CHAPTER 2

LITERATURE REVIEW

2.1 Shear Lag Problem

In the elementary beam theory, the normal stress in the longitudinal direction produced by bending deformation is assumed to be proportional to the distance from the neutral axis and therefore uniform across the flange width. However, as a flange gets wider, this assumption becomes invalid: the normal stress distribution is not uniform in the wide flange, but the stress takes the maximum value at the flange-web intersection in general, decreasing toward the middle of the flange. This phenomenon is called the shear lag.

The shear lag has been studied for many years. The existing studies may be classified into two groups based on the type of analysis such as analytical approach and finite element approach.

2.1.1 Analytical approach

1. Plane stress (Winter,1940)
2. Least work method (Reissner,1941)
3. Principle of minimum potential energy (Reissner,1946)
4. Plane stress elasticity method (Song and Scordelis, 1990)
5. Theory of thin-walled members (Sedlacek and Bild, 1993)
6. Single Fourier series approach (Tahan et al., 1997)

Winter (1940) conducted a study on stress distributions in thin-walled steel beams on the basis of theory of plane stress. The effective width of flanges was proposed in form of tables and curves for the use in practical design. The best-known achievement in the past is probably the one due to Reissner (1941, 1946). He assumed the parabolic and linear normal stress distributions in a flange and a web, respectively. In order to simplify the analysis, infinite elastic modulus in the transverse direction of the flange was further assumed. Song and Scordelis (1990) reduced the shear lag problem to that of plane stress problem by assuming that the flanges and webs were infinitely flexible out of their own plane, and the stress in the webs could be determined by elementary beam theory. They then solved this elasticity problem by a Fourier series. Sedlacek and Bild (1993) performed an analytical study of the shear lag effect in wide-flange structures based on the theory of thin-walled members. Simplified formulas were then proposed. Tahan et al. (1997) reduced a shear lag problem to a plane-stress problem where the flange was loaded at its longitudinal edges by shear loading. The shear loading was determined by the shear flow in the elementary beam theory. Then, they applied the single Fourier series method to evaluate the shear lag effect.

2.1.2 Finite element approach

1. Shear lag in steel box girder bridges (Moffatt and Dowling, 1975)
2. Shear lag in orthotropic beam flanges and plates with stiffeners (Tenchev, 1996)
3. Shear lag analysis by the adaptive finite element method (Lee and Wu, 2000)
4. Stress concentration due to shear lag in simply supported box girders (Lertsima et al., 2004)
5. Stress concentration and deflection of simply supported box girder including shear lag effect (Yamaguchi et al., 2008)

Moffatt and Dowling (1975) assumed that the webs behaved in accordance with the elementary theory of bending and that the diaphragms at each support cross-section had infinite in-plane rigidity but no out-of-plane rigidity. As an analysis tool, they employed the finite element method with rectangular third-order extensional-flexural element. Tenchev (1996) analyzed the shear lag in orthotropic beam flanges and plates with stiffeners by using two-dimensional plane stress finite element model. The empirical formula of shear lag coefficient was obtained in term of ratios of half flange width to half length of beam, Young’s modulus to shear modulus of flange, and thickness of flange to thickness of web. Longitudinal flange stiffener has been accounted for by modifying the ratio of Young’s modulus to shear modulus. Lee and Wu (2000) reduced a shear lag problem to a two-dimensional plane stress problem. The adaptive technique was used to reduce discretization error. Lertsima et al. (2004) and Yamaguchi et al. (2008) have recently studies about the shear lag effect in simply supported box girder by using the three-dimensional finite element analysis. They proposed the empirical formulas for the stress concentration factor to account the shear lag effect.

2.2 Effective Width Concept

The concept of effective width was first proposed by Von Karman (1924) so as to take care of the effect of shear lag in thin-walled structures. Owing to its simplicity, the effective width approach has been widely adopted for the evaluation of stress concentration due to shear lag. To provide a simple approach for stress evaluations due to shear lag, the effective width ratio has been provided in some current design codes, e.g. DIN (1981), BSI (1982) and JRA (2002). The definition of the effective width is given as follows (Moffatt and Dowling, 1975):

$$B_e = \frac{1}{2\sigma_{max}} \int_0^{2B} \sigma_y \, dx \tag{2.1}$$

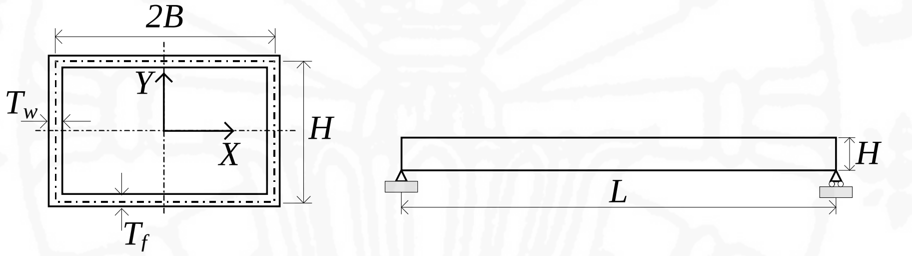
where B_e stands for the half effective width, and the numerator is the integration of the normal stress in the flange, σ_y, while the denominator is the actual maximum normal stress in the flange due to shear lag, σ_{max}. Eq. (2.1) simply implies that the analysis of shear lag is associated with the analysis of σ_y. In general, the effective width ratio, λ, which is defined as B_e to the half actual width (B) ratio is widely used since it can roughly notify how much the influence of the shear lag is (Eq. (2.2)).

$$\lambda = \frac{B_e}{B} \tag{2.2}$$

If the distribution of σ_y over the flange is approximately uniform, λ will be close to unity, which indicates that the degree of shear lag is very small. When the shear lag effect is severe, the distribution of σ_y across the flange is completely non-uniform with a very sharp gradient in the vicinity of web-flange intersections. In this way, σ_{max} arises considerably and results in significant reduction of B_e. As a result, λ will be much decreased and approaches zero.

2.3 Stress Concentration due to Shear Lag in Simply Supported Box Girder

Lertsima et al. (2004) and Yamaguchi et al. (2008) have recently studies about the shear lag effect in simply supported box girder by using the three-dimensional finite element analysis. The whole models were shell elements with two loading conditions of concentrated load at the mid-span and uniformly distributed load along the beam length. Figure 2.1 shows the structural geometry of simply supported box girder.



(a) Cross section (b) Side view

Figure 2.1 Structural geometry of simply supported box girder

They proposed the empirical formulas for the stress concentration that better than the effective width concept. Table 2.1 proved that the significant discrepancy was recognized, especially under concentrated load. K_c stands for the stress concentration factor defined by the ratio of the maximum normal stress in the flange to that of the elementary beam theory. The empirical formulas for K_c were proposed in Eq. (2.3), (2.4) and (2.5).

Table 2.1 *Kc* in literature for box girder with H/L = 1.0, B/H = 1.0 and T_f/T_w = 1.0

Literature	K_c	
	Concentrated load	Distributed load
Tenchev (1996)	1.31	1.07
British (1982)	1.39	1.05
Japan (2002)	1.23	1.09
Eurocode 3 (2003)	-	1.05
Song et al. (1990)	1.34	1.04
Sedlacek et al. (1993)	1.35	1.05
Tahan et al. (1997)	1.58	1.05
Lee et al.(2000)	1.56	1.05

Concentrated load (Load C-1):

$$K_c = a_1\left(\frac{H}{L}\right) + 1 \tag{2.3}$$

where

$$a_1 = b_1\left(\frac{B}{H}\right)^{c_1}$$

$$b_1 = 0.832\ln\left(\frac{T_f}{T_w}\right) + 2.77$$

$$c_1 = -0.034\ln\left(\frac{T_f}{T_w}\right) + 1.744$$

Concentrated load (Load C-2):

$$K_c = a_2\left(\frac{H}{L}\right) + 1 \tag{2.4}$$

where

$$a_2 = b_2\left(\frac{B}{H}\right)^{c_2}$$

$$b_2 = 1.756\ln\left(\frac{T_f}{T_w}\right) + 6.101$$

$$c_2 = 0.053\ln\left(\frac{T_f}{T_w}\right) + 1.202$$

Distributed load (Load D-1):

$$K_c = a_3\left(\frac{H}{L}\right)^2 + 1 \tag{2.5}$$

where

$$a_3 = b_3\left(\frac{B}{H}\right)^{c_3}$$

$$b_3 = 1.225\ln\left(\frac{T_f}{T_w}\right) - 0.494\left(\frac{T_f}{T_w}\right) + 6.001$$

$$c_3 = -0.041\ln\left(\frac{T_f}{T_w}\right) - 0.006\left(\frac{T_f}{T_w}\right) + 2.371$$

Lertsima et al. (2004) and Yamaguchi et al. (2008) concluded that the real stress can be much larger than those due to the beam theory. They showed that the existing studies including design codes may yield the stress quite different from each other and their result as well. The proposed formulas would be of some help to improve the current situation.

2.4 Shear Lag Effect in Simply Supported Box Girder with Longitudinal Stiffeners

The concept of effective width ratio was used to take care of the effect of shear lag. For the shear lag effect in simply supported box girder with longitudinal stiffeners, Tenchev (1996) provided the empirical formula for the calculation of the effective width ratio, λ by considering the loading and support conditions, the ratios *B/L*, *E/G* and T_f/T_w. The empirical formula was

$$\lambda = \frac{p}{C_1 C_2}\left(\frac{B}{\kappa L}\right)^q \left(\frac{\psi E}{G}\right)^r \left(\frac{T_f}{T_w}\right)^s \tag{2.6}$$

If $\lambda > 1; \qquad \lambda = 1$

where

$$C_1 = 1 + te^X$$
$$C_2 = 1 + ue^{vY}$$
$$X = -6.4\frac{B}{\kappa L}\sqrt{\frac{E}{G}}$$
$$Y = \left(\frac{E}{G}\right)\left(\frac{B}{\kappa L}\right)^{-1}$$
$$\psi = \frac{AE + A_s E_s}{AE}$$

in which *p, q, r, s, u, t, v* are the coefficients listed in Table 2.2. $e = 2.7183$ is the based on the natural logarithm, κL is the distance between the zero moment and the maximum moment cross section. As is the stiffeners total cross-sectional area at each flange, Es is Young's modulus of stiffeners and A, E are the cross-sectional area and Young's modulus of the flange, respectively.

Table 2.2 Value of the coefficient for simply supported beam

Load	κ	p	q	r	s	t	u	v
Concentrated load	1	0.384	-0.834	-0.389	-0.040	3.1	0.3	-1.0
Distributed load	1	0.570	-0.850	-0.416	0.0	5.0	0.31	-0.9

Eurocode 3 (2003) also determined the effective width ratio, λ from Eq. (2.7).

$$\left.\begin{array}{lll} \text{For} & \kappa \le 0.02\,; & \lambda = 1.0 \\ & 0.02 < \kappa \le 0.70\,; & \lambda = \dfrac{1}{1+6.4\kappa^2} \\ & \kappa > 0.70\,; & \lambda = \dfrac{1}{5.9\kappa} \end{array}\right\} \qquad (2.7)$$

Where $\kappa = \dfrac{\alpha_0 B}{L}$

$$\alpha_0 = \sqrt{1+\frac{A_s}{BT_f}}$$

The effective width B_e for shear lag can be determine from Eq. (2.2)