CHAPTER 3

DESIGN OF TUNED MASS DAMPER (TMD) AND MULTIPLE TUNED MASS DAMPERS (MTMD)

3.1 Tuned Mass Damper

3.1.1 Mathematical model and equation of motion

The TMD consists of a relatively small vibration system (mass, spring, and dashpot) as shown in Fig. 3.1, attached to a structure whose vibrations is designed to mitigate.

The energy of the system of first mode structure with TMD is shown [Mcnamara 1977] as

$$T_1 = \frac{1}{2}\left(m_s \dot{y}_s^2 + m_1 \dot{y}_1^2\right) \tag{3.1}$$

$$V_1 = \tfrac{1}{2}\left[k_s y_s^2 + k_1 (y_1 - y_s)^2\right] \tag{3.2}$$

$$D_1 = \frac{1}{2}\left[c_s \dot{y}_s^2 + c_1 (\dot{y}_1 - \dot{y}_s)^2\right] \tag{3.3}$$

where T_1, V_1, D_1 are the kinetic, potential, and dissipation energy, respectively, implying Lagrange equations to formulate the equation of motion

$$\frac{d}{dt}\left(\frac{\partial T_1}{\partial \dot{q}_i}\right) - \frac{\partial T_1}{\partial q_i} + \frac{\partial D_1}{\partial \dot{q}_i} + \frac{\partial V_1}{\partial q_i} = Q_i \quad ,i = 1,\ldots,n \tag{3.4}$$

where Q_i are the generalized external forcing functions corresponding to the generalized coordinates q_i.

Substituting and carrying out the differentiation:

$$\begin{aligned} &m_s \ddot{x}_s + c_s \dot{x}_s - c_1 \dot{x}_1 + k_s x_s - k_1 x_1 = F_s(t) \\ &m_1(\ddot{x}_1 + \ddot{x}_s) + c_1 \dot{x}_1 + k_1 x_1 = 0 \end{aligned} \tag{3.5}$$

The metric can be formulate as

$$\begin{bmatrix} m_s + m_1 & m_1 \\ m_1 & m_1 \end{bmatrix} \begin{bmatrix} \ddot{x}_s \\ \ddot{x}_1 \end{bmatrix} + \begin{bmatrix} c_s & 0 \\ 0 & c_1 \end{bmatrix} \begin{bmatrix} \dot{x}_s \\ \dot{x}_1 \end{bmatrix} + \begin{bmatrix} k_{s1} & 0 \\ 0 & k_1 \end{bmatrix} \begin{bmatrix} x_s \\ x_1 \end{bmatrix} = \begin{Bmatrix} k_s f(t) \\ 0 \end{Bmatrix} \tag{3.6}$$

where

$$
\begin{aligned}
c_s &= 2m_s\xi_s\omega_s, k_s = \omega_s^2 m_s \\
c_1 &= 2m_1\xi_1\omega_1, \quad k_1 = \omega_1^2\ m_1 \\
x_s &= y_s, x_1 = y_1 - y_s
\end{aligned}
\tag{3.7}
$$

where m_s, m_1 are first modal mass and TMD mass; k_s, k_1 are first modal stiffness and TMD stiffness; c_s, c_1 are first modal damping coefficient and TMD damping coefficient; ω_s, ω_1 are first modal natural frequency and TMD natural frequency; ξ_s, ξ_1 are first modal critical damping ratio and TMD critical damping ratio; y_s, y_1 are real first mode displacement and real TMD displacement; x_s, x_1 are relative first mode displacement and relative TMD displacement

Using the notation that the first mode structural force $F_s(t) = k_s f(t)$. $f(t) = e^{i\omega t}$ is assumed to calculate the complex frequency response H_{xs}. The equation can be rearranged as

$$
\begin{aligned}
&\ddot{x}_s + 2\xi_s\omega_s\dot{x}_s - 2\mu\xi_1\omega_1\dot{x}_1 + \omega_s^2 x_s - \mu\omega_1^2\ x_1 = \omega_s^2 f(t) \\
&\ddot{x}_1 + \ddot{x}_s + 2\xi_1\omega_1 x_1 + \omega_1^2 x_1 = 0
\end{aligned}
\tag{3.8}
$$

where mass ratio $\mu = m_1 / m_s$

3.1.2 Complex frequency response

Let x in terms of the complex frequency response is

$$
\begin{aligned}
x_s(t) &= H_{xs}(\omega)e^{i\omega t} \\
x_1(t) &= H_{x1}(\omega)e^{i\omega t}
\end{aligned}
\tag{3.9}
$$

Substitute these two parameters into the equation (3.8) then we get

$$
\begin{bmatrix} \omega_s^2 - \omega^2 + 2\xi_s\omega_s(i\omega) & -\mu\left(\omega_1^2 + 2\xi_1\omega_1(i\omega)\right) \\ -\omega^2 & \left(\omega_1^2 - \omega^2 + 2\xi_1\omega_1(i\omega)\right) \end{bmatrix} \begin{bmatrix} H_{xs}(\omega) \\ H_{x1}(\omega) \end{bmatrix} = \begin{bmatrix} \omega_s^2 \\ 0 \end{bmatrix}
\tag{3.10}
$$

Symbolical $[\Omega][H_{xi}(\omega)] = [F]$

$H_{xs}(\omega)$ can be solved for using standard metric technique. Thus

$$H_{xs}(\omega)=\frac{\left(-\omega_s^2\omega+2\xi_1\omega_s^2\omega_1 i\omega+\omega_s^2\omega_1^2\right)}{\Delta_\Omega} \tag{3.11}$$

where Δ_Ω is equal to the determinant of Ω and it is equal to

$$\begin{aligned}\Delta_\Omega &= \omega^4 - i\omega^3\left(2\xi_1\omega_1 + 2\xi_s\omega_s + 2\mu\xi_1\omega_1\right)\\ &- \omega^2\left(\omega_1^2 + 4\xi_s\xi_1\omega_s\omega_1 + \omega_s^2 + \mu\omega_1^2\right)\\ &+ i\omega\left(2\xi_s\omega_s\omega_1^2 + 2\xi_1\omega_s^2\omega_1\right) + \omega_s^2\omega_1^2\end{aligned} \tag{3.12}$$

3.1.3 Equivalent damping ratio

The mean square displacement response of the first mode structure is given as

$$E\left[H_{xs}^2\right]=E\left[H_{ys}^2\right]=\int_{-\infty}^{\infty}\left|H_{xs}(\omega)\right|^2 S(\omega)d\omega \tag{3.13}$$

where $S(\omega)$ = spectrum density function of input function, $F_s(t)$. For the parametric studies on TMD system, $S(\omega)$ is assumed to be constant from wide band characteristic [Vickery 1966 and Davenport 1967] so Eq. (3.13) can be reduced to

$$E\left[H_{ys}^2\right]=S_o\int_{-\infty}^{\infty}\left|H_{ys}(\omega)\right|^2 d\omega \tag{3.14}$$

Otherwise, the mean square acceleration response of one degree of freedom completely without second degree of freedom structure can be expressed as

$$E\left[H_{ys}^2\right]=\frac{\pi\, S_0\omega_s^2}{2\xi_1} \tag{3.15}$$

The performance of the fundamental mode structure with TMD as if the behavior of single degree of freedom structure can be calculated passing equivalent damping ratio as

$$E\left[H_{ys}^2\right]=\frac{\pi\, S_0\omega_s^2}{2\xi_e} \tag{3.16}$$

so the equivalent damping ratio is

$$\xi_e=\frac{\pi\, S_0\omega_s^2}{2E\left[H_{ys}^2\right]} \tag{3.17}$$

The MATLAB computer program for calculating the $E[H_{ys}^2]$ and ξ_e is shown in Appendix A.

3.1.4 Optimal equivalent damping ratio

Assume minima exist, two optimal parameters of damping ratio and tuning parameter may be determined for a given μ using the conditions

$$\frac{\partial\left(E\left[H_{ys}^2(\omega)\right]\right)}{\partial\xi_1}=0 \tag{3.18}$$

$$\frac{\partial\left(E\left[H_{ys}^2(\omega)\right]\right)}{\partial f_1}=0 \tag{3.19}$$

By neglecting the structural damping ratio to simplify the analysis, optimal damping ratio (ξ_l) and tuned parameter ($f_l=\omega_l/\omega_s$) can be obtained as [Ankireddi and Yang 1996].

$$\xi_{1opt}=\sqrt{\frac{\mu(2\mu+4)}{8(\mu+1)(\mu+2)}} \tag{3.20}$$

$$f_{1opt}=\sqrt{\frac{\mu+2}{2(\mu+2)^2}} \tag{3.21}$$

Alternately, Luft (1979) shows that with negligible errors, the following approximate relation can be used.

$$\xi_{1opt}=\frac{\sqrt{\mu}}{2} \tag{3.22}$$

$$f_{1opt}=\frac{1}{1+\mu} \tag{3.23}$$

$$\xi_{eopt}\approx\frac{\sqrt{\mu}}{4}+0.8\xi_s>\xi_s \tag{3.24}$$

3.1.5 Displacement of TMD mass

In design a TMD system, allowance must be made for the peak displacements (stroke) of TMD mass. Let the displacement of TMD with respective to mass m_s be denoted by $x_1=y_1-y_s$. The complex frequency response function of x_1 can be written as [Mcnamara 1977]

$$H_{x1}(\omega)=\frac{\left(\omega_s^2\omega^2\right)}{\Delta_\Omega} \tag{3.25}$$

Because the maximum roof displacement of the building due to dynamic effect only can be determined by $\Delta_g = C_g - 1$, the peak stroke of TMD can be obtained as

$$\Delta_{TMD} = \Delta_g \left(E\left[H_{x1}^2(\omega)\right] / E\left[H_{ys}^2(\omega)\right] \right)^{1/2} \tag{3.26}$$

where $\left(E\left[H_{x1}^2(\omega)\right] / E\left[H_{ys}^2(\omega)\right] \right)^{1/2}$ is the ratio of the standard deviation of the stroke of TMD and the roof displacement of the building.

3.1.6 Effect of complex frequency response on equivalent damping ratio

The reduction of the complex response of 400-m. height building ($\omega = 0.09$, $\xi_s = 0.01$) with and without TMD ($\mu = 0.01$) by applying optimal ξ_1 and f_1 is shown in Fig. 3.2. By applying TMD, the complex response of controlled system is high reduced. The flatness of the response increases equivalent damping ratio.

3.2 Multiple Tuned Mass Dampers (MTMD)

3.2.1 Mathematical model and equation of motion

The MTMD consists of a relatively small vibration system (a number of masses, springs, and dashpots as shown in Fig. 3.3 attached to a structure whose vibrations is designed to mitigate

The energy of the system of first mode structure with MTMD that shown in Fig. 3.3 as

$$T_2 = \frac{1}{2}\left(m_s \dot{y}_s^2 + m_1 \dot{y}_1^2 + \ldots + m_k \dot{y}_k^2\right) \tag{3.27}$$

$$V_2 = \tfrac{1}{2}\left[k_s y_s^2 + k_1 (y_1 - y_s)^2 + \ldots + k_k (y_k - y_s)^2\right. \tag{3.28}$$

$$D_2 = \frac{1}{2}\left[c_s \dot{y}_s^2 + c_1 (\dot{y}_1 - \dot{y}_s)^2 + \ldots + c_k (\dot{y}_k - \dot{y}_s)^2\right] \tag{3.29}$$

where T_2, V_2, D_2 are the kinetic potential, and dissipation energy, respectively, implying Lagrange equations to formulate the equation of motion (Eq. (3.4)). Substituting and carrying out the differentiation:

$$M\ddot{Y} + C\dot{Y} + kY = F_0(t) \tag{3.30}$$

Where

$$Y = \left[Y_s \; Y_1 .. Y_k\right]$$

$$M = \begin{bmatrix} m_s & 0 & 0 & 0 & 0 \\ 0 & m_1 & 0 & 0 & 0 \\ 0 & 0 & m_2 & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & m_n \end{bmatrix}, \quad C = \begin{bmatrix} c_s + \sum_{k=1}^{n} c_k & -c_1 & -c_2 & \cdots & c_n \\ -c_1 & c_1 & 0 & \cdots & 0 \\ -c_2 & 0 & c_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -c_n & 0 & 0 & \cdots & c_n \end{bmatrix}$$

$$K = \begin{bmatrix} k_s + \sum_{k=1}^{n} k_k & -k_1 & -k_2 & \cdots & k_n \\ -k_1 & k_1 & 0 & \cdots & 0 \\ -k_2 & 0 & k_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -k_n & 0 & 0 & \cdots & k_n \end{bmatrix}$$

$$F_0(t) = \left[F_s(t) \; 0 \ldots 0\right]$$

(3.31)

where the mass metric, M is composed of the first mode structural mass, m_s and each TMD mass coefficient, m_k. The damping metric, C is composed of the first mode structural damping, c_s and each TMD damping, c_k. The stiffness metric, K is composed of the first mode structural stiffness, k_s and each TMD stiffness, k_k. The absolute response metric Y is composed of the first mode structural response y_s and each TMD response y_k. The force metric $F_0(t)$ is composed of the first mode structural force $F_s(t)$.

3.2.2 Complex frequency response

Let the notation that $F_s(t) = K_s f(t)$. $f(t) = e^{i\omega t}$ is assumed to calculate the complex frequency response H_{ys}. Where y in terms of the complex frequency response is

$$\begin{aligned} y_s(t) &= H_{ys}(\omega) e^{i\omega t} \\ y_k(t) &= H_{yk}(\omega) e^{i\omega t} \end{aligned} \tag{3.32}$$

ξ_e can be obtained by the same principal as TMD case, but $H_{ys}(\omega)$ can be expressed as [Yamaguchi and Harnpornchai 1993]

$$H_{ys}(\omega) = \frac{1}{\mathrm{Re}(z) + \mathrm{Im}(z) i} \tag{3.33}$$

3.2.3 Equivalent damping ratio

The mean square acceleration response of the first mode structure is

$$|H_{ys}(\omega)| = \left[\frac{1}{\mathrm{Re}^2(z) + \mathrm{Im}^2(z)} \right]^{1/2} \tag{3.34}$$

where $\mathrm{Re}(z)$ and $\mathrm{Im}(z)$ are

$$\mathrm{Re}\ (z) = 1 - \left(\frac{\omega}{\omega_s} \right)^2 - \sum_{k=1}^{n} \frac{\mu_k \left(\frac{\omega}{\omega_s} \right)^2 \left[\gamma_k^2 \left\{ \gamma_k^2 - \left(\frac{\omega}{\omega_s} \right)^2 \right\} + \left\{ 2\xi_k \gamma_k \frac{\omega}{\omega_s} \right\}^2 \right]}{\left\{ \gamma_k^2 - \left(\frac{\omega}{\omega_s} \right)^2 \right\}^2 + \left\{ 2\xi_k \gamma_k \frac{\omega}{\omega_s} \right\}^2}$$

and

$$\mathrm{Im}(z) = 2\xi_s \frac{\omega}{\omega_s} + \sum_{k=1}^{n} \frac{2\mu_k \xi_k \gamma_k \left(\frac{\omega}{\omega}\right)^5}{\left\{\gamma_k^2 - \left(\frac{\omega}{\omega_s}\right)^2\right\}^2 + \left\{2\xi_k \gamma_k \frac{\omega}{\omega_s}\right\}^2} \tag{3.35}$$

$$\mu_k = \frac{m_k}{m_s}, k = 1,2,\ldots,n \quad \text{is mass ratio} \tag{3.36}$$

$$\gamma_k = \frac{\omega_k}{\omega_s} \quad \text{is tuning parameter} \tag{3.37}$$

$$\xi_k = \frac{c_k}{2m_k\omega_k} \quad \text{is damping ratio} \tag{3.38}$$

The MATLAB computer program for calculating the equivalent damping ratio of MTMD is shown in Appendixes B.

3.2.4 Robustness on natural frequency error

The natural frequencies for MTMD are distributed around natural frequency of first mode structure with the natural frequency error as shown in Fig. 3.4. The robustness on natural frequency error of the 400 m. high-rise building that has damping ratio equal to 0.01, ω =0.09 and attached with TMD (μ=0.01, ξ_{1opt}=0.05), compared to the case of structure that has damping ratio equal to 0.01 and attached by 5 TMDs (each MTMD consists of $\delta\mu$=0.002, $\delta\xi_\kappa$=0.01, and $\Delta\gamma$=0.172) is expressed in Fig. 3.5. In Fig. 3.5, it is shown that the robustness on natural frequency error in case of MTMD is better than TMD, as the result of the interaction between each TMD (in MTMD) can suppress vibration response of the structure.

3.2.5 Robustness on critical damping ratio error

The robustness on critical damping ratio error for 400 m. high-rise building that has damping ratio equal to 0.01, ω =0.09 and attached with TMD (μ_{1opt}=0.01, ξ_{1opt}=0.05) is compared to the case of same building attached with optimal MTMD (each MTMD consists of μ_k=0.002, ξ_k=0.02, $\delta\gamma$=0.035), expressed in Fig. 3.6. In Fig. 3.6, it is shown that the robustness on critical damping ratio in case of MTMD is better than TMD, however not so high. It can be viewed that total damping ratio error of MTMD can distribute into each individual TMD.

3.2.6 Optimal equivalent-damping ratio

Because of the lack of close form equation, the procedure to determine optimal MTMD is trial and error. Firstly, select the number of TMD, however, there are not large number of TMDs in order to control the displacement of each TMD. Secondly, change the spacing between each TMD's natural frequency in order to obtain optimal equivalent damping ratio. Fig. 3.7 shows the graph of 400-m tall building (ξ_s=0.01,

ω_s =0.09) attached with cases of MTMD (each MTMD consists of μ_k=0.002, ξ_k=0.01, $\Delta\gamma$=0.08, $\Delta\gamma$=0.148, $\Delta\gamma$=0.172, and $\Delta\gamma$=0.2.). It is clearly seen that too high or too small frequency ranges result in small value of ξ_e. In addition, when there is too small frequency range, ξ_e is sensitive to the error of natural frequency of structure. The results show that $\Delta\gamma$=0.148 for 5TMD is optimal. The results also show that the total optimal damping ratio in MTMD with 1% total mass is about twice of that in TMD. After the distribution of the natural frequency is tuned to decrease the flatness of complex frequency response and wideness the frequency area, finally, the damping ratio of each TMD is tuned to the optimum to reduce the secondary peak of complex frequency response. As shown in Fig. 3.8, the complex frequency response of the 400-m. height building (ω_s = 0.09, ξ_s = 0.01) is attached with 5TMDs (ξ = 0.02 , $\delta\gamma$ = 0.015, 0.03, 0.035, 0.04, 0.055), and each TMD has mass ratio μ = 0.01 and Fig. 3.9, the complex frequency response of the 400-m. height building (ω_s = 0.09, ξ_s = 0.01) is attached with 5TMDs (ω = 0.09, ξ = 0.005, 0.015, 0.02, 0.025, 0.035), and each TMD has mass ratio μ = 0.01.

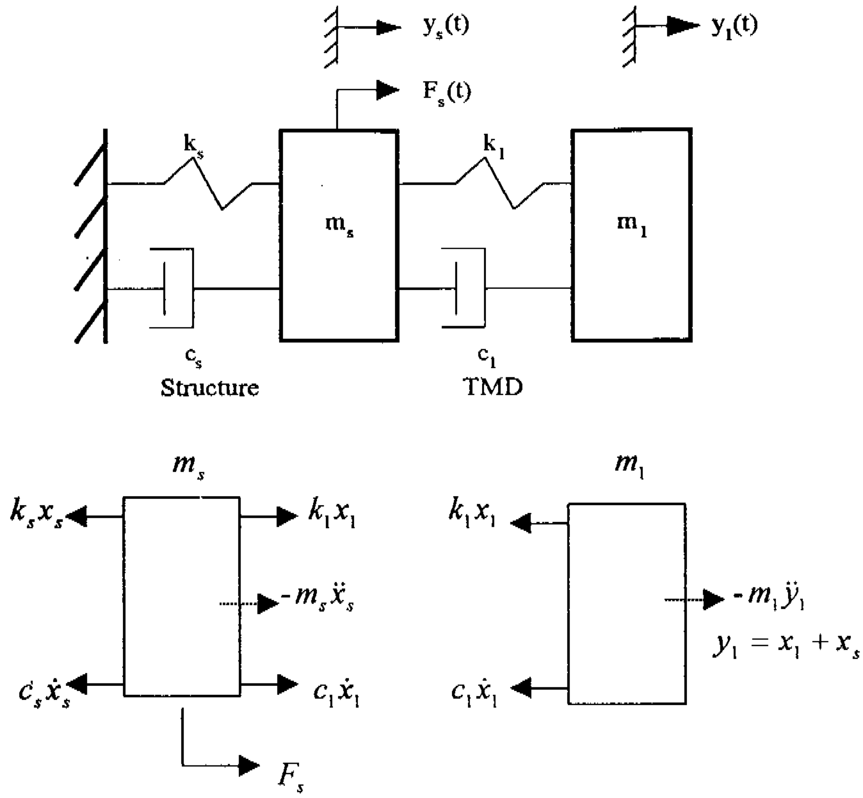

Fig. 3.1 Mathematical model of a first mode structure attached with TMD and free body diagrams

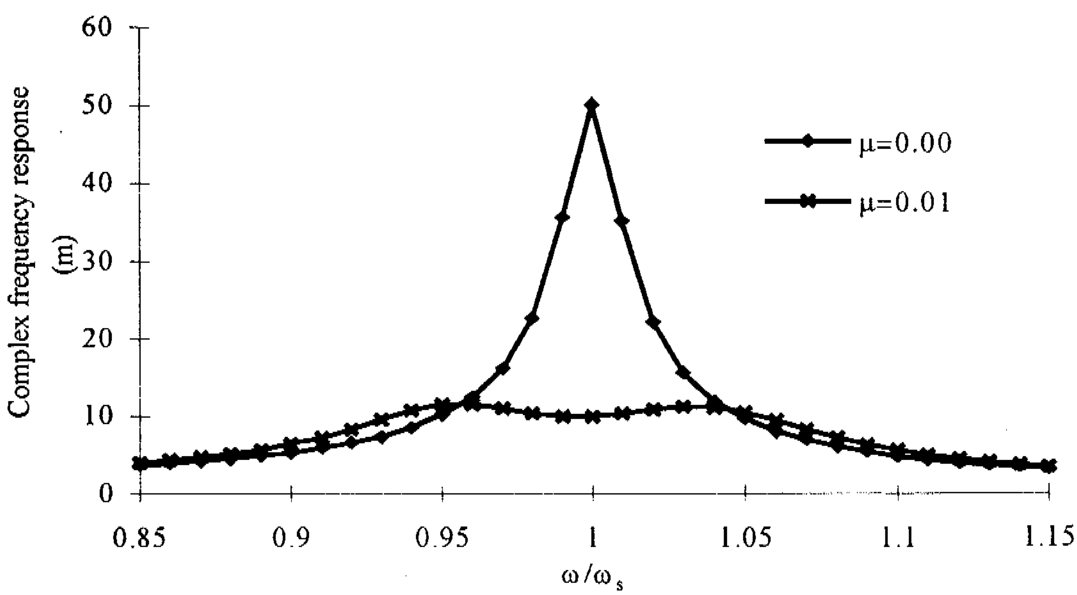

Fig. 3.2 Complex frequency response of 400-m high building with and without TMD

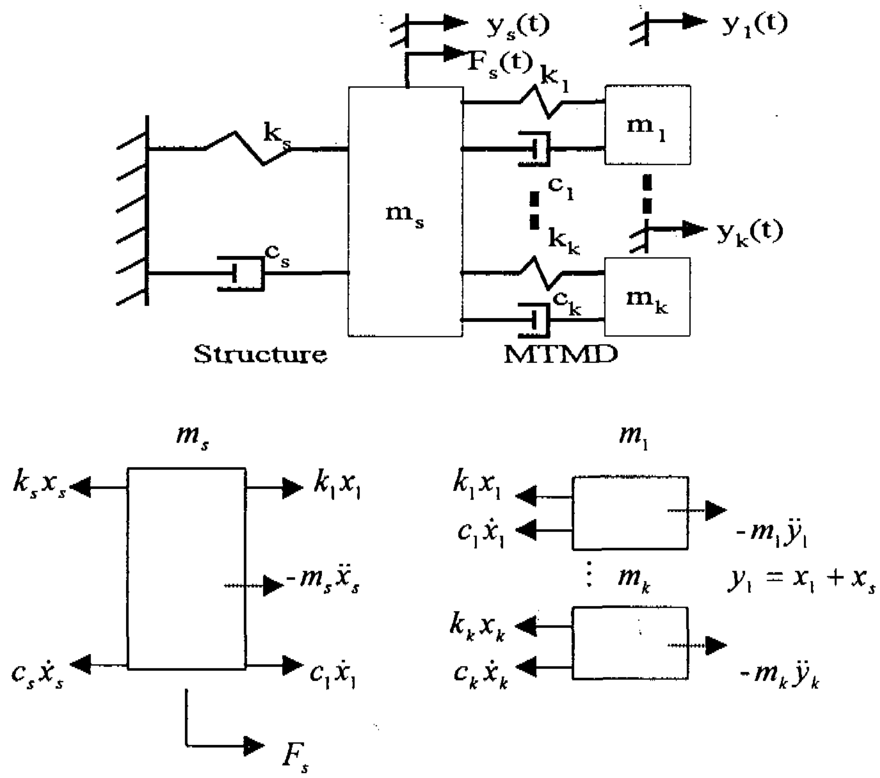

Fig. 3.3 Mathematical model of first mode structure attached with MTMD and free body diagrams

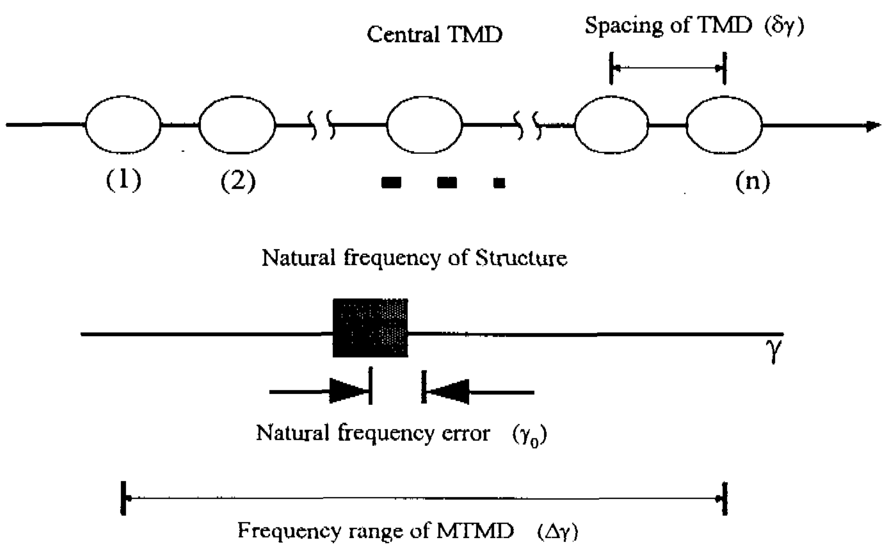

Fig. 3.4 Frequency distribution of MTMD around structural natural frequency

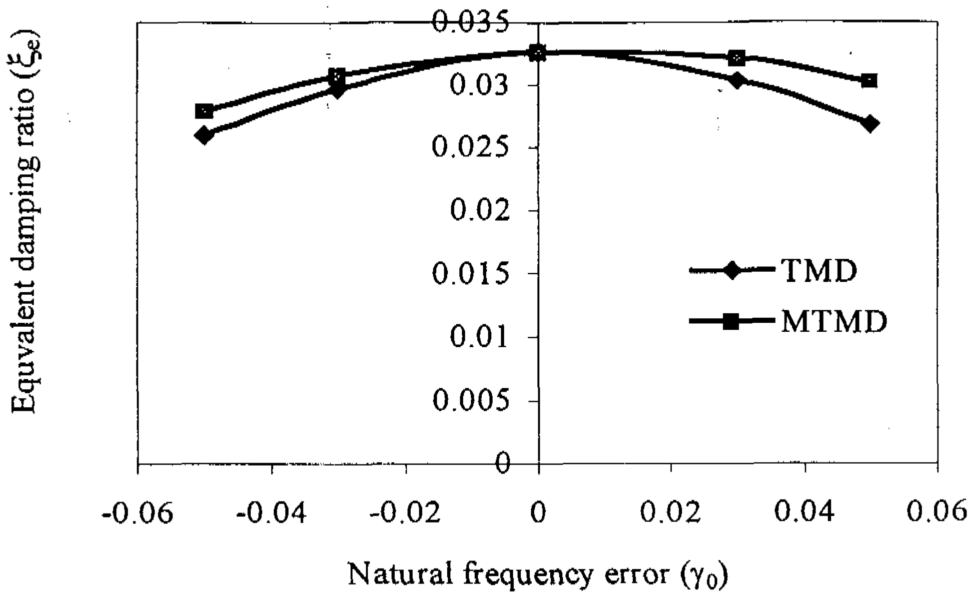

Fig. 3.5 Robustness of TMD and MTMD on natural frequency error (γo) for 400 m. high-rise building

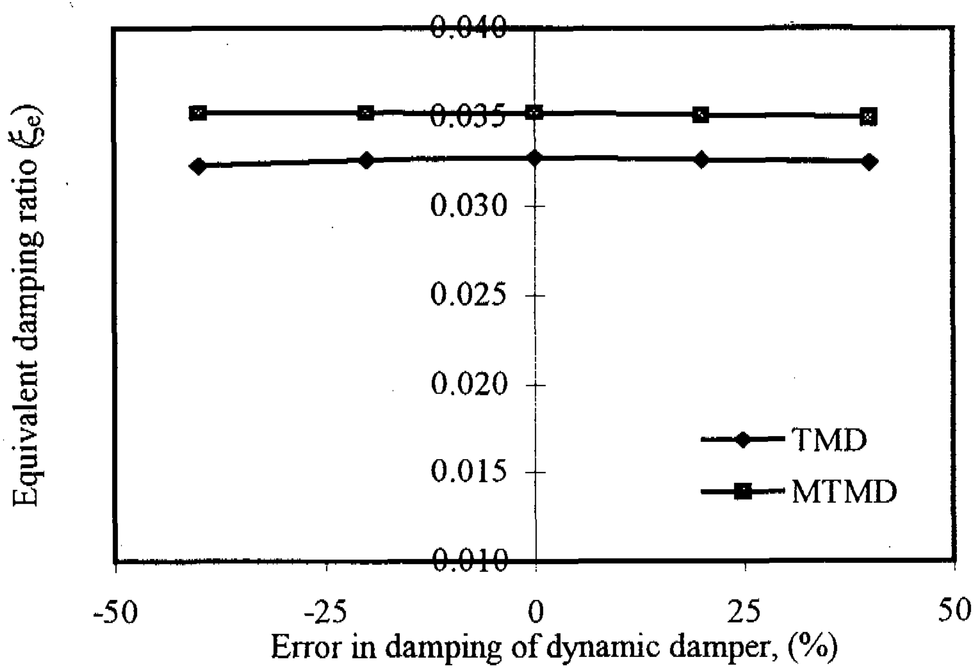

Fig. 3.6 Robustness of TMD and MTMD on total critical damping ratio error of dynamic damper

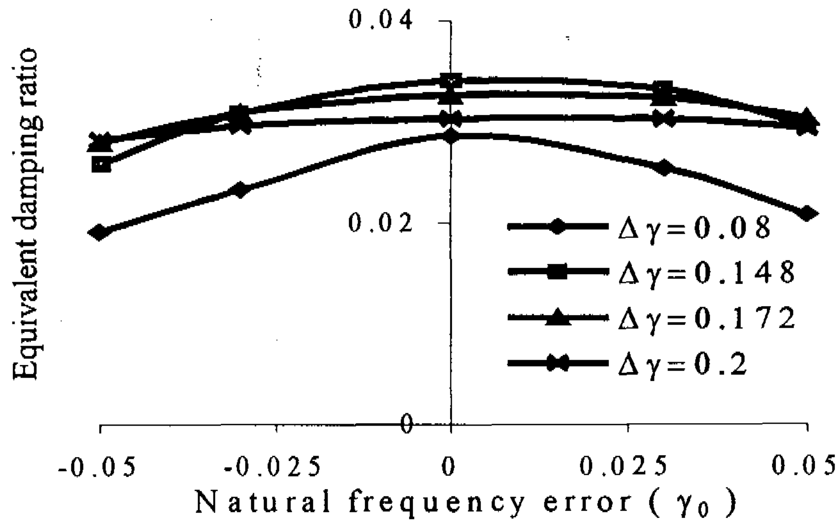

Fig. 3.7 Effectiveness and robustness of MTMD for 400 m high-rise building

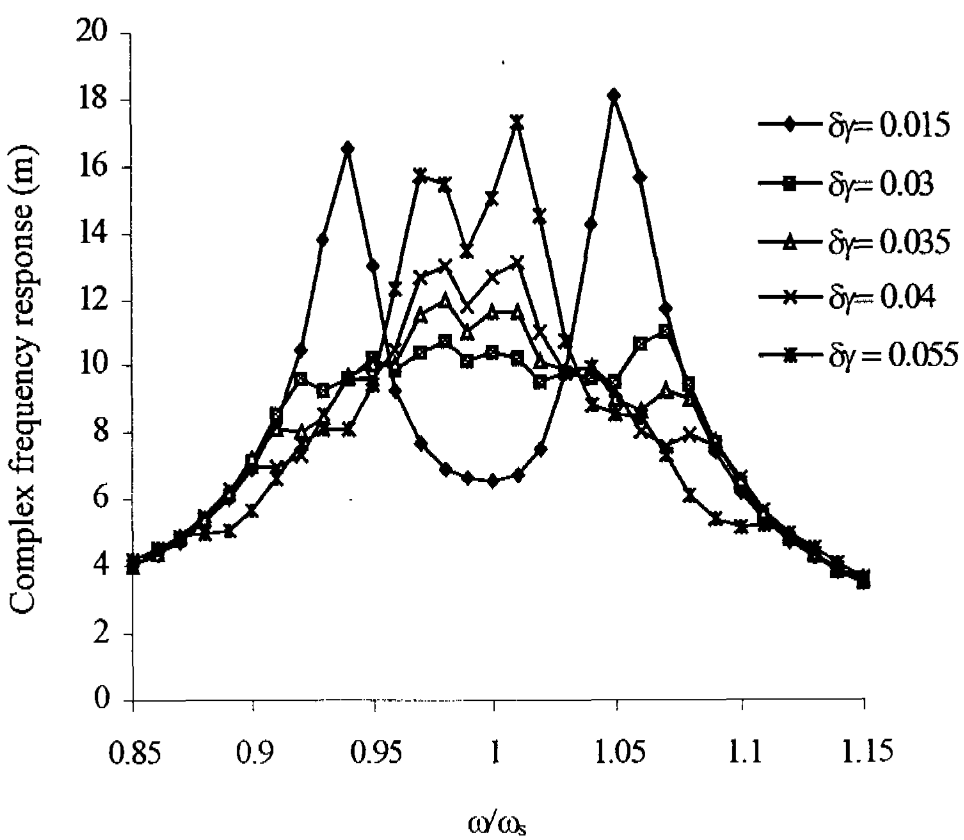

Fig. 3.8 Harmonic frequency response of 400-m building with a number of MTMDs with different natural frequency distribution

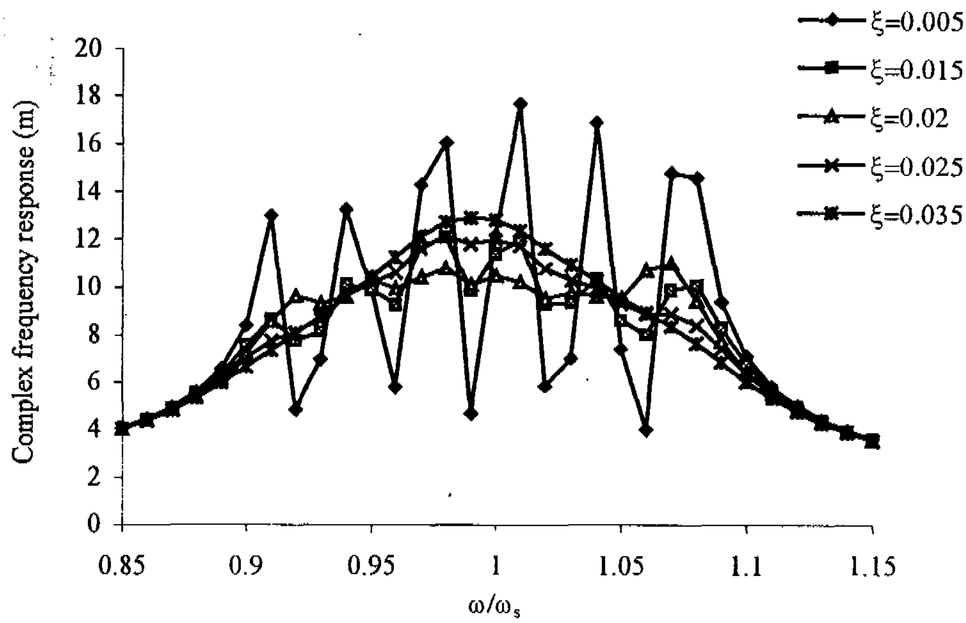

Fig. 3.9 Complex frequency response of 400-m.building with a number of MTMDs with different damping ratio