CHAPTER 8

CONCLUSIONS

In this study, an analysis method for cracking localization in quasi-brittle materials is presented. The proposed analysis method is an incremental analysis method and it is composed of two key processes. The first key process involves locating bifurcation points and the second one involves tracing the actual equilibrium path at any bifurcation point. For the first process, the analysis method employs the smeared crack finite elements with a mixed formulation for the stability investigation of crack patterns. In the mixed finite element formulation employed, the discretization is performed on not only the displacement field but also on the crack strain field. The reason why the smeared crack finite element analysis with the mixed formulation is selected is that the smeared crack approach is suitable for problems with many cracks, compared with the discrete crack approach. However, the stability analysis of crack patterns cannot be performed easily with the conventional smeared crack finite element analysis since the irreversible parameter in the model—the crack strain—is not discrete, but continuous. This drawback can be avoided by discretizing the crack strain variable in the mixed formulation. As a result, the formulation will allow the stability analysis of crack patterns to be done easily even when the smeared crack approach is used. By using the stability analysis of crack patterns, it is possible to identify when a current crack pattern becomes unstable. When that happens, it means that a bifurcation point is reached. At the bifurcation point, a fan of many equilibrium paths can be observed. Each equilibrium path represents an equilibrium path for each different crack pattern.

The second key process traces the actual equilibrium path from a bifurcation point incrementally by finding the path with the minimum total potential energy increment. The search for the minimum total potential energy increment is done by employing both the exhaustive and GA search algorithms, depending on the size of the problem being solved. If the size of the problem is small, the exhaustive search, which directly compares the total potential energy increments of all possible crack patterns, is still possible. In this case, the crack pattern with the minimum total potential energy increment obtained from the search is numerically exact with respect to the discretization being used. However, when the size of the problem becomes large and there are subsequently many possible crack patterns to be investigated, performing the exhaustive search become impossible and the GA is employed instead. From the obtained results in this study, it is found that GAs can be efficiently used for this search.

The major problems used to invalidate the proposed analysis method in this study are the four-point bending problems of plain concrete and steel-fiber-reinforced concrete. The results obtained from the four-point bending problem of plain concrete clearly show that the true localized solutions are very much different from the solution obtained by assuming one localized crack at the center of the span. Furthermore, the true localized solutions are also very much different from the solution obtained without the localization consideration. It is also found that there are two major localized cracks that are not localized into one crack until at a much later loading stage. The behavior of the beam is therefore governed by these two cracks.

This clearly illustrates that assuming only one localized crack from the beginning may lead to erroneous results. Finally, it is found that, for the four-point bending test of plain concrete, neglecting the self-weight does not have significant effect on the obtained results. With self-weight or without self-weight, there are two main localized cracks. Although these two cracks are slightly closer when the self-weight is considered, the difference between the obtained responses from both cases are negligible. The results for the steel-fiber-reinforced concrete beam also exhibit large difference between the solution with the localization consideration and the solution that assumes one localized crack at the center of the span from the beginning. Similar to the case of plain concrete, it is also found that there are two major localized cracks. These two cracks are expected to finally localize into one crack at a later loading stage. However, since the nonlinear behavior of the material in compression is not considered in this study, the analysis is stopped before the localization into one crack happens.

In this study, the displacement solutions are obtained by using the conventional smeared crack finite element analysis where the primary unknowns are the nodal displacements. The mixed finite element formulation is used only for the stability analysis.