developed to solve this problem (Bazant and Oh 1983, and Oliver 1989).

When Bazant and Oh (1983) introduced the single fixed crack approach to the smeared crack finite element analysis for concrete, the crack is modeled by changing the element stiffness matrix instead of changing topological connectivity of the mesh. The concept of this model is obvious that the orientation of the crack plane is fixed to be the direction of the first crack in the element defined on the principal tensile plane even though the principal stress might rotate during the analysis. The problems of the single fixed crack approach occur when stresses rotate due to change in load pattern or shift in the principal stress directions. Therefore, the multidirectional fixed crack approach was developed. This approach is based on the idea that one element can accommodate more than one fixed crack and any single crack holds the same assumptions as the single fixed crack approach (de Borst and Nauta 1985). The material behavior is found from coupling multiple planes, each represents a different crack orientation. After the first crack initiates, the following crack plane initializations are described on the trespassing of a threshold value intensity and/or orientation.

Although the smeared crack model has proven to be attractive when used in the finite element fracture analysis, when the crack orientation does not coincide with the mesh of finite elements, the stress locking occurs. This condition is often found in the smeared crack model. Rots (1989) indicated that when stress locking occurs, the result will be stiffer than the actual result. He also stated that the coaxial rotating crack concept and the fixed crack concept which omit shear retention (zero mode II crack stiffness) provide the results that suffer least from stress locking. Jirasek and Zimmermann (1998) combined the standard rotating crack with scalar damage concept. The combination of two models keeps the anisotropic character of the rotating crack but does not transfer spurious stress across widely open crack.

The other problem of the smeared crack concept is the characteristic length or crack band width. Whenever the finite element mesh becomes irregular mesh, the characteristic length becomes more complicated. Oliver (1998) proposed a general approach for calculation of the characteristic length. For two-dimensional domain, the idea of his work is that the crack is modeled as a limiting case of two singular lines that coincide with the boundary of elements covering the crack path. Across two lines, the displacement is continuous while the displacement gradient is not. The expression for the characteristic length is obtained by analyzing the dissipated energy in the band bounded by these two singular lines. Finally, the mathematical expression for the characteristic length, which depends on mesh size, crack direction and spatial position can be deduced.

2.3 Localizations and Bifurcations

In consideration of the post-peak behavior of quasi-brittle materials like concrete, the solution may not always be unique. A bifurcation of the equilibrated solution occurs. Therefore, seeking only the equilibrated solution is not enough. The analysis of stability of the possible solution in order to select the stable one is needed. It turns out that the stable solution is the localized one. In other words, the cracking localization occurs when the crack pattern becomes unstable and the localized cracks prevail. After that, the localized cracks continue to open while all other cracks unload.

Riks (1979) used the arc length of the equilibrium path to check the stability of the equilibrium solutions. The stability of the solution is judged by the sign of the second derivative of the potential energy with respect to the arc length of the equilibrium path. If the sign of derivative is positive, the stability of the equilibrium solution is ensured. Otherwise, the instability occurs.

Nemat-Nasser et al. (1980) showed that the interacting tension cracks, associated with small imperfections, can substantially reduce the level of straining at which a crack growth pattern became unstable. He used the minimum elastic energy theory to select the actual equilibrium path from a fan of bifurcation paths. He stated that the bifurcation paths with smaller total stored elastic energy, hence, are defined more stable states than the corresponding ones on the equilibrium path.

de Borst (1987) studied the post-peak bifurcation and post-failure behavior of strain-softening solids. He proposed a numerical approach to detect the bifurcation point and trace post-bifurcation behavior of the solid. It is a combination of an incremental loading procedure and eigenvalue analysis of the tangent stiffness matrix. The bifurcation points occurring in the solid can be located by eigenvalue analysis and appear whenever the lowest eigenvalue of the tangent stiffness matrix becomes slightly negative.

Nguyen (1987) proposed the mathematical framework for the bifurcation and post-bifurcation analysis. In his work, he defines reversible variables as a function of irreversible variables. The stability of the solution is determined from the sign of the second derivative of the potential energy with respect to the irreversible variables.

Valente (1992) extended the cohesive crack models to enable the possible bifurcations of the equilibrium path to be detected and analyzed by considering the positive definiteness of the stiffness of the domain or the eigenvalue of the effective stiffness. In his work, when there are more than one stable equilibrium paths, he selected the actual equilibrium path by using the minimum stored elastic energy theory that was stated in Nemat-Nasser et al. (1980).

2.4 Genetic Algorithms

Holland (1975) developed the genetic algorithms to design the artificial system software that retains the important mechanisms of natural systems and also abstracts and rigorously explains the adaptive processes of natural systems.

Since Holland's seminal work in 1975, there have been a growing number of applications of genetic algorithms to variety of optimization and learning tasks. Holland's theorem of schema describes the actual behavior of genetic algorithms. In the same year, De Jong (1975) compared genetic algorithms with the best gradient techniques. He devised five test functions and proposed two methods for assessing the performance. He called these two measures on-line (ongoing) and off-line (convergence) performance, respectively. Simply stated, the on-line performance is the average performance of all tested strings over the courses of the search and off-line performance is the average of the best performances. He concluded that higher population size, higher crossover rate and lower mutation rate generally yield good results.

Grefenstette and Baker (1989) suggested a more robust approach when they used a genetic algorithm to address a control parameter problem by treating it as a meta-problem. The idea of their work is that the appropriate value of control parameters of GAs are searched by using GAs themselves. The problem they used is based on a set of De Jong's test functions. He recommended a smaller population size and much higher rate of applying the genetic operators than De Jong proposed.

After Grefenstette's and Baker's work, many efforts have been continued to find either an optimal set of control parameters or better searching operations for genetic algorithms applied to various engineering optimization problems. Schaffer et al. (1989) examined all available methods of crossover by two types of biases: the positional bias and the distribution bias. All of them drew the same conclusion that the traditional one point crossover is the least efficient method than other methods of crossover.