Chapter 4

Results

The validity and effectiveness of the proposed ACO algorithm is investigated by solving several diverse examples. The examples are selected to cover most of the areas incorporated in the field of geotechnical engineering in terms of slope stability analysis. To demonstrate the robustness of the proposed algorithm, problems with homogeneous and inhomogeneous soils are included. Some of the problems consider the effects of a horizontal earthquake force. In order to examine both the quality and uniformity of the obtained results, the algorithm is run for more than 100 times for each problem. The best solution of a run is the best solution found in that run. Consequently, since there are n runs for each problem, there will be n best solutions obtained. Among these best, for each problem, the minimum, average, maximum factors of safety of these best solutions are reported with the standard deviation SD. The Figure 4.1 schematically shows the process to obtain statistics of a problem.

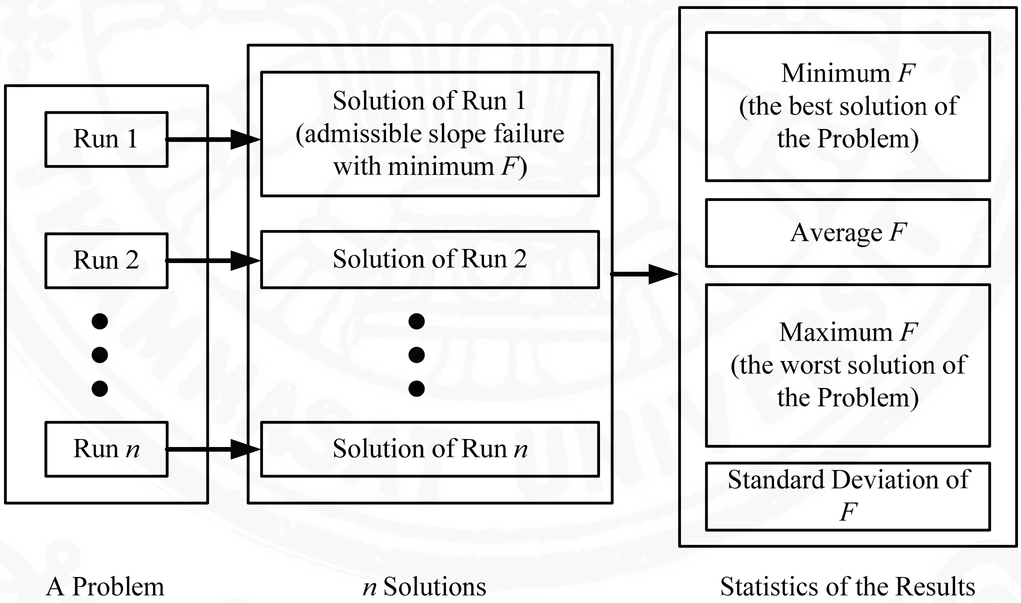

Figure 4.1 Results and the extracted information

In all the problems, the scaling constant Z in the bilinear scaling shown in Figure 3.7 is set to two. This means that the quality value of the best solution found in a tour is scaled to be equal to Z times those of the average solutions.

The values of the ACO parameters used in this study are obtained from a parametric investigation on a homogeneous soil slope. The investigation is performed by varying all the ACO parameters and the combination of these parameters that provides good results is used to solve the problems in this study. As it has come out, the values of the ACO parameters that give good results are similar to those employed by Nanakorn et al. (2002). In this study, all slices have the same width. The slice width has to be sufficiently small in order that accurate results are obtained. The slice widths used in all the examples have been selected in such a way that there is no meaningful improvement when smaller widths are used.

4.1 Example 1

The first example is abstracted from the work by Yamagami and Ueta (1988), where it was analyzed by using several nonlinear programming techniques. Figure 4.2 shows the geometry of the slope. The soil is homogeneous with the soil parameters given in Table 4.1. In the original research by Yamagami and Ueta (1988), the Morgenstern-Price method is used in conjunction with the DFP, BFGS, Powell and simplex methods. In their work, the function $f(x)$ in Equation (3.17) is also assumed to be one. Besides Yamagami and Ueta (1988), many researchers have engaged in this problem by employing different methods and they have also come up with satisfactory results. Among them, Greco (1996) used the Spencer method, which is equivalent to the Morgenstern-Price method with $f(x)=1$, together with the pattern search and Monte-Carlo methods for finding the factor of safety. Similarly, Malkawi et al. (2001) used the ordinary method of slice and the Monte-Carlo technique for the solutions. Further, Solati and Habibagahi (2006) and Jianping et al. (2008) used GA to solve this problem.

The ACO parameters used in this problem are shown in Table 4.2. In addition, 120 vertical slices with the width of 0.25 m, are used for the failing soil mass. The ranges of choices for the variables $\hat{x}$, α_1, and $\Delta\alpha_i$ are shown in Table4.3. For all examples in this study, the value ranges for α_1 are approximately taken from the Rankine failure angle range, which are between $45° - \frac{\phi}{2}$ to $45° + \frac{\phi}{2}$. As aforementioned, $\Delta\alpha_i$ is always assumed negative in order to avoid unrealistic slip surfaces. As a result, the available choices for $\Delta\alpha_i$ are set in a negative range.

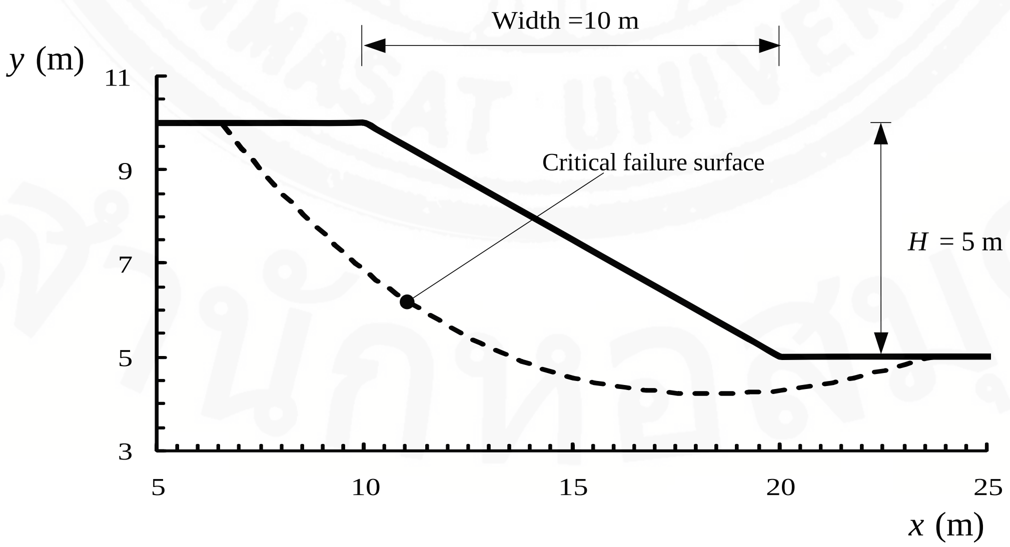

Figure 4.2 Slope geometry and critical failure surface—Example 1

Table 4.1 Soil parameters—Example 1

Coefficient of cohesion c (kPa)	9.8
Friction angle ϕ (degree)	10.0
Density γ (kN/m^3)	17.64

Table 4.2 ACO parameters—Example 1

Number of ants N_{ants}	300
Number of tours N_{tour}	200
Evaporation rate ρ	0.3
Scaling constant Z	2.0

Table 4.3 Search parameters—Example 1

	Min	Max	Interval size
$\hat{x}$ (m)	0	10	0.25
α_1 (degree)	40	50	0.1
$\Delta\alpha_i$ (degree)	-4.0	0.0	0.04

For this problem, the algorithm is run for 500 times. The statistics of the 500 solutions obtained from the 500 runs are shown in Table 4.4. It can be seen that the minimum factor of safety of 1.311 is obtained for this problem. The obtained standard deviation results in the coefficient of variation of only 0.192, which demonstrates the uniformity of the solutions.

Table 4.4 Statistics of the results—Example 1

Minimum F	1.311
Average F	1.556
Maximum F	2.966
SD	0.298

Table 4.5 shows the comparison of the results from this study and the literature. It can be seen that the result obtained in this study comparable with the results found in the literature. Note that different limit equilibrium methods have different accuracy and will normally give different factors of safety even for the same slope. As a result, objective performance comparisons of different optimization methods used in slope stability analysis can be obtained only when the same limit equilibrium method is used. The factor of safety obtained in this study is found to be smaller and, therefore, better than all others that are obtained by the Morgenstern-Price method.

Table 4.5 Result comparison—Example 1

Source	Limit equilibrium method	Optimization method	F
Yamagami and Ueta (1988)	Morgenstern-Price [$f(x)=1$]	BFGS	1.338
		DFD	1.338
		Powell	1.338
		Simplex	1.339–1.348
Greco (1996)	Morgenstern-Price [$f(x)=1$]	Pattern search	1.326-1.330
		Monte Carlo	1.327-1.333
Malkawi et al. (2001)	Ordinary method of slice	Monte Carlo	1.238
Solati and Habibagahi (2006)	Janbu	GA	1.380
Jianping et al. (2008)	Morgenstern-Price [$f(x)=1$]	GA+Spline	1.321
		GA+Line	1.324
This study	Morgenstern-Price [$f(x)=1$]	ACO	1.311

Figure 4.3 shows the convergence of the solution. It can be seen that the convergence starts at around the 50th iteration and the factor of safety converges to its minimum value in the 107th iteration.

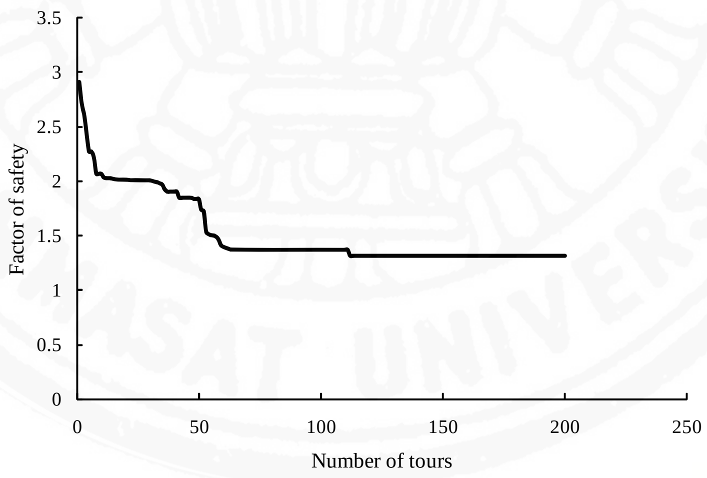

Figure 4.3 Convergence of the factor of safety—Example 1

4.2 Example 2

This example is abstracted from Fredlund and Krahn (1977) and Figure 4.4 shows the geometry of the slope used. Recently, Solati and Habibagahi (2005) successfully analyzed this slope using a genetic algorithm combined with the Janbu method of limit equilibrium. The soil is assumed homogeneous and the related soil parameters are given in Table 4.6. The ACO parameters used for the analysis are given in Table 4.7 and the search parameters are given in Table 4.8. Furthermore, a pseudo-static coefficient for horizontal earthquake loading a_h of 0.1 is used for the analysis and the results are compared with the available literatures.

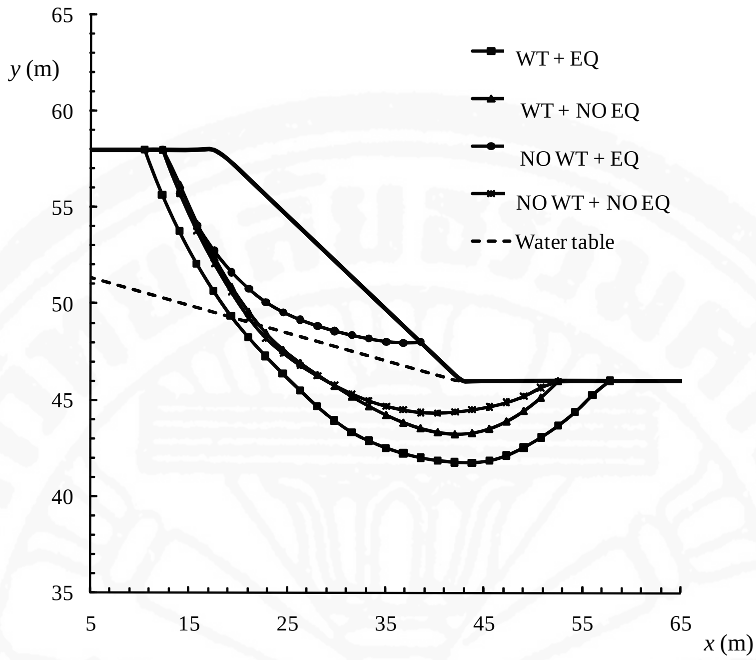

Figure 4.4 Slope geometry and critical failure surface—Example 2

Table 4.6 Soil parameters—Example 2

Coefficient of cohesion c (kPa)	29.0
Friction angle ϕ (degree)	20.0
Density γ (kN/m^3)	19.4

Table 4.7 ACO parameters—Example 2

Number of ants N_{ants}	300
Number of tours N_{tour}	100
Evaporation rate ρ	0.3
Scaling constant Z	2.0

Table 4.8 Search parameters—Example 2

	Min	Max	Interval size
$\hat{x}$ (m)	0	20	0.25
α_1 (degree)	35	55	0.2
$\Delta\alpha_i$ (degree)	-2.5	0.0	0.025

For this problem, the algorithm is run for 100 times. The statistics of the 100 solutions obtained from the 100 runs are shown in Table 4.9. The standard deviations are comparatively small and the coefficients of variations for each respective case are 0.129, 0.051, 0.202, and 0.061. This indicates the uniformity of the obtained solutions.

Table 4.9 Statistics of the results—Example 2

	WT + EQ	WT + NO EQ	NO WT + EQ	NO WT + NO EQ
Minimum *F*	1.432	1.801	1.799	1.996
Average *F*	1.711	1.842	2.294	2.067
Maximum *F*	2.451	2.489	3.415	2.602
SD	0.221	0.094	0.464	0.126

The convergence of the factor of safety is shown in Figure 4.5. In addition, Table4.10 gives the required numbers of iteration for the convergence.

Table 4.10 The required numbers of iteration for the convergence—Example 2

	Required number of iteration
EQ + WT	80
EQ + NO WT	66
NO EQ + WT	49
NO EQ + NO WT	56

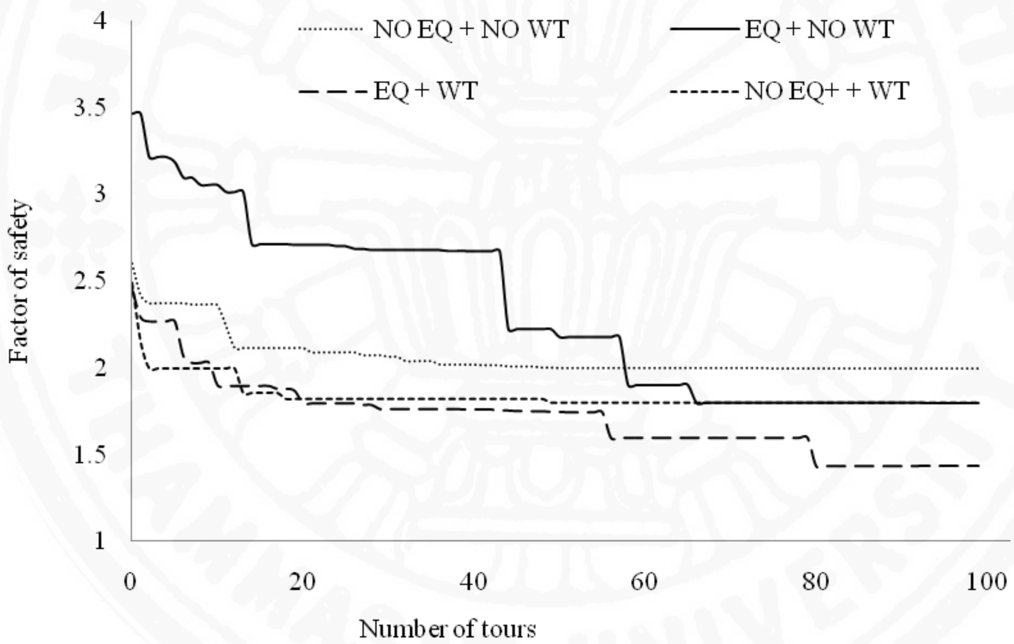

Figure 4.5 Convergence of the factor of safety—Example 2

Table 4.11 shows the comparison of the results from this study and the literature. In this connection, for all cases, it can be observed that the result obtained in this study is comparable with the results found in the literature. The factor of safety achieved in this study is found to be smaller compared with those who used the Morgenstern–Price method as the limit equilibrium method. As it is obviously expected, an increase of water pressure decreases the value of factor of safety and application of pseudo-static horizontal earthquake loading decreases the factor of safety drastically for a given slope even in homogenous soil.

Table 4.11 Result comparison—Example 2

Source	Limit equilibrium method	Optimization method	*F* Without groundwater (NO WT)	*F* With groundwater (WT)
Fredlund and Krahn (1977) Without earthquake (NO EQ)	Ordinary method of slice	Best fit regression	1.928	1.693
	Simplified Bishop		2.080	1.834
	Janbu’s rigorous		2.008	1.776
	Morgenstern-Price [$f(x)=1$]		2.073	1.830
	Janbu’s simplified		2.041	1.827
Solati and Habibagahi (2006) Without earthquake (NO EQ)	Janbu's rigorous	GA	2.079	1.840
This study Without earthquake (NO EQ)	Morgenstern-Price [$f(x)=1$]	ACO	1.996	1.801
With earthquake (EQ)		ACO	1.799	1.432

Further, Fredlund and Krahn (1977) used circular failures instead of non-circular failures which might give higher values of factor of safety even though it had been implemented with a solid theoretical numerical technique combined with a graphical method. Therefore, the geometry of the failure surface is a critical factor in slope stability analysis. In the case of Solati and Habibagahi (2006), a power heuristic optimization method of GA has been used for finding the critical failure surface. In addition, the same optimization method is employed to minimise an objective function which includes *F* in order to solve the equations from Janbu’s equilibrium method. It should be noted that GAs are optimization algorithm. Solving equations using a GA, though possible, may not yield accurate results. As a matter of fact, there are better algorithms for solving nonlinear equations, for example, the Newton-Raphson method.

4.3 Example 3

A multilayer slope consisting of a sand layer (Layer 1) over a clay layer (Layer 2) with a water table shown in the Figure 4.6 is analyzed. There is a firm hard layer located underneath the clay layer. The example is taken from Joseph Spigolon (2001). The soil parameters for the sand layer and clay layer are given in the Table 4.12. Two loading cases with and without the effect of earthquake loading are considered. The effect of pseudo-static horizontal earthquake loading is considered as 0.1 of the weight of the slice. The slice width is taken as 1 ft. Also, the values of all ACO parameters as well as the ranges of $\hat{x}$, α_1, and $\Delta\alpha_i$ used for this problem are shown in Tables 4.13 and 4.14. The obtained results are presented in Table 4.16. Similar to the previous examples, good agreement can be observed in the comparison with the results from the literature.

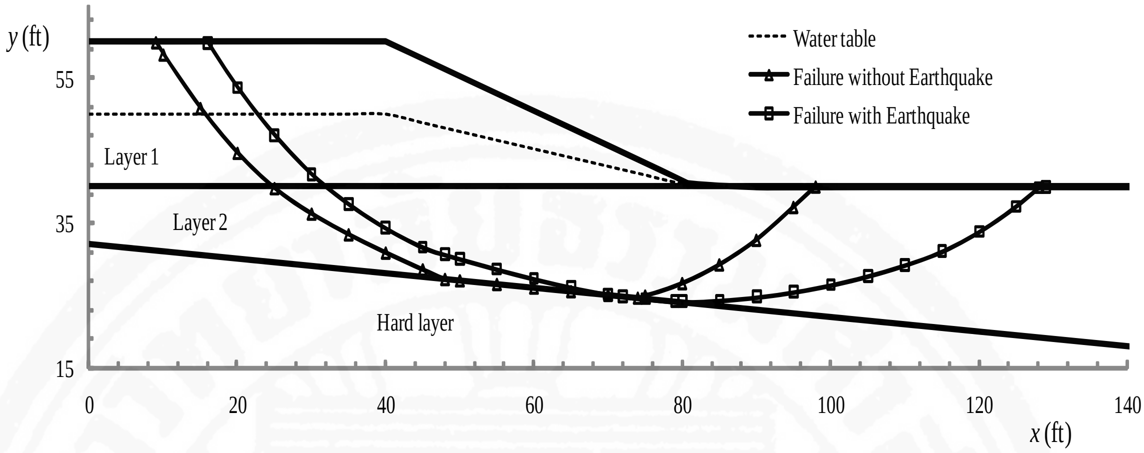

Figure 4.6 Slope geometry and critical failure surface—Example 3

Table 4.12 Soil parameters—Example 3

	Layer 1	Layer 2
Coefficient of cohesion *c* (psf)	0.0	600.0
Friction angle ϕ (degree)	25.0	25.0
Density γ (pfc)	120.0	92.0

Table 4.13 ACO parameters—Example 3

Number of ants N_{ants}	300
Number of tours N_{tour}	100
Evaporation rate ρ	0.3
Scaling constant *Z*	2.0

Table 4.14 Search parameters—Example 3

	Min	Max	Interval size
$\hat{x}$ (ft)	0	40	1.0
α_1 (degree)	32	57	0.25
$\Delta\alpha_i$ (degree)	-2.0	0.0	0.02

For this problem, the algorithm is run for 100 times. The statistics of the solutions obtained from the 100 runs are shown in Table 4.15. The SD value for the case with earthquake has a slightly higher value than the case without earthquake and the coefficients of variation for both cases are 0.115 and 0.354.

Table 4.15 Statistics of the results—Example 3

	Without EQ	With EQ
Minimum *F*	1.096	0.841
Average *F*	1.208	1.337
Maximum *F*	1.769	5.301
SD	0.139	0.473

For this example, the convergence starts for the case without earthquake (NO EQ) around the 10th iteration and the minimum value of F =1.096 is obtain in the 26th iteration. For the case with earthquake (EQ), the convergence starts around the 43rd iteration. The factor of safety converges to its minimum value of F =0.841 at the 67th iteration. Figure 4.7 shows the convergence in each case.

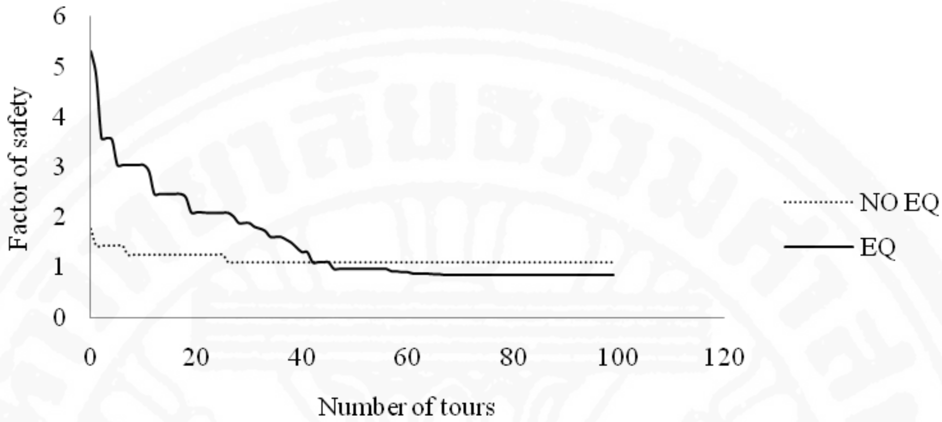

Figure 4.7 Convergence of the factor of safety—Example 3

The obtained results are also presented in Table 4.16. Similar to the previous examples, good agreement can be observed in the comparison with the result by Joseph Spigolon (2001). In this particular problem Joseph Spigolon (2001) used the wedge method to calculate the factor of safety by considering only 3 wedges. It is evident from his result that the wedge method can be used only for rough estimation of slopes rather than precise estimation.

The effect of earthquake force can be clearly seen in the reduction of factor of safety as well as the change in the position of the failure line as shown in Figure 4.6. Under the earthquake force, the failing mass becomes massive and will cause massive damages in the real case scenario.

Table 4.16 Result comparison—Example 3

Source	Limit equilibrium method	Optimization method	F
Joseph Spigolon (2001)	Method of slice	—	1.270
This study (without earthquake)	Morgenstern-Price [$f(x) = 1$]	ACO	1.096
This study (with earthquake)		ACO	0.841

4.4 Example 4

This example is also from the work by Yamagami and Ueta (1988). Figure 4.8 shows the geometry of the slope. The slope consists of four layers of different soils and the boundaries between the layers are horizontal. The soil and ACO parameters can be found in Table 4.17 and Table 4.18, respectively. Same as Example 1, in addition to Yamagami and

Ueta (1988), Greco (1996), Malkawi et al. (2001), Solati and Habibagahi (2006), and Jianping et al. (2008) also solved this problem. In the present study, 500 slices, with the width of 0.5 m, are used for the failing soil mass. The ranges of choices for the variables $\hat{x}$, α_1, and $\Delta\alpha_i$ are shown in Table 4.19.

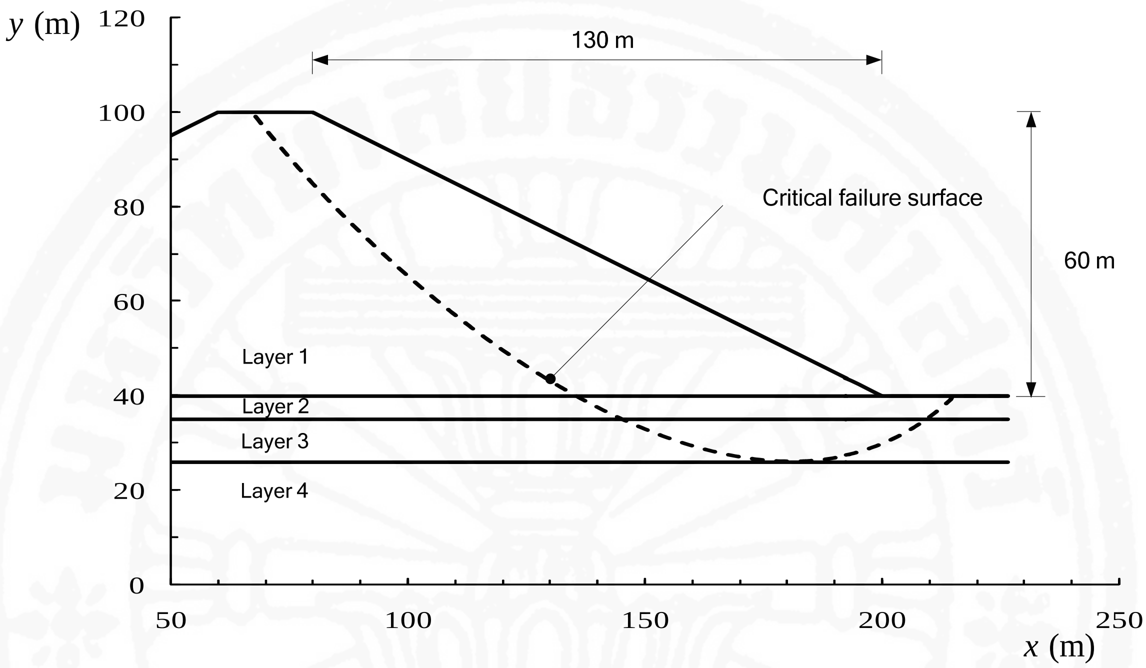

Figure 4.8 Slope geometry and critical failure surface—Example 4

Table 4.17 Soil parameters—Example 4

	Layer 1	Layer 2	Layer 3	Layer 4
Coefficient of cohesion c (kPa)	49.0	0.0	7.84	0.0
Friction angle ϕ (degree)	29.0	30.0	20.0	30.0
Density γ (kN/m^3)	20.38	17.64	20.38	17.64

Table 4.18 ACO parameters—Example 4

Number of ants N_{ants}	500
Number of tours N_{tour}	1000
Evaporation rate ρ	0.3
Scaling constant Z	2.0

Table 4.19 Search parameters—Example 4

	Min	Max	Interval size
$\hat{x}$ (m)	40	80	0.5
α_1 (degree)	30	60	0.3
$\Delta\alpha_i$ (degree)	-2.0	0.0	0.02

For this problem, the algorithm is run for 500 times. The statistics of the results are shown in the Table 4.20 and it can be seen that the minimum factor of safety of 1.348 is obtained for this problem. The obtained standard deviation results in the coefficient of variation of only 0.112. Therefore, it can be seen that the result obtained from the proposed algorithm is in good agreement with the results from the literatures.

Table 4.20 Statistics of the results—Example 4

Minimum *F*	1.348
Average *F*	1.614
Maximum *F*	2.166
SD	0.181

Figure 4.9 shows the convergence of *F* with respect to the number of tours. The factor of safety converges at the 237th iteration with the value of 1.348.

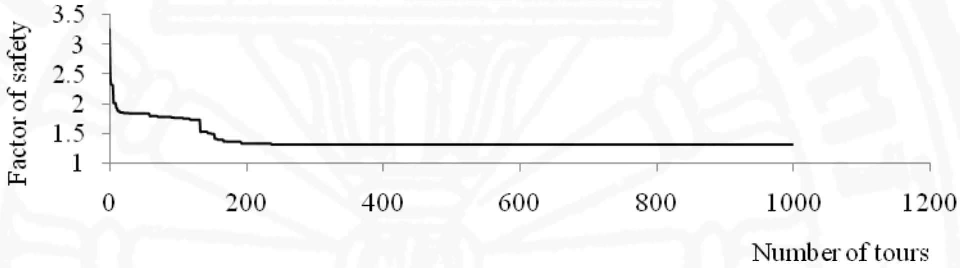

Figure 4.9 Convergence of the factor of safety—Example 4

Table 4.21 shows the comparison of the results from this study and the literature. It can be seen that the result obtained in this study is far better than the results found in the available literatures. The factor of safety obtained in this study is found to be smaller and, therefore, better than all others that are obtained by the Morgenstern-Price method.

Table 4.21 Result comparison—Example 4

Source	Limit equilibrium method	Optimization method	*F*
Yamagami and Ueta (1988)	Morgenstern-Price [$f(x)=1$]	BFGS	1.423
		DFD	1.453
		Powell	1.402
		Simplex	1.405
Greco (1996)	Morgenstern-Price [$f(x)=1$]	Pattern search	1.400
		Monte Carlo	1.401
Malkawi et al. (2001)	Ordinary method of slice	Monte Carlo	1.330
Solati and Habibagahi (2006)	Janbu	GA	1.460
Jianping et al. (2008)	Morgenstern-Price [$f(x)=1$]	GA+Spline	1.395
		GA+Line	1.395
This study	Morgenstern-Price [$f(x)=1$]	ACO	1.348

4.5 Example 5

The fifth example is shown in Figure 4.10. Similar to the previous problem, there are four layers of different soils. Nevertheless, the boundaries between the soil layers in this problem are not horizontal. The problem was analyzed by Zolfaghari et al. (2005) and Cheng et al. (2008). Zolfaghari et al.(2005) and Cheng et al. (2008) both used the Morgenstern-Price method with $f(x)=1$ as their limit equilibrium methods. However, Zolfaghari et al.(2005) employed a GA while Cheng et al. (2008) used a fish swarms algorithm for the minimization of the factor of safety.

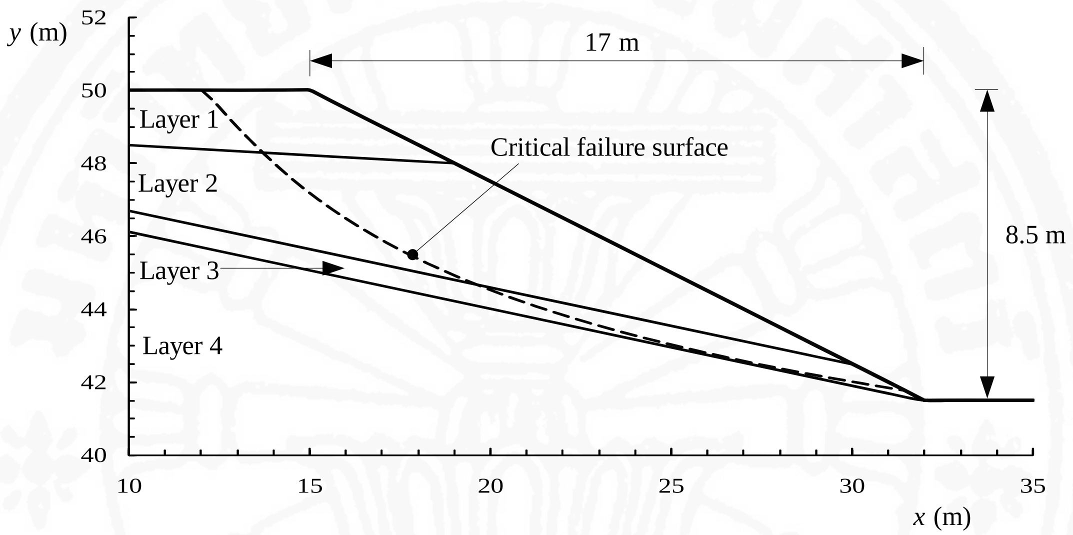

Figure 4.10 Slope geometry and critical failure surface—Example 5

The soil and ACO parameters used in this study can again be found in Table 4.22 and Table 4.23. In the analysis, the total search domain is dived into 150 slices, each with the width of 0.25 m. The ranges of choices for the search parameters $\hat{x}$, α_1, and $\Delta\alpha_i$ can be found in Table 4.24.

Table 4.22 Soil parameters—Example 5

	Layer 1	Layer 2	Layer 3	Layer4
Coefficient of cohesion c (kpa)	14.7	16.7	4.9	34.3
Friction angle ϕ (degree)	20	21	10	28
Density γ (kN/m^3)	18.63	18.63	18.63	18.63

Table 4.23 ACO parameters—Example 5

Number of ants N_{ants}	500
Number of tours N_{tour}	1000
Evaporation rate ρ	0.3
Scaling constant Z	2.0

Table 4.24 Search parameters—Example 5

	Min	Max	Interval size
$\hat{x}$ (m)	10	15	0.25
α_1 (degree)	35	55	0.2
$\Delta\alpha_i$ (degree)	-2.0	0.0	0.02

For this problem, the algorithm is run for 500 times. Table 4.25 shows the statistics of the results and the minimum factor of safety of 1.361 is obtained for this problem. The standard deviation and the coefficient of variation which is equal of 0.195 indicate that the results are distributed within a narrow range.

Table 4.25 Statistics of the results—Example 5

Minimum *F*	1.361
Average *F*	1.662
Maximum *F*	3.401
SD	0.316

The results in Table 4.26 show that the proposed result is found to be better than the result from the GA by Zolfaghari et al.(2005).

The factor of safety converges at the 412th iteration with the value of 1.361. Figure 4.11 shows the convergence of *F* with respect to the number of tours.

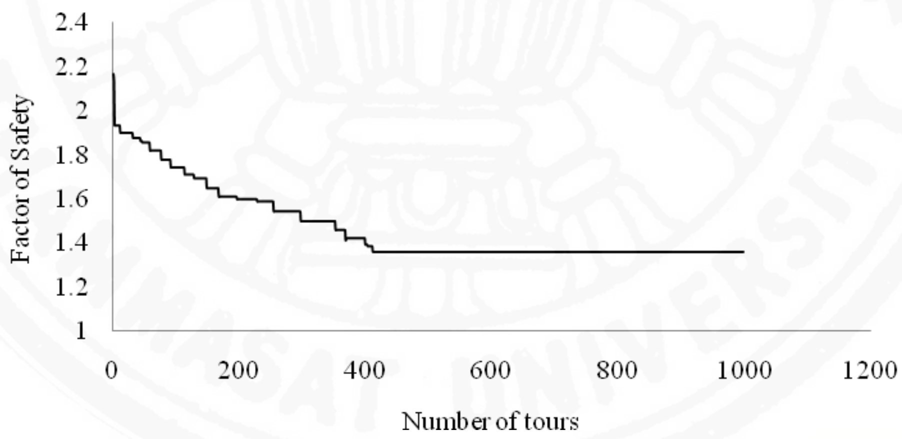

Figure 4.11 Convergence of the factor of safety—Example 5

Table 4.26 Result comparison—Example 5

Source	Limit equilibrium method	Optimization method	*F*
Zolfaghari et al. (2005)	Morgenstern-Price [$f(x)=1$]	GA	1.500
Cheng et al. (2008)	Morgenstern-Price [$f(x)=1$]	AFSA	1.110
This study	Morgenstern-Price [$f(x)=1$]	ACO	1.361

Similar to the previous case, the non-circular failure surface is drawn towards the weakest layer, resulting in a lower factor of safety. It can be seen from the failure shown in Figure 4.10 that the ACO search strategy is clearly able to find the path through the weaker layer, resulting in the lower value of *F*.

4.6 Example 6

The slope for the last example is shown in Figure 4.12. It consists of four layers of different soils as well as a water table. Except for the boundary between the first and second layers where slight inclination of the boundary exists, the other layer boundaries are horizontal. In addition to the groundwater, the pseudo-static coefficient for horizontal earthquake loading of 0.1 will also be considered. Four loading cases are considered for the analysis: water pressure and earthquake loading (WT+EQ), only water pressure without earthquake loading (WT+NO EQ), earthquake loading with no effect of water pressure (NO WT+EQ), and finally the normal situation where there is no effect of water pressure and earthquake load (NO WT+NO EQ).

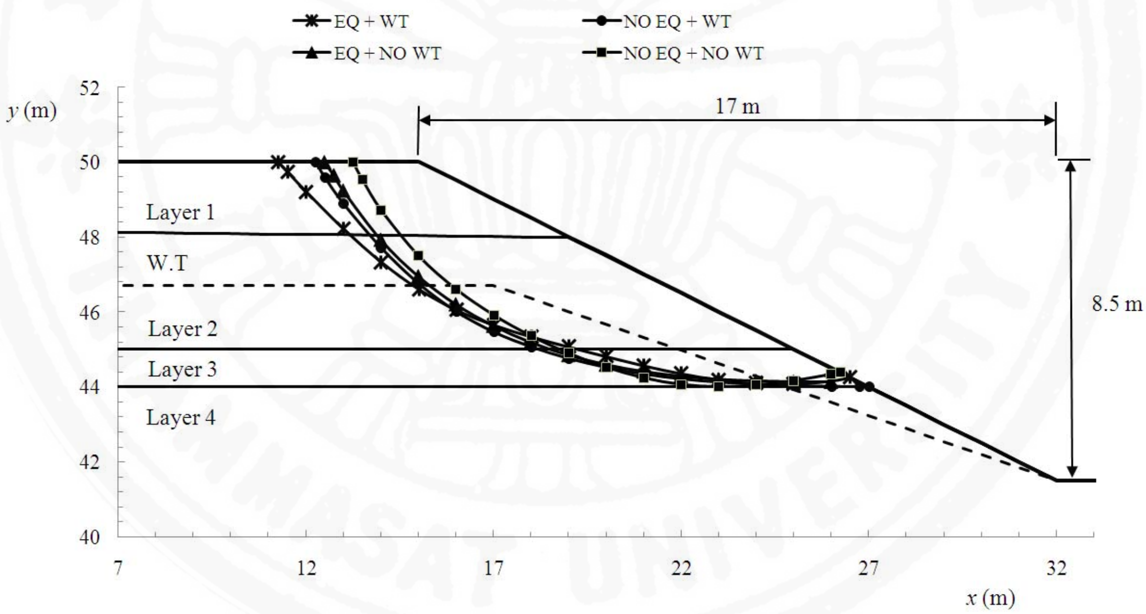

Figure 4.12 Slope geometry and critical failure surface—Example 6

This example was also analyzed by Zolfaghari et al. (2005) by using the same method as that used in the previous example. The soil and ACO parameters used in this study are shown in Table 4.27 and Table 4.28. In the analysis, 120 slices, each with the width of 0.25 m, are used in total search area. In addition, the ranges of choices for the variables $\hat{x}$, α_1, and $\Delta\alpha_i$ is given in Table 4.29.

Table 4.27 Soil parameters—Example 6

	Layer 1	Layer 2	Layer 3	Layer4
Coefficient of cohesion c (kpa)	14.7	16.7	4.9	34.3
Friction angle ϕ (degree)	20	21	10	28
Density γ (kN/m^3)	18.63	18.63	18.63	18.63

Table 4.28 ACO parameters—Example 6

Number of ants N_{ants}	500
Number of tours N_{tour}	1000
Evaporation rate ρ	0.3
Scaling constant Z	2.0

Table 4.29 Search parameters—Example 6

	Min	Max	Interval size
$\hat{x}$ (m)	10	15	0.25
α_1 (degree)	35	55	0.2
$\Delta\alpha_i$ (degree)	-2.0	0.0	0.02

As mentioned above the problem is solved with and without the groundwater as well as the horizontal earthquake loading. For this problem, the algorithm is run for 500 times. Table 4.30 shows the statistics of the obtained results and the coefficients of variation in all cases are 0.102, 0.185, 0.226, 0.171, which fall under a satisfactory range.

Table 4.30 Statistics of the results—Example 6

	EQ+WT	NO EQ+WT	EQ+NO WT	NO EQ+NO WT
Minimum F	0.846	1.377	1.091	1.501
Average F	1.504	1.678	1.374	1.875
Maximum F	1.446	2.355	2.977	2.938
SD	0.153	0.310	0.311	0.321

Table 4.31 gives the required numbers of iteration for the convergence while Figure 4.13 graphically shows each convergence with respect to the number of tours. In fact, it can be recommended from the results obtained from all the examples that, when at least 300 ants are used, at least 100 tours should be allowed in the calculation.

Table 4.31 The required numbers of iteration for the convergence—Example 6

	Required number of iteration
EQ + WT	477
EQ + NO WT	346
NO EQ + WT	411
NO EQ + NO WT	431

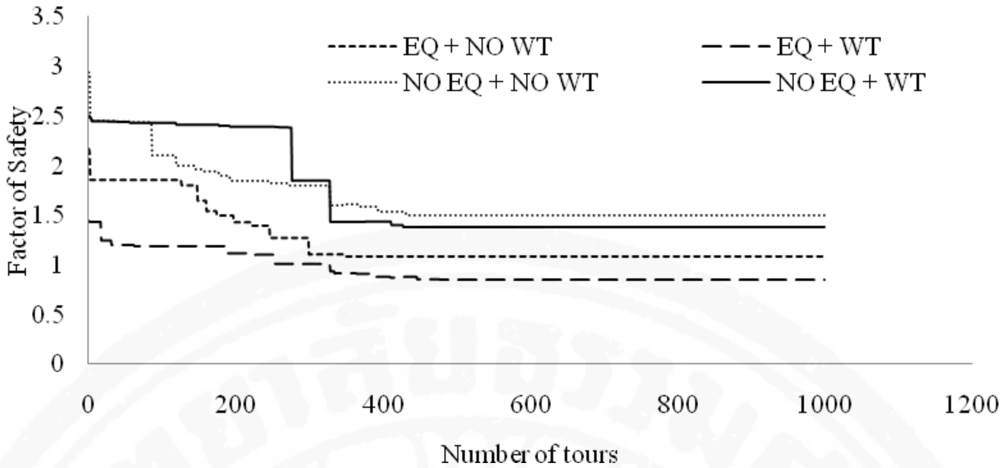

Figure 4.13 Convergence of the factor of safety—Example 6

The whole surfaces of failure for all cases are shown in Figure 4.12. It can be seen that the horizontal earthquake loading and the groundwater result in larger failing soil masses. Similar to the all other cases of more than one layer, the failure occurs from the weak layer between hard soil layers. Table 4.32 shows that, in all cases, the proposed results are in a satisfactory level with the results from the GA by Zolfaghari et al. (2005).

Table 4.32 Result comparison—Example 6

Source	Limit equilibrium method	Optimization method	*F* NO WT
Zolfaghari et al. (2005)			
EQ + WT	Morgenstern-Price [$f(x)=1$]	GA	0.98
NO EQ +WT			1.36
EQ + NO WT			1.37
NO EQ + NO WT			1.48
This study			
EQ + WT	Morgenstern-Price [$f(x)=1$]	ACO	0.846
NO EQ +WT			1.377
EQ + NO WT			1.091
NO EQ + NO WT			1.501