CHAPTER 6

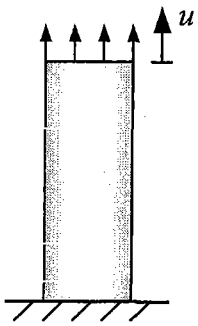
EQUILIBRIUM PATH WITH THE MINIMUM TOTAL POTENTIAL ENERGY

When the equilibrium path reaches a bifurcation point, a fan of many possible equilibrium paths emanates from the bifurcation point. Among these equilibrium paths, the actual equilibrium path is the path that contains the minimum total potential energy (Bazant and Cedolin 1991) as well as the minimum elastic strain energy (Nemat-Nasser 1979). These two conditions are actually the same (Nemat-Nasser 1979). In this study, the minimum total potential energy criterion is employed in the selection of the actual equilibrium path. However, since the analysis is performed incrementally and the total potential energy is written in the incremental form [see Eq. (5.1)], the stable path with the minimum total potential energy increment is the desired solution path.

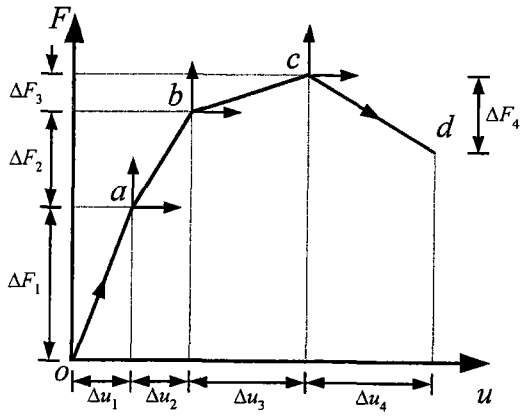
In order to obtain the solution path with the minimum total potential energy increment, energy increments of all possible equilibrium paths, which depend on their crack patterns, can be compared. This approach of comparing all possible solutions is essentially an exhaustive search. The algorithm for this search approach is simple and straightforward. Nevertheless, it is obvious that the technique is expensive and suitable only for small problems where the complete search is still possible. In the case of larger problems where many cracks occur in the domain and, as a result, many crack patterns are possible, the exhaustive search may not be practical and it is advisable to employ an appropriate optimization technique to find the minimum energy path. In this study, a genetic algorithm (GA) (GoldBerg 1989) is used for this purpose because this optimization technique is suitable for problems with discrete variables. Variables in the minimization problem of the total potential energy increment are discrete statuses of cracks that can be either opening or unloading. Since GAs do not require the evaluation of the gradient of the function being minimized or maximized, the evaluation of the total potential energy increment is enough for the minimization process.

6.1 Minimization of Total Potential Energy Increment

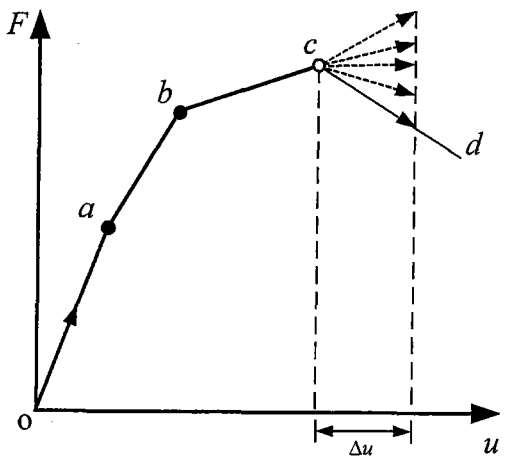
The analysis of cracking localization in this study is in the form of piecewise-linear incremental steps. Each step is ended when a new cracked element is initiated by the stress criterion or when the incremental crack constitutive law of one of the existing cracked elements needs to be updated. The incremental constitutive law of a crack needs to be updated when the slope of its tension-softening curve changes or when the crack switches from loading to unloading or vice versa. During each incremental step, the behavior of the system is actually linear. As an example, Fig. 6.1 schematically illustrates a uniaxial tension test and its load-deformation curves with the bifurcation paths. At the end of the analysis in the first step (point *a*), cracks are initiated. Next, the analysis is continued to the second step (path *ab*), the system stiffness is updated due to the initiation of new cracks. Thereafter, the rest of analysis



(a) Uniaxial tension test



(b) Load-deformation curve



(c) Equilibrium and bifurcation paths

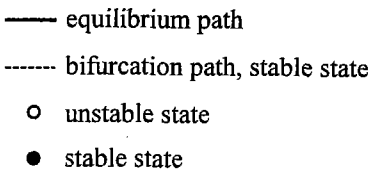


Fig. 6.1 Uniaxial tension test and its load-deformation curve

will continue in this manner to complete the whole response of the system (see Fig. 6.1b).

In Fig. 6.1c, it is assumed that at point *c* the bifurcation occurs. The foregoing discussion of the minimum total potential energy increment can be instructively derived from the diagram in the figure. At the bifurcation point *c*, it is further assumed that there are five bifurcation paths. The total potential energy increment of all possible bifurcation paths is then compared in order to obtain the path with the minimum total potential energy increment which is the actual equilibrium path. In order to compare the energy of the bifurcation paths, all paths must be done under the same controlled parameter. For this problem, the controlled displacement increment Δu is considered as the controlled parameter. For small problems, the exhaustive search can be used but, for large problems, the genetic algorithm will be more suitable. Since the optimization problem being solved here is the minimization of the total potential energy increment, the objective function $f(\mathbf{x})$ for the GA is the total potential energy increment itself, i.e.,

$$f(\mathbf{x}) = \Delta\Pi = \frac{1}{2}\int \Delta\boldsymbol{\varepsilon}^T \overline{\mathbf{D}} \Delta\boldsymbol{\varepsilon} dV \tag{6.1}$$

where $\Delta\boldsymbol{\varepsilon}$ is the total strain increment and $\overline{\mathbf{D}}$ is the constitutive matrix of the smeared material. Here, $\mathbf{x}$ is the variable representing crack patterns. Note that there is no potential terms from the external forces because loading by the controlled displacement is employed.

6.1.1 Coding and fitness

In general, GAs do not directly work with the parameters themselves. The algorithms start with coding of the parameter set. For coding, binary strings are most popular and convenient. Each point in a search space, often called "individual" in the GA terminology, is represented by a single string of number 0's and 1's. The optimization problem of this study is to minimize the total potential energy increment. The total potential energy increment to be minimized is a function of crack patterns. Therefore, each crack pattern will be coded as a binary string. The idea of the coding is to have each bit in a binary string represent the status of one particular crack. If the value of the bit is one (1), it indicates that its corresponding crack is opening. If the value of the bit is zero (0), the corresponding crack is unloading. The objective function is set as $f(\mathbf{x})$ where $x_i \in \{0,1\}$ which represents a status of the particular crack. Fig. 6.2 shows examples of the coding of two different crack patterns. The number of bits used in the string is equal to the number of the existing crack paths.

Since genetic algorithms mimic the survival of the fittest principle of nature. They are naturally suitable for solving maximization problems. Therefore the minimization problems have to be transformed into maximization problems by using a suitable transformation equation Eq. (3.26).

In fact, the objective, which is the total potential energy increment might become negative due to the dissipation of energy of system that usually occurs in the post-peak region. The fitness transformation function Eq. (3.26) is invalid. Nevertheless, the fitness transformation function can be modified by adjusting the constants in numerator and denominator to handle the negative objective values. For example, the minimum of total potential energy increment can be predicted and observed that it will not be less than –1,000 (unit is arbitrary). The modified fitness function is written as

$$F(\mathbf{x}) = \frac{1000}{(1000 + \Delta\Pi)}. \tag{6.2}$$

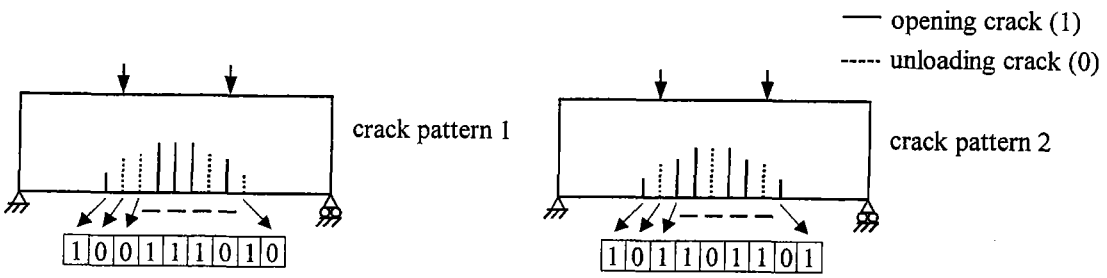


Fig. 6.2 Examples of coding of crack patterns

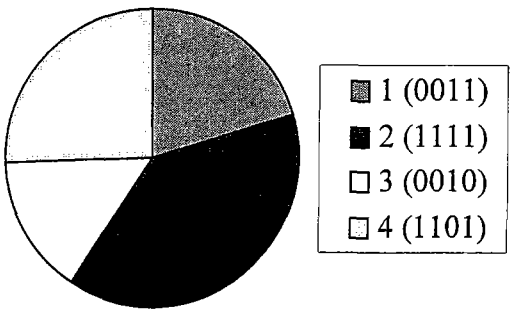
After the fitness value of each crack pattern is obtained, the genetic algorithm's operators, i.e., reproduction, crossover and mutation operators, will be carried out accordingly.

6.1.2 Genetic algorithm's operators

In this study, the simple GA is employed. It is composed of three different operators, i.e., reproduction, crossover and mutation operators. These three operators are based on the same basic elements in the real natural genetics. The details of this technique can be found in the literatures (Goldberg 1989, and Deb 1995).

In GAs, the reproduction operator defines a process in which individuals are selected for mating based on their fitness values relative to that of the population. Fitness is defined as a figure of merit. Individuals with higher fitness values have higher probabilities of being selected for mating and subsequent genetic actions. Consequently, highly fit individuals live and reproduce, and less fit individuals die. According to the objective of this optimization, a crack pattern that results in a smaller total potential energy increment will be given a higher fitness value.

By following Deb (1995), reproduction operator is based on the proportional selection which can be illustrated as a roulette wheel. Fig. 6.3 shows the roulette wheel for four individuals having different assumed total potential energy increment. Since the second crack pattern has the lower total potential energy increment than the other, it is expected that the roulette wheel selection will choose the second crack pattern more than any other crack pattern. As a result, good individuals in population are probabilistically assigned a large number of copies and a mating pool is formed. It is important to note that no new strings are created in the reproduction phase but they are created in the crossover and the mutation phases.



Individual (crack pattern)	$\Delta\Pi$	Fitness	Probability	Cumulative
1 (0011)	95.97	0.010	0.204	0.204
2 (1111)	50.48	0.019	0.385	0.589
3 (0010)	130.16	0.008	0.151	0.740
4 (1101)	75.31	0.013	0.260	1.000
Summation		0.050	1.000	

Fig. 6.3 A roulette wheel marked for four individuals according to their $\Delta\Pi$

In the crossover operator, new strings are created by exchanging information among strings. Many crossover operators exist in the literature (Goldberg 1989). Generally, two strings are selected at random as a crossover pair and some portions of the two strings are exchanged. The two strings participating in the crossover are known as parent strings and the resulting strings are known as children strings. In this study, three types of crossover operator are employed, i.e., one-point, two-point and uniform crossover operators. Fig. 6.4 shows an example of the one-point crossover. In

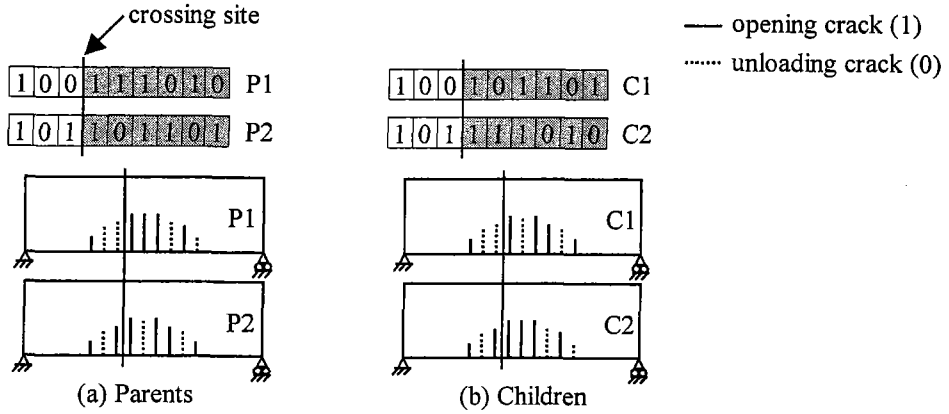


Fig. 6.4 One-point crossover

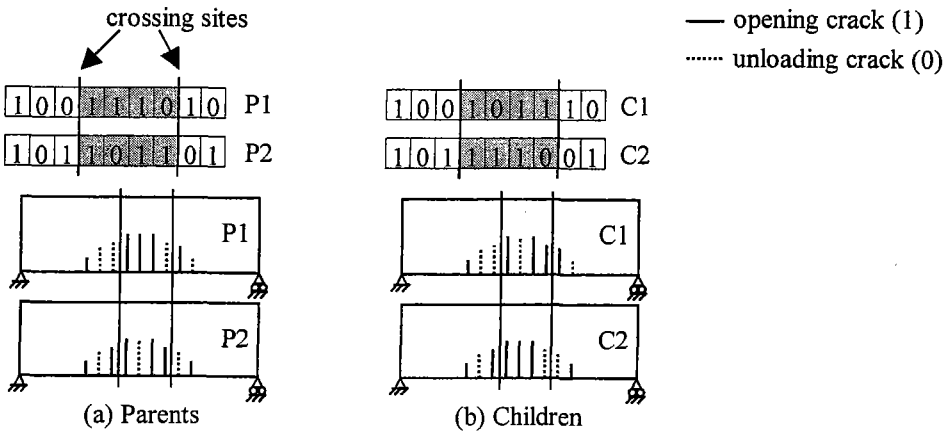


Fig. 6.5 Two-point crossover

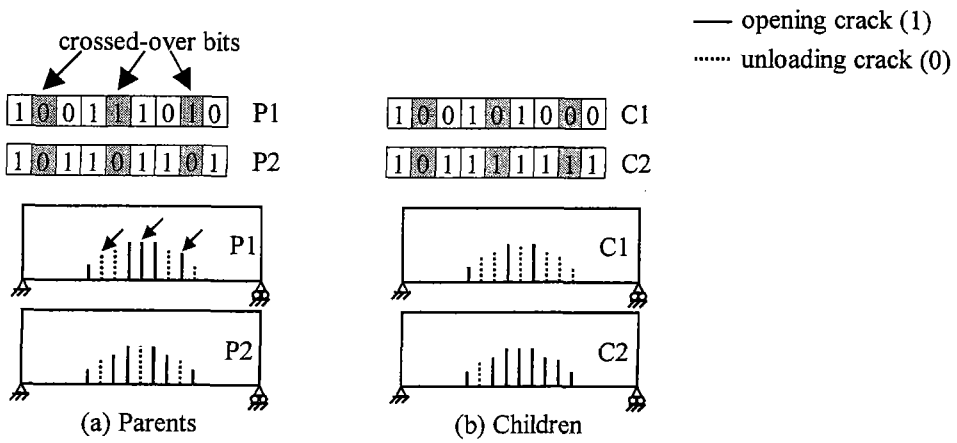


Fig. 6.6 Uniform crossover

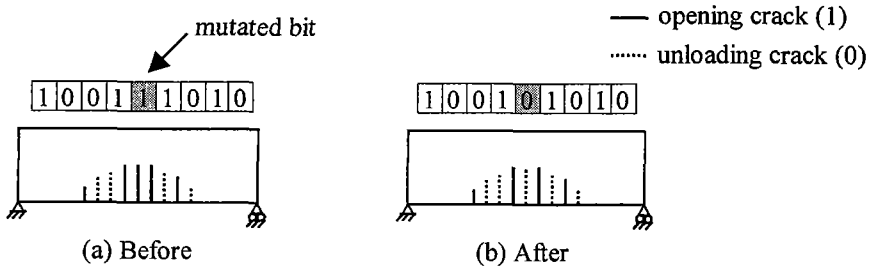


Fig. 6.7 Mutation crossover

this study, the one-point crossover is performed by randomly selecting a crossing site along the parent strings and by exchanging all bits on the right hand side of the selected crossing site. Finally, the new crack patterns are obtained. Note that this crossover is the simplest and the most effective type among all crossover operators.

In the case of the two-point crossover, two crossing sites are randomly selected and all the bits between the two crossing sites of the two parent strings are exchanged as shown in Fig. 6.5. For the uniform crossover, the number of bits to be crossed over and their positions are randomly determined. Fig. 6.6 shows an example of this type of crossover in this study. Since there are three types of crossover using in the minimization of total potential energy increment, the every pair of the parent will be randomly selected the crossover type which is equally probability for each types of crossover. It is clear that the crossover operator may yield better or worse children strings. To be able to adjust the degree of the uncertainty of the crossover phase, it is not necessary to use all individuals in the mating pool in the operator. This is done by adjusting the probability that a crossover operator is performed (crossover probability).

The last genetic algorithm operator is the mutation operator. Fig. 6.7 shows an example of the mutation operator employed in this study. The mutation operator changes 1 to 0 and vice versa at a randomly chosen bit. The operator is used sparingly with a small probability (mutation probability).

6.2 Analysis Procedure

This analysis of cracking localization is developed by implementing the mixed finite element formulation and the minimization of total potential energy increment into the finite element analysis program, FEAP** written and developed by Assoc. Prof. Pruettha Nanakorn. The input of this program consists of the geometry of the specimen, thickness, Young's modulus, Poison's ratio, tension-softening parameters and other material properties (if necessary). The tension-softening diagram may be linear, bilinear, or other piecewise linear diagrams.

In the analysis, the specimen under consideration is analyzed by using the conventional smeared crack model. Cracks are initiated when the maximum tensile stress reaches the tensile strength of the material. After that, the cracks follow the tension-softening curve, which is treated as one of the material properties. The tension-softening curve is the relationship between the tensile stress transferred across

the crack surfaces and the crack opening displacement. Note that, in this study, shear retention of cracks is assumed negligible. As mentioned earlier, the analysis is done incrementally. In each step, the stability of the obtained crack pattern will be investigated by performing eigenvalue analysis of the matrix $\mathbf{K}^{cr}$ obtained from the mixed smeared crack finite element formulation (Nanakorn and Soparat 2000). If the crack pattern is found to be stable, the analysis is continued to the next step. However, if the crack pattern is unstable, the search for the crack pattern with the minimum total potential energy increment must be performed. Here, if the number of possible crack patterns is not very large, an exhaustive search can be employed; otherwise, a GA will be used, instead. It must be noted that, if a GA or another optimization technique is used, the obtained crack pattern may have a near-minimum total potential energy increment, not the true minimum one for the finite element discretization being currently used. In order to compare total potential energy increments of different crack patterns, the energy for cases with different crack patterns must be evaluated under the same controlled parameter. In this study, the controlled displacement is used. After the crack pattern with the minimum or near-minimum total potential energy increment is obtained, the analysis is carried on to the next step. The same process is then repeated and the actual equilibrium path can be traced.