Chapter 2

Literature Review

2.1 Slope Stability Analysis

As per the forgone discussion, most failure analysis methods are developed from the limit equilibrium analysis. Those methods evaluate the slope at its limit state and consider the equilibrium of either force, moment or both. Further, all limit equilibrium methods involve two basic steps. Firstly, a trial surface has to be selected and secondly the factor of safety for the selected failure surface is calculated. The most critical failure is the one along which failure is most likely to occur and for which the factor of safety is the lowest.

During the past 70 years, considerable attention has been given to finding a method of calculating the factor of safety. The initial method was developed by Swedish State Railways in 1922 based on circular failure in undrained soils. This method assumes no friction ($\phi = 0$). As a result, it is not really useful because, in most cases, the friction angle ϕ is greater than zero. In order to improve this method, Fellenius (1936) transformed it in to a statically determinate problem by dividing the failure surface into a finite number of vertical slices and assuming that the resultant of normal and shear forces acting on the two sides of each slice are equal in magnitude and co-linear. Consequently, the side forces cancel each other; hence their magnitudes and positions are not needed to be known. This method is known by the ordinary method of slice or the Fellenius method. Later, it was found out that, for shallow circles, the computed factors of safety were approximately 20 percent less than the correct values. For deep and small-radius circles that extend well below the groundwater table, even larger errors are observed (Wright, 1985). Later, Bishop (1955) addressed this problem by assuming that the shear forces on both sides of each slice are equal and the normal forces on both sides of each slice are co-linear, but not necessary equal. Recent researches have shown that this method is sufficiently precise if the critical failure surface is close to circular but has a larger error when the critical failure surface becomes non-circular (Wright, 1985).

Many failures are actually non-circular and various analysis methods have been developed to accommodate irregular shaped failure surfaces. Morgenstern and Price (1965), Spencer (1967), Sarma (1973) and others have developed additional analysis methods to deal with non-circular failure surfaces. Since non-circular failure surfaces are considered, the mathematical simplification of convenient geometry is lost and thus these methods are more complex and difficult to implement unless by a computer. One of the sources of the complexities is statically indeterminacy. Different simplifying assumptions are used to overcome statically indeterminacy by each method.

Among the methods aforementioned, the Morgenstern-Price method is very popular among geotechnical engineers because it combines high precision with less assumptions. It uses a simple assumption regarding the interslice forces, which involves least numerical difficulties. It assumes that the angle of the resultant thrust forces can be describe by an arbitrary function $\lambda f(x)$, where λ is a scaling parameter which has to be calculated simultaneously with the factor of safety *F*. Function $f(x)$ is a user-defined function which reflects the shape of the failure line. For the force equilibrium equations, the Morgenstern-Price method uses differential forms and, for the moment equilibrium equation, it uses an integral form. Later, a method was proposed also by Morgenstern and Price (1965) to solve these equations using the Newton-Raphson method. With those assumptions, several researches have derived various forms of force and moment equilibrium equations and several schemes for solving λ and F have come out. This rigorous limit equilibrium method, which satisfies both force and moment equilibrium simultaneously, is recommended for general shaped failure surfaces.

Spencer (1967) simplified the Morgenstern-Price method using one special case of it. The Spencer method assumes that the interslice forces for the each slice are parallel to each other but not necessary co-linear. Therefore, the angles of the interslice resultant forces are equal for all the slices through the failure surface. In effect, $f(x)$ in the Morgenstern-Price method is set to one.

Another method was proposed by Sarma (1973) as a means of computing the critical horizontal acceleration that is required to bring the mass of soil bounded by the slip surface and the free surface to the state of limiting equilibrium. This critical acceleration is considered as a measure of the static factor of safety. Though it has a simple iteration process, it has some difficulties when dealing with complex inhomogeneous conditions and the reliability of results (Wright, 1985). Therefore, this method is not popular among the geotechnical engineers.

Above mentioned are a few methods famous among researchers to calculate the factor of safety for a given failure surface. Later, some developments have been introduced but they are mostly based on the aforementioned methods. Searching for the critical failure surface with the lowest factor of safety is always tedious. The numbers of trial failures are chosen by a process of trial and error with thoughtful search pattern and the analysis is repeated for each until the lowest F is obtained. For a simple homogeneous slope where soil parameters are uniform throughout, D.W. Tailor (1948) made a series of charts for determining the minimum factor of safety by using total stress analysis and ignoring the possibilities of tension crack. The charts are produced based on the slope geometry for approximation of the circular failure surface.

Analytically, indentifying the critical failure surface from all possible slip surfaces requires the calculus of variations. Baker and Garber (1978) used the calculus of variations to locate the critical failure surface and thereby calculated the minimum factor of safety. With the advent of high performance computers, use of optimization techniques is becoming an effective means of finding the critical failure surface. In late 70’s and 80’s, researchers were more keen on traditional optimization techniques. Baker (1980) used dynamic programming to determine the critical failure surface. The Spencer method is used in conjunction with a minimization scheme based on dynamic programming to construct a computation procedure by which both the critical failure surface and the minimal factor of safety are determined simultaneously. Yamagami and Ueta (1988) enhanced Baker's approach by combining it with the finite element stress analysis to calculate the factor of safety more accurately. They assumed the critical failure surface to be a chain of linear segments connecting two state points located in two successive stages. The resisting and actuating forces used in the analysis are determined from stresses interpolated from Gaussian points within the domain of the problem. Recently, Pham and Fredlund (2003) also proposed a dynamic programming method that is combined with a finite element stress analysis to determine the stress state in the soil mass and then determine the shape and location of the critical slip surface and the corresponding factor of safety. Ngugen (1985) developed a method where the factor of safety is formulated as a multivariate function $F(x)$ with the independent variable x describing the geometry of the failure surface. He employed the simplex method as the optimization technique. Li and White (1987) also proposed a more efficient one-dimensional optimization technique associated with Tailor's optimization procedure. In this case, the failure surface is located by using the alternating variable method. Greco (1996) proposed a Monte-Carlo based technique, where it locates the critical slip surface by the random walking method. By generating trial failures close to the current best solution by slightly modifying it at random, the most critical failure is obtained. However, too many constrains are required for implementation of this method as an automatic search. Later, Malkawi (2001) also developed an approach based on the Monte-Carlo technique to find the critical failure surface. A large number of trail failure surfaces are generated to ensure the minimum factor safety by avoiding local optima. The main draw back in this method is that the search space must be tightly controlled in order to reduce the amount of unproductive computation. Even with this remedy, the method still needs a large number of solutions to guarantee the minimum factor of safety (Sengupta and Upadhyay, 2008).

In the recent development of evolutionary algorithms, the genetic algorithms (GAs) have been receiving a lot of attention and successfully used by several researchers. They are inspired by the basic mechanisms of natural evolution. A simple GA solves problems by simulating the natural evolution processes and uses only three genetic operators namely, reproduction, crossover and mutation. Among the researchers who used GAs, Goh (2000) and McCombie and Wilkinson (2002) proposed GA based methods in conjunction with the Bishop simplified method to analyse soil slopes. Three-dimensional chromosome coding, containing the x and y coordinates of the centre of the circular failure surface and its radius, is used. The Bishop simplified method is used to analyse the factor of safety. Later, Zolfaghari et al. (2005) has extended the GA search for the general cases including non-circular failures also. The definition of the surface is done by the change of angle at points along rather than using y coordinates. Das (2005) also used a GA for analysis of slope stability problems. The objective function is derived using the wedge method by dividing the failure mass into total three wedges. The minimum factor of safety is obtained by solving six nonlinear equations using a GA. The application of the proposed method is limited to few exceptional cases. Jianping et al. (2008) proposed a method that combines spline curves with a GA. A potential slip surface is divided into segments by a number of nodal points and each pair of nodal points is connected by a spline to generate a smooth curve and a GA is used to find the critical failure surface. In addition, the Spencer method is employed to calculate the factor of safety.

Even though GAs might have been the one of the better optimization tools used in the recent past, GAs have their own drawbacks. For example, GAs always have to work with binary strings instead of real parameter sets, which can be inconvenient in slope stability problems. Recently, few evolutionary algorithms have been developed such as Particle Swam Optimization (PSO), Fish Swam Algorithm (FSM). Cheng et al. (2007) proposed a method based on PSO and later same proposed another method based on FSO. Both the methods do not have significant differences except for the number of iteration criteria. In addition, both the methods have similar approaches to locate the critical slip surface. A Cartesian system is used to represent the problem domain and ground profile which are represented by functions of x and y. The method of generating failure surfaces is similar to that method proposed by Greco (1996), Malkawi et al. (2001).

It is noticeable that there are many types of optimization algorithm that have been used with different limit equilibrium methods in solving slope stability analysis. ACO has been used very limitedly in the field of civil engineering applications, especially in the geotechnical engineering field.

2.2 Ant Colony Optimization

Ants first evolved million years ago, taking form in over ten thousand different species. They are considered one of the most adaptable insects due to their highly organized and socialized colonies consisting thousands or sometimes millions of ants. One significant aspect of ants is their ability to create “ant paths”, which are long, bi-directional lane of single file pathways in which they navigate landscape in order to reach food sources and their nest in the optimal time. These tasks are made possible by the use of pheromone which guides them to find the shortest path. This technique allows an adaptive routing system which is updated until the more optimal path is found.

Based on the aforementioned foraging behaviour of ants, ACO was proposed by Colorni, Dorigo and Maniezzo (1991) for discreet optimizations. The main idea is that of a parallel search over several constructive computational threads based on local problem data and on a dynamic memory structure containing information on quality of previously obtained results. Since the ACO technique utilizes information from many search points at the same time, the ACO technique can be considered as a global-search algorithm. Recently, the ACO technique is becoming popular among researchers in the field of heuristic optimization. The land mark problem is the travelling salesman problem (Dorigo and Gambardella, 1997). The travelling salesman problem (TSP) is the problem of finding the shortest closed tour which visits all the cities in a given set. Since it is identically fit for the model of food foraging behaviour of ant colonies, the ACO is utilized to find the optimal rout for the travelling salesmen.

Later the ACO technique has been applied to various other disciplines of problems. Among them, ACO is extensively used in the fields of transportation and logistics management, telecommunication, and computer networking. For example, McMullen (2001) used the ACO approach to address a just-in-time sequencing problem with multiple objectives in manufacturing logistic problems. In this work, setups and stability of material usage rate in production sequence is optimized using an ACO algorithm. Gravel et al. (2002) used ACO for finding the solution of an industrial scheduling problem in an aluminium casting centre. They have represented an efficient representation of continuous horizontal casting process which takes account of a number of objectives that are important to scheduler. Lee et al. (2002) introduced ACO to solve a weapon and target assignment problem successfully. The algorithm is successfully implemented to find a proper assignment of weapons to targets with the objective of minimizing the expected damage of own-force assets. Moreover, frequency assignment problem by Maniezzo and Carbonaro (2000), optimization problems for designing and scheduling of batch plants by Jayaraman et al. (2000), quadratic assignment problem by Talbi et al. (2001), two-machine flow-shop scheduling problem by T’kindt et al. (2003), optimization of the keyboard arrangement by Eggers et al. (2003) and solving the mesh-partitioning problem by Korošec et al. (2004) are a fewer examples in which the ACO algorithms are used successfully.

As mentioned earlier, the application of this technique in the field of civil engineering is still rare. Among them, Abbaspour (2001) introduced an adaptation and application of the ACO technique to the inverse elimination of unknown parameters in unsaturated flow models. Maier (2003) used ACO for the optimal design of water distribution system. Also, Nanakorn et al. (2002) proposed an ACO based optimization method for structural design optimization. They have applied this technique to sizing optimization of trusses and come up with better results compared to other optimizations. Camp et al.(2004) used this algorithm to develop a discrete optimization algorithm for steel frames. The objective function considered is the total weight (or cost) of the structure subjected to serviceability and strength requirements. The design of steel frames is mapped into a modified travelling salesman problem (TSP) where the configuration of the TSP network reflects the structural topology, and the resulting length of the TSP tour corresponds to the weight of the frame. The resulting system was minimized using an ACO algorithm with a penalty function to enforce strength and serviceability constraints. Jalali et al. (2006) used ACO for multi reservoirs operation. They have approached this problem by considering a finite horizon with a time series of inflow, classifying the reservoir volume to several intervals, and deciding for releases at each period with respect to a predefined optimality criterion. It can been seen from these examples that the ACO technique has been successfully used in a wide range of problems. It has also been reported that, compared to other heuristic optimization methods, ACO is competitive and advantageous in large problems.