CHAPTER 5

DESIGN OF ACTIVE TUNED MASS DAMPER (ATMD)

5.1 Mathematical Model and Equation of Motion

To improve the performance of the passive damper, an active control mechanism is included between the SDOF corresponding to the building model and the damper mass, as shown schematically in Fig. 5.1 and the mechanism of the close loop active damper control is also shown in Fig. 5.2. The controller exerts a force of magnitude u on the both building and damper mass. In practice such a device consists of hydraulic actuator mechanism installed between the roof masonry and then passive damper. The dynamics of system are then described by the relations [Ankireddi 1996]

$$\begin{bmatrix} m_s & 0 \\ 0 & m_1 \end{bmatrix} \begin{bmatrix} \ddot{y}_s \\ \ddot{y}_1 \end{bmatrix} + \begin{bmatrix} c_s + c_1 & -c_1 \\ -c_1 & c_1 \end{bmatrix} \begin{bmatrix} \dot{y}_s \\ \dot{y}_1 \end{bmatrix} + \begin{bmatrix} k_s + k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \begin{bmatrix} y_s \\ y_1 \end{bmatrix} = \begin{Bmatrix} F_s(t) - u \\ u \end{Bmatrix} \tag{5.1}$$

Firstly, it is assumed that the passive damper properties (c_1, k_1) are at their optimal values (for a given μ), as obtained earlier. When optimally tuned, the passive damper causes the entire system to have optimal effective stiffness, damping and inertia. For designing the controller, choices are displacement feedback, velocity feedback and acceleration feedback, or combination of these. If either displacement, or velocity, or acceleration, it would disturb the effectiveness system characteristics so that it would no longer be operating at optimal point. Such an argument also holds if pairwise combinations (of displacement, velocity and acceleration) were considered for feed back control. Hence, in this study, and active control involving feedback of all three quantities (displacement, velocity, and acceleration) is considered. Such a controller (which will be refer to, for lack of a better term, as one based on complete feedback), if properly design, can be used the system to another operation points where the effectiveness of stiffness, damping and inertia are optimal. Furthermore, by choosing controller gains appropriately, the response characteristic can be made better than those using an optimally tuned passive damper alone.

5.2 The Controller

Assume that the feedback controller have the form

$$u = m_c(\ddot{y}_s - \ddot{y}_1) + c_c(\dot{y}_s - \dot{y}_1) + k_c(y_s - y_1) \tag{5.2}$$

where m_c, c_c, and k_c = gains of the acceleration feedback, velocity feedback, and displacement feedback, respectively. Such a feedback control law ensures that the mean control force is zero. The effective of any two of these gains would be to disturb the optimal, passively damped system characteristics. But it should be possible to choose the third gain appropriately to move the system over to another optimal operating point. This is the philosophy of current design.

From $y_s = x_s$ and $y_1 = x_1 + x_s$, Eq. (5.1) can be expanded as

$$m_s\ddot{x}_s + c_s\dot{x}_s - c_1\dot{x}_1 + k_s x_s - k_1 x_1 + u = F_s(t) \tag{5.3}$$
$$m_1(\ddot{x}_1 + \ddot{x}_s) + c_1\dot{x}_1 + k_1 x_1 - u = 0 \tag{5.4}$$

To formulate the metric form, Eq. (5.3) is taken to plus Eq. (5.4) then substitute $u = -m_c\ddot{x}_1 - c_c\dot{x}_1 - k_c x_1$ into Eq. (5.4). The following equations are obtained.

$$(m_s + m_1)\ddot{x}_s + m_1\ddot{x}_1 + c_s\dot{x}_s + k_s x_s = F_s(t) \tag{5.5}$$
$$m_1\ddot{x}_s + (m_1 + m_c)\ddot{x}_1 + (c_1 + c_c)\dot{x}_1 + (k_1 + k_c)x_1 = 0 \tag{5.6}$$

Using the notation that the first mode structural force $F_s(t) = k_s f(t)$. $f(t) = e^{i\omega t}$ is assumed to calculate the complex frequency response H_{xs}. Eq. (5.5) and (5.6) and be formulate the metric forms as

$$\begin{bmatrix} m_s + m_1 & m_1 \\ m_1 & m_1 + m_c \end{bmatrix}\begin{bmatrix} \ddot{x}_s \\ \ddot{x}_1 \end{bmatrix} + \begin{bmatrix} c_s & 0 \\ 0 & c_1 + c_c \end{bmatrix}\begin{bmatrix} \dot{x}_s \\ \dot{x}_1 \end{bmatrix} + \begin{bmatrix} k_s & 0 \\ 0 & k_1 + k_c \end{bmatrix}\begin{bmatrix} x_s \\ x_1 \end{bmatrix} = \begin{Bmatrix} k_s f(t) \\ 0 \end{Bmatrix} \tag{5.7}$$

5.3 Complex Frequency Response

Let $H_{xs}(\omega)$, and $H_{x1}(\omega)$ be the complex frequency response functions for the system of equation (5.7), and let it be defined that,

$$\mu_0 = m_c / m_s,\ \varepsilon = (c_1 + c_c)/m_s,\ \psi = (k_1 + k_c)/m_s \tag{5.8}$$

where μ_0 is called the acceleration feedback gain coefficient. Substitute Eq. (5.8) into Eq. (5.7) for the frequency response functions and divide by m_s lead to

$$\begin{bmatrix} 1+\mu & \mu \\ \mu & \mu + \mu_0 \end{bmatrix}\begin{bmatrix} \ddot{x}_s \\ \ddot{x}_1 \end{bmatrix} + \begin{bmatrix} 2\omega_s\xi_s & 0 \\ 0 & \varepsilon \end{bmatrix}\begin{bmatrix} \dot{x}_s \\ \dot{x}_1 \end{bmatrix} + \begin{bmatrix} \omega_s^2 & 0 \\ 0 & \psi \end{bmatrix}\begin{bmatrix} x_s \\ x_1 \end{bmatrix} = \begin{Bmatrix} \omega_s^2 f(t) \\ 0 \end{Bmatrix} \tag{5.9}$$

After some simplification the following Equation is obtained.

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{12} & A_{22} \end{bmatrix}\begin{Bmatrix} H_{xs}(\omega) \\ H_{x1}(\omega) \end{Bmatrix} = \begin{Bmatrix} \omega_s^2 \\ 0 \end{Bmatrix} \tag{5.10}$$

$$H_{xs}(\omega) = H_{ys}(\omega) = \frac{\omega_s^2 A_{22}}{\Delta} \tag{5.11}$$

and

$$\Delta = A_{11}A_{22} - A_{12}^2; \tag{5.12}$$

$$A_{11} = \left[-\omega^2(1+\mu)+\omega_s^2\right]+i\left[2\omega\xi_s\omega_s\right];$$
$$A_{12} = -\omega^2\mu$$
$$A_{22} = \left[-\omega^2(\mu+\mu_0)+\psi\right]+i(\omega\varepsilon); \tag{5.13}$$

5.4 Equivalent Damping Ratio

The effect of ATMD can be viewed as being equivalent to changing the damping ratio of structure (ξs) to equivalent damping ratio (ξe). Thus the dynamic response of the first mode of structure can be calculated as mentioned in the part of TMD. The mean square structural displacement can be calculated in the closed form equation as [Ankireddi 1996]

$$E\left[H_{ys}^2(\omega)\right] = S_o\int_{-\infty}^{\infty}\left|H_{ys}\right|^2 d\omega = \omega_s^4\,\frac{\left[\left(\dfrac{B_0^2}{A_0}\right)(A_2A_3 - A_1A_4)+A_3\left(B_1^2-2B_0B_2\right)+A_1B_2^2\right]\pi S_o}{\left[A_1(A_2A_3-A_1A_4)-A_0A_3^2\right]} \tag{5.14}$$

$$A_0 = \omega_s^2;\ A_1 = \omega_s^2\varepsilon+2\xi_s\omega_s;\ A_2 = (\mu+\mu_0)\omega_s^2+(1+\mu)\psi+2\xi_s\omega_s\varepsilon;$$
$$A_3 = (1+\mu)\varepsilon+2\xi_s\omega_s(\mu+\mu_0);\ A_4 = \mu+\mu_0+\mu\mu_0;\ B_0 = \psi;$$
$$B_1 = \varepsilon;\quad B_2 = \mu+\mu_0 \tag{5.15}$$

Alternatively, the mean square displacement can be calculated directly in the equation by using the MATLAB computer program as shown in Appendix C. The equivalent damping ratio of ATMD is also shown in Appendix C.

5.5 Optimal Equivalent Damping Ratio

Assuming minima exist, the optimal parameters ε and ψ may by determined for a given μ_0 using the conditions

$$\frac{\partial E\left[H_{ys}^2(\omega)\right]}{\partial\varepsilon} = 0 \tag{5.16}$$

$$\frac{\partial E\left[H_{ys}^2(\omega)\right]}{\partial\psi} = 0 \tag{5.17}$$

To simplify the analysis, if it is assumed that the damping of the building is negligible, the two conditions in Eq. (5.16) and Eq. (5.17) yield

$$\varepsilon_{opt} = \omega_s \sqrt{\frac{\mu^2 \mu_o}{(1+\mu)^2} + \frac{\mu^3 (3\mu + 4)}{4(1+\mu)^3}} \tag{5.18}$$

$$\psi_{opt} = \omega_s^2 \left(\frac{\mu^2 + 2\mu + 2\mu_o + 2\mu\mu_o}{2(\mu+1)^2} \right) \tag{5.19}$$

It is noted that if $\mu_0 = 0$, then the equivalent damping ratio corresponds to optimal passive damper case. From Eqs. (5.18), and (5.19), ε_{opt} and ψ_{opt} are increased with increasing μ_0.

5.6 Stability Analysis for ATMD Design

For the optimal ATMD design, the characteristic equation is of the form

$$A_4 s^4 + A_3 s^3 + A_2 s^2 + A_1 s + A_0 = 0 \tag{5.20}$$

with the polynomial coefficients being given by Eq. (5.15). According to Routh's stability criterion, [Ankireddi 1996], the necessary and sufficient conditions that a polynomial equation of the proceeding kind must satisfy are that all its coefficients be positive and, in addition, that all terms in the first column of the Routh's array have the same signs. For Eq. (5.20), the criterion gives the conditions,

$$A_0 > 0, A_1 > 0, A_2 > 0, A_3 > 0, A_4 > 0, (A_2 A_3 - A_1 A_4) > 0, (A_1 A_2 A_3 - A_1^2 A_4 - A_o A_3^2) > 0 \tag{5.21}$$

A necessary condition for the system to satisfy these criteria can be shown to be

$$\frac{-\mu}{1+\mu} < \mu_o \leq 0 \tag{5.22}$$

5.7 Displacement and Active Force of ATMD

Let the displacement of ATMD with respect to mass m_s be noted $x_l = y_l - y_s$. The complex frequency response function of x_1, $H_{x1}(\omega)$, can be written as [Ankireddi 1996]

$$H_{x1}(\omega) = \frac{-A_{12}\omega_s^2}{\Delta_\Omega} \tag{5.23}$$

Then, similar to the case of TMD, the stroke of ATMD, Δ_{ATMD}, can be determined from

$$\Delta_{ATMD} = \Delta_g \left(E\left[H_{x1}^2(\omega)\right] / E\left[H_{ys}^2(\omega)\right] \right)^{1/2} \tag{5.24}$$

where $\Delta_g = C_g - 1$.

The active force can be computed from controller force and the complex frequency response function as

$$u(\varpi) = m_c\left(\ddot{H}_{x1}(\omega)\right) + c_c\left(\dot{H}_{x1}(\omega)\right) + k_c\left(H_{x1}(\omega)\right) \tag{5.25}$$

Then, root mean square of the complex frequency active force can be determined as

$$E\left[u^2(\omega)\right] = S_0 \int_{-\infty}^{\infty} \left|u(\omega)\right|^2 d\omega \tag{5.26}$$

Finally, similar to the case of the stroke, active force of ATMD, Δ_u, can be obtained from

$$\Delta_u = \Delta_g \left(E\left[u^2(\omega)\right] / E\left[H_{ys}^2(\omega)\right] \right)^{1/2} \tag{5.27}$$

where Δ_u is active force of ATMD.

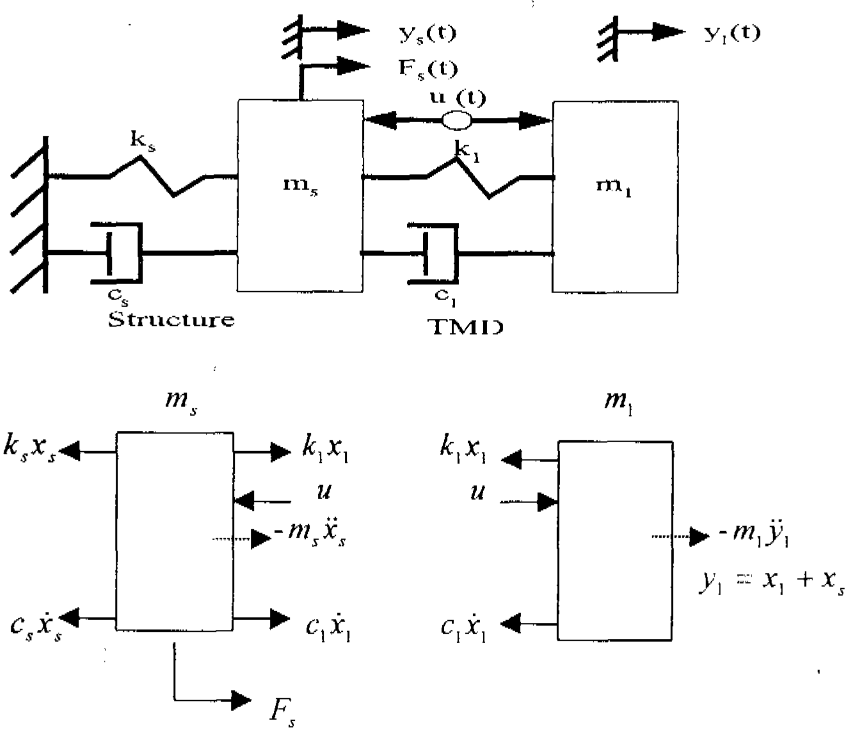

Fig. 5.1 Mathematical model of first mode structure attached with ATMD and free body diagrams

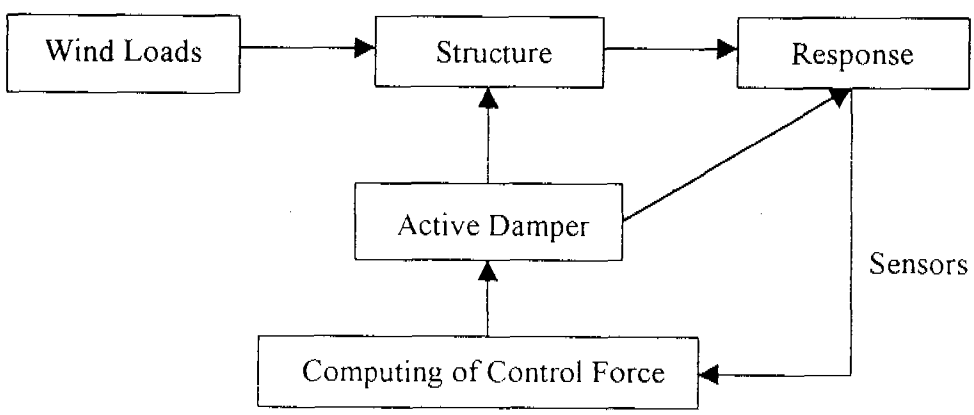

Fig. 5.2 Close loop active damper control