APPENDIX A

PRIMARY DESIGN OF EJECTOR USING 1-DIMENSIONAL THEORY

The design method based on 1-dimensional analysis for compressible-gas flow in ejector using the models of stream mixing at constant pressure was first proposed by Keenan and Neumann [1]which was later become a classical theory. Eames et al. [2] proposed a set of the 1-D equation in designing the ejector based on Keenan and Neumann's theory [1]. The loss coefficient at the primary nozzle, the mixing chamber and the diffuser were accounted. In this thesis, the 1-D theory of Eames et al. [2] was selected to use for the design of the ejector.

A.1 Theoretical assumptions

List of simplified assumptions used in Aphornratana's theory is given:

1) Friction Losses are accounted to the base equations of Keenan by applying appropriate loss efficiencies to the primary nozzle (η_N), the diffuser (η_d) and the mixing process (η_m).
2) Kinetic energies of both the primary fluid and secondary fluid at the ejector inlet and the diffuser outlet are negligible (zero velocity).
3) The static pressure at the primary nozzle exit plane where the two fluid streams first met is assumed to be uniform
4) The two streams completely mix before a normal shock wave occurs at the end of the mixing chamber.

A.2 Governing Equations

The analysis of 1-D theory of Keenan [xx] was based on the ideal gas assumption combined with the principles of mass, momentum, and energy conservation. For a study flow process, the equations are given as follows:

Energy Equation for Adiabatic Process:

$$\sum \dot{m}_i \left(h_i + \frac{V_i^2}{2} \right) = \sum \dot{m}_e \left(h_e + \frac{V_e^2}{2} \right) \tag{A.1}$$

Momentum Equation:

$$F + P_i A_i + \sum \dot{m}_i V_i = P_e A_e + \sum \dot{m}_e V_e \tag{A.2}$$

Continuity Equation:

$$\sum \rho_i V_i A_i = \sum \rho_e V_e A_e \tag{A.3}$$

Referring to Figure 2.2, applying energy equation and introducing an assumed isentropic efficiency (η_N) of the primary nozzle between state P_0 and 2′, the speed of the primary fluid leaving the primary nozzle exit is:

$$V_{2'}^2 = 2\eta_N \left(h_{P_0} - h_{2'} \right) \tag{A.4}$$

The Mach number of the primary fluid at the nozzle exit plane, therefore, can be calculated using:

$$M_{2'} = \sqrt{\frac{2\eta_N}{k-1}\left[\left(\frac{P_{P_0}}{P_2} \right)^{\frac{k-1}{k}} - 1 \right]} \tag{A.5}$$

Similar to equation (A.5), between state S_0 and 2″, the Mach number of the secondary fluid at the nozzle exit plane is determined by:

$$M_{2''} = \sqrt{\frac{2}{k-1}\left[\left(\frac{P_{S_0}}{P_2} \right)^{\frac{k-1}{k}} - 1 \right]} \tag{A.6}$$

Introducing the isentropic efficiency (η_m) during the mixing process of the two streams, between state 2 and 5, the momentum equation can be written in the following form:

$$\eta_m \left(P_2.A_2 + \dot{m}_P.V_{2'} + \dot{m}_S.V_{2''} \right) = P_5.A_5 + V_5 \left(\dot{m}_p + \dot{m}_s \right) \tag{A.7}$$

The speed of the mixed stream at state 5 can then be given as:

$$V_5 = \eta_m \left(\frac{\dot{m}_p.V_{2'} + \dot{m}_s.V_{2''}}{\dot{m}_p + \dot{m}_s} \right) \tag{A.8}$$

Introducing the relation between M and M*

$$M^* = \sqrt{\frac{(k+1).\left(\frac{M^2}{2}\right)}{1+(k-1).\left(\frac{M^2}{2}\right)}} \tag{A.9}$$

Equation (A.8) can be written in term of Mach number as:

$$M_5{}^* = \frac{M_{2'}{}^* + Rm.M_{2''}{}^*.\sqrt{\frac{T_S}{T_P}}}{\sqrt{(1+Rm).\left(1+Rm.\sqrt{\frac{T_S}{T_P}}\right)}} \tag{A.10}$$

Assuming that the normal shock wave occur between state 5 and 6, the Mach number of the mixed fluid immediately after the normal shock wave is determined from:

$$M_6 = \sqrt{\frac{M_5{}^2 + \frac{2}{(k+1)}}{\left(\frac{2.k}{(k-1)}.M_5{}^2\right)-1}} \tag{A.11}$$

The pressure ratio across the normal shock wave is calculated from:

$$\frac{P_6}{P_5} = \frac{1+k.M_5{}^2}{1+k.M_6{}^2} \tag{A.12}$$

If assumed that the flow speed is brought to stagnation state at the end of the diffuser and a loss coefficient of the diffuser (η_d) is applied, the pressure ratio across the subsonic diffuser can be written as:

$$\frac{P_b}{P_6} = \left[\left(\frac{\eta_d.(k-1)}{2}.M_6^{\ 2}\right)+1\right]^{\left(\frac{k}{k-1}\right)} \tag{A.13}$$

From equation (A.1) to (A.13), if the isentropic efficiencies are initially given and the entrainment ratio is first assumed, the Mach number of each position and the pressure ratios across the ejector can be iteratively calculated.

After the pressure ratios are obtained, the cross sectional areas at each location of ejector can be determined using equation (A.14) to (A.19) with a given diameter of the primary nozzle throat.

The mass flow rate of the primary fluid ($\dot{m}_p$) is calculated using theory of compressible gas flow through convergent-divergent nozzle which is the critical mass flow rate or maximum mass flow rate for a given throat diameter of the nozzle.

$$\dot{m}_p = A_1.P_{P_0}\sqrt{\frac{k}{RT_{P_0}}\left(\frac{2}{k+1}\right)^{(k+1)/(2(k-1))}} \tag{A.14}$$

Thus, the cross sectional area at the throat of the primary nozzle can be written as:

$$A_1 = \frac{\dot{m}_p}{P_{P_0}}\sqrt{\frac{T_{P_0}.R}{k}\left(\frac{k+1}{2}\right)^{\frac{k+1}{k-1}}} \tag{A.15}$$

Applying continuity equation of an ideal gas between state 1 and 2′, the cross sectional area at the primary nozzle exit can be calculated using the following equation:

$$\frac{A_{2'}}{A_1} = \frac{\left(\frac{2}{k+1}\right)^{\frac{1}{k-1}}\left(\frac{P_{P_0}}{P_2}\right)^{\frac{1}{k}}}{\sqrt{\frac{k+1}{k-1}\left(1-\left(\frac{P_2}{P_{P_0}}\right)^{\frac{k-1}{k}}\right)}} \tag{A.16}$$

Where as the annular cross sectional area of the secondary fluid at the inlet of mixing chamber (primary nozzle exit plane) can be evaluated as:

$$\frac{A_{2''}}{A_1} = \frac{Rm.\sqrt{\frac{T_{S_0}}{T_{P_0}}}.\left(\frac{2}{k+1}\right)^{\frac{1}{k-1}}.\left(\frac{P_{P_0}}{P_{S_0}}\right).\left(\frac{P_{S_0}}{P_2}\right)^{\frac{1}{k}}}{\sqrt{\frac{k+1}{k-1}.\left(1-\left(\frac{P_2}{P_{S_0}}\right)^{\frac{k-1}{k}}\right)}} \quad (A.17)$$

If a zero thickness of the primary nozzle exit's wall is assumed, the mixing chamber inlet diameter can be expressed as:

$$A_2 = A_{2'} + A_{2''} \quad (A.18)$$

The area ratio (*AR*) between the cross sectional area of constant area's throat to the cross sectional area of the primary nozzle's throat can finally be obtained as follows:

$$\frac{A_6}{A_1} = \frac{\sqrt{(1+Rm)(1+Rm.T_s/T_p)}.\left(\frac{2}{k+1}\right)^{\frac{1}{k-1}}.\left(\frac{P_{P_0}}{P_b}\right).\left(\frac{P_b}{P_6}\right)^{\frac{1}{k}}}{\sqrt{\frac{k+1}{k-1}.\left(1-\left(\frac{P_6}{P_b}\right)^{\frac{k-1}{k}}\right)}} \quad (A.19)$$