Appendix A

Derivation of Equations

A.1 Properties Regarding Conditional Probabilities

We define $p(a)$ as the probability that a random variable A takes on the value a, and we define $p(a, b)$ as the probability that a random variable A takes on the value a *and* a random variable B takes on the value b. We then define the *conditional probability* $p(a \mid b)$ as

$$p(a \mid b) = \frac{p(a,b)}{p(b)}, \tag{A.1}$$

where $p(b)$ has a non-vanishing probability. $p(a \mid b)$ is the probability that A takes on the value a given that B takes on the value b. From (A.1), we have $p(a,b) = p(a \mid b)\, p(b)$ and $p(b,a) = p(b \mid a)\, p(a)$. Since $p(a,b) = p(b,a)$, we obtain

$$p(a \mid b) = \frac{p(b \mid a)\, p(a)}{p(b)}, \tag{A.2}$$

which is referred to as *Bayes theorem.*

In this thesis, we frequently use the following formulae:

$$\begin{aligned} p(a \mid b, c) &= \frac{p(c \mid a, b)\, p(a \mid b)}{p(c \mid b)}, & \text{(A.3)} \\ p(a, b \mid c) &= p(a \mid b, c)\, p(b \mid c). & \text{(A.4)} \end{aligned}$$

(A.3) is an extension of the Bayes theorem. Derivation of (A.3) is as follows:

$$\begin{aligned} p(a \mid b, c) &= \frac{p(a,b,c)}{p(b,c)} \\ &= \frac{p(c,a,b)}{p(a,b)} \frac{p(a,b)}{p(c,a,b)} \frac{p(a,b,c)}{p(b,c)} \\ &= p(c \mid a, b) \frac{p(a,b)}{p(b,c)} \\ &= p(c \mid a, b) \frac{p(b)\, p(a \mid b)}{p(b)\, p(c \mid b)} \\ &= \frac{p(c \mid a, b)\, p(a \mid b)}{p(c \mid b)}. \end{aligned} \tag{A.5}$$

To derive (A.4), we first apply (A.1) to $p(a, b \mid c)$,

$$p(a, b \mid c) = \frac{p(a,b,c)}{p(c)}. \tag{A.6}$$

Applying (A.3) to $p(a, b, c)$ yields

$$p(a, b \mid c) = \frac{p(a \mid b, c)\, p(b, c)}{p(c)}. \tag{A.7}$$

Finally, applying (A.1) to $p(c)$ yields

$$\begin{aligned} p(a, b \mid c) &= \frac{p(a \mid b, c)\, p(b, c)}{p(c, b)/p(b \mid c)} \\ &= p(a \mid b, c)\, p(b \mid c). \end{aligned} \tag{A.8}$$

A.2 Bayes Filter Equation for SLAM Posterior $p(\boldsymbol{s}_t, \Theta \mid \boldsymbol{z}_{0:t}, \boldsymbol{u}_{1:t}, n_{1:t})$

We first apply (A.3) to $p(\boldsymbol{s}_t, \Theta_t \mid \boldsymbol{z}_{0:t}, \boldsymbol{u}_{1:t}, n_{1:t})$ to obtain

$$p(\boldsymbol{s}_t, \Theta_t \mid \boldsymbol{z}_{0:t}, \boldsymbol{u}_{1:t}, n_{1:t}) = \frac{p(\boldsymbol{z}_t \mid \boldsymbol{s}_t, \Theta_t, \boldsymbol{z}_{0:t-1}, \boldsymbol{u}_{1:t}, n_{1:t})\, p(\boldsymbol{s}_t, \Theta_t \mid \boldsymbol{z}_{0:t-1}, \boldsymbol{u}_{1:t}, n_{1:t})}{p(\boldsymbol{z}_t \mid \boldsymbol{z}_{0:t-1}, \boldsymbol{u}_{1:t}, n_{1:t})}. \tag{A.9}$$

Since $p(\boldsymbol{z}_t \mid \boldsymbol{z}_{0:t-1}, \boldsymbol{u}_{1:t}, n_{1:t})$ gives a constant probability, we have

$$p(\boldsymbol{s}_t, \Theta_t \mid \boldsymbol{z}_{0:t}, \boldsymbol{u}_{1:t}, n_{1:t}) = \eta\, p(\boldsymbol{z}_t \mid \boldsymbol{s}_t, \boldsymbol{\theta}_{n_t}, n_t)\, p(\boldsymbol{s}_t, \Theta_t \mid \boldsymbol{z}_{0:t-1}, \boldsymbol{u}_{1:t}, n_{1:t}), \tag{A.10}$$

where η is a normalizer. From the theorem of total probability we obtain

$$\begin{aligned} p(\boldsymbol{s}_t, \Theta_t \mid \boldsymbol{z}_{0:t}, \boldsymbol{u}_{1:t}, n_{1:t}) &= \eta\, p(\boldsymbol{z}_t \mid \boldsymbol{s}_t, \boldsymbol{\theta}_{n_t}, n_t) \\ \iint p(\boldsymbol{s}_t, \Theta_t \mid \boldsymbol{z}_{0:t-1}, \boldsymbol{u}_{1:t}, n_{1:t}, \boldsymbol{s}_{t-1}, \Theta_{t-1})\, & p(\boldsymbol{s}_{t-1}, \Theta_{t-1} \mid \boldsymbol{z}_{0:t-1}, \boldsymbol{u}_{1:t}, n_{1:t})\, d\boldsymbol{s}_{t-1}\, d\Theta_{t-1}. \end{aligned} \tag{A.11}$$

Since we assume that the environment is time-independent, i.e. $\Theta_t = \Theta$ for any t, the first factor inside the double integral vanishes unless $\Theta_{t-1} = \Theta_t = \Theta$. Therefore,

$$\begin{aligned} p(\boldsymbol{s}_t, \Theta_t \mid \boldsymbol{z}_{0:t}, \boldsymbol{u}_{1:t}, n_{1:t}) &= \eta\, p(\boldsymbol{z}_t \mid \boldsymbol{s}_t, \boldsymbol{\theta}_{n_t}, n_t) \\ \int p(\boldsymbol{s}_t, \Theta \mid \boldsymbol{z}_{0:t-1}, \boldsymbol{u}_{1:t}, n_{1:t}, \boldsymbol{s}_{t-1}, \Theta)\, & p(\boldsymbol{s}_{t-1}, \Theta \mid \boldsymbol{z}_{0:t-1}, \boldsymbol{u}_{1:t}, n_{1:t})\, d\boldsymbol{s}_{t-1}. \end{aligned} \tag{A.12}$$

Finally, eliminating unnecessary conditional variables in the integrands based on Markov assumption yields

$$\begin{aligned} p(\boldsymbol{s}_t, \Theta \mid \boldsymbol{z}_{0:t}, \boldsymbol{u}_{1:t}, n_{1:t}) &= \eta\, p(\boldsymbol{z}_t \mid \boldsymbol{s}_t, \boldsymbol{\theta}_{n_t}, n_t) \\ & \int p(\boldsymbol{s}_t \mid \boldsymbol{s}_{t-1}, \boldsymbol{u}_t)\, p(\boldsymbol{s}_{t-1}, \Theta \mid \boldsymbol{z}_{0:t-1}, \boldsymbol{u}_{1:t-1}, n_{1:t-1})\, d\boldsymbol{s}_{t-1}. \end{aligned} \tag{A.13}$$

A.3 Bayes Filter Equation for SLAM Posterior $p(s_{0:t}, \Theta \mid z_{0:t}, \boldsymbol{u}_{1:t}, n_{1:t})$

We first apply (A.3) to $p(s_{0:t}, \Theta_t \mid z_{0:t}, \boldsymbol{u}_{1:t}, n_{1:t})$ to obtain

$$p(s_{0:t}, \Theta \mid \boldsymbol{z}_{0:t}, \boldsymbol{u}_{1:t}, n_{1:t}) = \frac{p(\boldsymbol{z}_t \mid s_{0:t}, \Theta, z_{0:t-1}, \boldsymbol{u}_{1:t}, n_{1:t})\, p(s_{0:t}, \Theta \mid z_{0:t-1}, \boldsymbol{u}_{1:t}, n_{1:t})}{p(\boldsymbol{z}_t \mid z_{0:t-1}, \boldsymbol{u}_{1:t}, n_{1:t})}. \tag{A.14}$$

We have a constant probability $p(z_t \mid z_{0:t-1}, u_{1:t}, n_{1:t})$. We eliminate unnecessary conditional variables in numerators based on Markov assumption. So we can simplify the equation as

$$p(s_{0:t}, \Theta \mid z_{0:t}, u_{1:t}, n_{1:t}) = \eta\, p(z_t \mid s_t, \theta_{n_t}, n_t)\, p(s_{0:t}, \Theta \mid z_{0:t-1}, u_{1:t}, n_{1:t-1}), \quad (A.15)$$

where η is a normalizer. We then apply (A.4) to $p(s_{0:t}, \Theta \mid z_{0:t-1}, u_{1:t}, n_{1:t-1})$ to have

$$\begin{aligned} p(s_{0:t}, \Theta \mid z_{0:t}, u_{1:t}, n_{1:t}) &= \eta\, p(z_t \mid s_t, \theta_{n_t}, n_t)\, p(s_t \mid s_{0:t-1}, \Theta, z_{0:t-1}, u_{1:t}, n_{1:t-1}) \\ &\quad p(s_{0:t-1}, \Theta \mid z_{0:t-1}, u_{1:t}, n_{1:t-1}). \end{aligned} \quad (A.16)$$

Finally, by eliminating unnecessary conditional variables again, we obtain

$$\begin{aligned} p(s_{0:t}, \Theta \mid z_{0:t}, u_{1:t}, n_{1:t}) &= \eta\, p(z_t \mid s_t, \theta_{n_t}, n_t) \\ &\quad p(s_t \mid s_{t-1}, u_t)\, p(s_{0:t-1}, \Theta \mid z_{0:t-1}, u_{1:t-1}, n_{1:t-1}). \end{aligned} \quad (A.17)$$

A.4 Factorization of SLAM Posterior $p(s_{0:t}, \Theta \mid z_{0:t}, u_{1:t}, n_{1:t})$

First, using (A.4), we rewrite the posterior $p(s_{0:t}, \Theta \mid z_{0:t}, u_{1:t}, n_{1:t})$ as

$$p(s_{0:t}, \Theta \mid z_{0:t}, u_{1:t}, n_{1:t}) = p(s_{0:t} \mid \Theta, z_{0:t}, u_{1:t}, n_{1:t})\, p(\Theta \mid s_{0:t}, z_{0:t}, u_{1:t}, n_{1:t}). \quad (A.18)$$

We then eliminate non-contributing conditional variables in conditional probabilities to obtain

$$p(s_{0:t}, \Theta \mid z_{0:t}, u_{1:t}, n_{1:t}) = p(s_{0:t} \mid z_{0:t}, u_{1:t}, n_{1:t})\, p(\Theta \mid s_{0:t}, z_{0:t}, n_{1:t}). \quad (A.19)$$

Hence we derive the factorization of the map posterior

$$p(\Theta \mid s_{0:t}, z_{0:t}, n_{1:t}) = \prod_{n=1}^{N} p(\theta_n \mid s_{0:t}, z_{0:t}, n_{1:t}). \quad (A.20)$$

in the following.

If landmark θ_n is not observed at time step t, i.e. $n \neq n_t$, then observation at time t has no effect on the posterior distribution of θ_n, so

$$p(\theta_n \mid s_{0:t}, z_{0:t}, n_{1:t}) = p(\theta_n \mid s_{0:t-1}, z_{0:t-1}, n_{1:t-1}). \quad (A.21)$$

On the other hand, if landmark θ_n is observed at time t, i.e. $n = n_t$, we can rewrite its posterior distribution using (A.3) as

$$\begin{aligned} p(\theta_{n_t} \mid s_{0:t}, z_{0:t}, n_{1:t}) &= \frac{p(z_t \mid \theta_{n_t}, s_{0:t}, z_{0:t-1}, n_{1:t})\, p(\theta_{n_t} \mid s_{0:t}, z_{0:t-1}, n_{1:t})}{p(z_t \mid s_{0:t}, z_{0:t-1}, n_{1:t})} \\ &= \frac{p(z_t \mid s_t, \theta_{n_t}, n_t)\, p(\theta_{n_t} \mid s_{0:t-1}, z_{0:t-1}, n_{1:t-1})}{p(z_t \mid s_{0:t}, z_{0:t-1}, n_{1:t})}. \end{aligned} \quad (A.22)$$

We prove (A.20) by mathematical induction. When $t = 0$, (A.20) is obviously true since all landmarks are conditionally independent when they are observed in the beginning. Now let (A.20) be true for time $t-1$, i.e.

$$p(\Theta \mid s_{0:t-1}, z_{0:t-1}, n_{1:t-1}) = \prod_{n=1}^{N} p(\theta_n \mid s_{0:t-1}, z_{0:t-1}, n_{1:t-1}). \quad (A.23)$$

Using (A.3), we rewrite $p(\Theta \mid s_{0:t}, z_{0:t}, n_{1:t})$ as

$$\begin{aligned} p(\Theta \mid s_{0:t}, z_{0:t}, n_{1:t}) &= \frac{p(z_t \mid \Theta, s_{0:t}, z_{0:t-1}, n_{1:t})\, p(\Theta \mid s_{0:t}, z_{0:t-1}, n_{1:t})}{p(z_t \mid s_{0:t}, z_{0:t-1}, n_{1:t})} \\ &= \frac{p(z_t \mid s_t, \theta_{n_t}, n_t)\, p(\Theta \mid s_{0:t-1}, z_{0:t-1}, n_{1:t-1})}{p(z_t \mid s_{0:t}, z_{0:t-1}, n_{1:t})}. \end{aligned} \tag{A.24}$$

Inserting (A.23) into (A.24), we obtain

$$\begin{aligned} p(\Theta \mid s_{0:t}, z_{0:t}, n_{1:t}) &= \frac{p(z_t \mid s_t, \theta_{n_t}, n_t)}{p(z_t \mid s_{0:t}, z_{0:t-1}, n_{1:t})} \prod_{n=1}^{N} p(\theta_n \mid s_{0:t-1}, z_{0:t-1}, n_{1:t-1}) \\ &= \frac{p(z_t \mid s_t, \theta_{n_t}, n_t)}{p(z_t \mid s_{0:t}, z_{0:t-1}, n_{1:t})} p(\theta_{n_t} \mid s_{0:t-1}, z_{0:t-1}, n_{1:t-1}) \\ &\quad \prod_{n \neq n_t}^{N} p(\theta_n \mid s_{0:t-1}, z_{0:t-1}, n_{1:t-1}). \end{aligned} \tag{A.25}$$

Since we have (A.21) and (A.22), we can furthermore rewrite (A.25) as

$$\begin{aligned} p(\Theta \mid s_{0:t}, z_{0:t}, n_{1:t}) &= p(\theta_{n_t} \mid s_{0:t}, z_{0:t}, n_{1:t}) \prod_{n \neq n_t}^{N} p(\theta_n \mid s_{0:t}, z_{0:t}, n_{1:t}) \\ &= \prod_{n=1}^{N} p(\theta_n \mid s_{0:t}, z_{0:t}, n_{1:t}). \end{aligned} \tag{A.26}$$

Hence, the map posterior is also factored at time t. This completes the proof of (A.20) by mathematical induction.

A.5 Kalman Filter Update Equations

The posterior over the landmark $p(\theta_{n_t} \mid n_{1:t}, s_{0:t}^{[m]}, z_{0:t})$ is calculated as

$$\begin{aligned} & p(\theta_{n_t} \mid n_{1:t}, s_{0:t}^{[m]}, z_{0:t}) \\ &= \frac{p(z_t \mid n_{1:t}, \theta_{n_t}, s_{0:t}^{[m]}, z_{0:t-1})\, p(\theta_{n_t} \mid n_{1:t}, s_{0:t}^{[m]}, z_{0:t-1})}{p(z_t \mid s_{0:t}^{[m]}, n_{1:t}, z_{0:t-1})} \\ &= \eta\, p(z_t \mid \theta_{n_t}, n_t, s_t^{[m]}) p(\theta_{n_t} \mid n_{1:t-1}, s_{0:t-1}^{[m]}, z_{0:t-1}). \end{aligned} \tag{A.27}$$

From (3.15) and (3.16), we can rewrite this using Gaussian functions as

$$\begin{aligned} & p(\theta_{n_t} \mid n_{1:t}, s_{0:t}^{[m]}, z_{0:t}) \\ &= \eta\, \exp\Big\{ -\frac{1}{2} \left(z_t - g(\theta_{n_t}, s_t^{[m]})\right)^T \mathrm{R}_t^{-1} \left(z_t - g(\theta_{n_t}, s_t^{[m]})\right) \\ &\quad -\frac{1}{2} \left(\theta_{n_t} - \mu_{n_t,t-1}^{[m]}\right)^T (\Sigma_{n_t,t-1}^{[m]})^{-1} \left(\theta_{n_t} - \mu_{n_t,t-1}^{[m]}\right) \Big\}. \end{aligned} \tag{A.28}$$

By linearizing $g(\theta_{n_t}, s_t^{[m]})$ with respect to θ_{n_t} as

$$g(\theta_{n_t}, s_t^{[m]}) \approx \hat{z}_t^{[m]} + \mathrm{G}_t^{[m]}(\theta_{n_t} - \mu_{n_t,t-1}^{[m]}), \tag{A.29}$$

where

$$\hat{z}_t^{[m]} = g(\mu_{n_t,t-1}^{[m]}, s_t^{[m]}), \tag{A.30}$$

we obtain

$$p(\theta_{n_t} \mid n_{1:t}, s_{0:t}^{[m]}, z_{0:t}) = \eta \exp(-J_t), \tag{A.31}$$

where J_t is a quadratic form in terms of θ_{n_t}:

$$\begin{aligned} J_t &= \frac{1}{2}\left(z_t - \hat{z}_t^{[m]} - G_t^{[m]}(\theta_{n_t} - \mu_{n_t,t-1}^{[m]})\right)^T R_t^{-1}\left(z_t - \hat{z}_t^{[m]} - G_t^{[m]}(\theta_{n_t} - \mu_{n_t,t-1}^{[m]})\right) \\ &+\frac{1}{2}\left(\theta_{n_t} - \mu_{n_t,t-1}^{[m]}\right)^T (\Sigma_{n_t,t-1}^{[m]})^{-1}\left(\theta_{n_t} - \mu_{n_t,t-1}^{[m]}\right). \end{aligned} \tag{A.32}$$

First and second derivatives of J_t are

$$\begin{aligned} \frac{\partial J_t}{\partial \theta_{n_t}} &= -G_t^{[m]T} R_t^{-1}\left(z_t - \hat{z}_t^{[m]} - G_t^{[m]}(\theta_{n_t} - \mu_{n_t,t-1}^{[m]})\right) + (\Sigma_{n_t,t-1}^{[m]})^{-1}(\theta_{n_t} - \mu_{n_t,t-1}^{[m]}), \\ \frac{\partial^2 J_t}{\partial \theta_{n_t}^2} &= G_t^{[m]T} R_t^{-1} G_t^{[m]} + (\Sigma_{n_t,t-1}^{[m]})^{-1}. \end{aligned} \tag{A.33}$$

The new covariance $\Sigma_{n_t,t}^{[m]}$ is given by the inverse of the second derivative:

$$\Sigma_{n_t,t}^{[m]} = \left(G_t^{[m]T} R_t^{-1} G_t^{[m]} + (\Sigma_{n_t,t-1}^{[m]})^{-1}\right)^{-1}. \tag{A.34}$$

Since the new mean $\mu_{n_t,t}^{[m]}$ is the point that maximizes J_t, we solve

$$\left.\frac{\partial J_t}{\partial \theta_{n_t}}\right|_{\theta_{n_t}=\mu_{n_t,t}^{[m]}} = 0, \tag{A.35}$$

i.e.

$$-G_t^{[m]T} R_t^{-1}\left(z_t - \hat{z}_t^{[m]} - G_t^{[m]}(\mu_{n_t,t}^{[m]} - \mu_{n_t,t-1}^{[m]})\right) + (\Sigma_{n_t,t-1}^{[m]})^{-1}(\mu_{n_t,t}^{[m]} - \mu_{n_t,t-1}^{[m]}) = 0. \tag{A.36}$$

Reordering some terms gives

$$\left(G_t^{[m]T} R_t^{-1} G_t^{[m]} + (\Sigma_{n_t,t-1}^{[m]})^{-1}\right)(\mu_{n_t,t}^{[m]} - \mu_{n_t,t-1}^{[m]}) = G_t^{[m]T} R_t^{-1}(z_t - \hat{z}_t^{[m]}). \tag{A.37}$$

By (A.34), the equation becomes

$$(\Sigma_{n_t,t}^{[m]})^{-1}(\mu_{n_t,t}^{[m]} - \mu_{n_t,t-1}^{[m]}) = G_t^{[m]T} R_t^{-1}(z_t - \hat{z}_t^{[m]}). \tag{A.38}$$

Therefore, the new mean $\mu_{n_t,t}^{[m]}$ is given as

$$\mu_{n_t,t}^{[m]} = \mu_{n_t,t-1}^{[m]} + K_t^{[m]}(z_t - \hat{z}_t^{[m]}), \tag{A.39}$$

where the Kalman gain $K_t^{[m]}$ is given as

$$K_t^{[m]} = \Sigma_{n_t,t}^{[m]} G_t^{[m]T} R_t^{-1}. \tag{A.40}$$

Next, we rewrite this $K_t^{[m]}$ in terms of $\Sigma_{n_t,t-1}^{[m]}$ as follows:

$$\begin{aligned}
K_t^{[m]} &= \Sigma_{n_t,t}^{[m]} G_t^{[m]T} R_t^{-1} \\
&= \Sigma_{n_t,t}^{[m]} G_t^{[m]T} R_t^{-1} \left(G_t^{[m]} \Sigma_{n_t,t-1}^{[m]} G_t^{[m]T} + R_t \right) \left(G_t^{[m]} \Sigma_{n_t,t-1}^{[m]} G_t^{[m]T} + R_t \right)^{-1} \\
&= \Sigma_{n_t,t}^{[m]} \left(G_t^{[m]T} R_t^{-1} G_t^{[m]} \Sigma_{n_t,t-1}^{[m]} G_t^{[m]T} + G_t^{[m]T} \right) \left(G_t^{[m]} \Sigma_{n_t,t-1}^{[m]} G_t^{[m]T} + R_t \right)^{-1} \\
&= \Sigma_{n_t,t}^{[m]} \left(G_t^{[m]T} R_t^{-1} G_t^{[m]} + (\Sigma_{n_t,t-1}^{[m]})^{-1} \right) \Sigma_{n_t,t-1}^{[m]} G_t^{[m]T} \left(G_t^{[m]} \Sigma_{n_t,t-1}^{[m]} G_t^{[m]T} + R_t \right)^{-1} \\
&= \Sigma_{n_t,t}^{[m]} (\Sigma_{n_t,t}^{[m]})^{-1} \Sigma_{n_t,t-1}^{[m]} G_t^{[m]T} \left(G_t^{[m]} \Sigma_{n_t,t-1}^{[m]} G_t^{[m]T} + R_t \right)^{-1} \\
&= \Sigma_{n_t,t-1}^{[m]} G_t^{[m]T} \left(G_t^{[m]} \Sigma_{n_t,t-1}^{[m]} G_t^{[m]T} + R_t \right)^{-1} \\
&= \Sigma_{n_t,t-1}^{[m]} G_t^{[m]T} (Q_t^{[m]})^{-1}, \qquad \text{(A.41)}
\end{aligned}$$

where

$$Q_t^{[m]} = G_t^{[m]} \Sigma_{n_t,t-1}^{[m]} G_t^{[m]T} + R_t \qquad \text{(A.42)}$$

Finally, we simplify (A.34) using the *matrix inversion lemma*

$$(C^{-1} + BAB^T)^{-1} = C - CB(A^{-1} + B^T CB)^{-1} B^T C \qquad \text{(A.43)}$$

as

$$\begin{aligned}
\Sigma_{n_t,t}^{[m]} &= \left(G_t^{[m]T} R_t^{-1} G_t^{[m]} + (\Sigma_{n_t,t-1}^{[m]})^{-1} \right)^{-1} \\
&= \Sigma_{n_t,t-1}^{[m]} - \Sigma_{n_t,t-1}^{[m]} G_t^{[m]T} \left(G_t^{[m]} \Sigma_{n_t,t-1}^{[m]} G_t^{[m]T} + R_t \right)^{-1} G_t^{[m]} \Sigma_{n_t,t-1}^{[m]} \\
&= \Sigma_{n_t,t-1}^{[m]} - \Sigma_{n_t,t-1}^{[m]} G_t^{[m]T} (Q_t^{[m]})^{-1} G_t^{[m]} \Sigma_{n_t,t-1}^{[m]} \\
&= \left(I - K_t^{[m]} G_t^{[m]} \right) \Sigma_{n_t,t-1}^{[m]}. \qquad \text{(A.44)}
\end{aligned}$$