CHAPTER 3

STRESS CONCENTRATION DUE TO SHEAR LAG IN SIMPLY SUPPORTED BOX GIRDERS WITH LONGITUDINAL STIFFENERS

3.1 Introduction

The early studies on shear lag are based on analytical approach. Timoshenko and Goodier (1970) have documented one of the earliest researches due to von Karman (1924). Reissner (1941, 1946) used the principle of minimum potential energy to formulate the governing differential equation and boundary conditions. The previous study confirmed by the present FEA that the shear lag depends on loadings conditions (Reissner, 1941, Moffatt and Dowining 1975, Song and Scordelis 1990, Lertsima et al. 2004, Yamaguchi et al. 2008). Although many research focus on shear lag problem, there are only a few research on the shear lag in stiffened box girder. Tenchev (1996) and Eurocode 3 (1993) presented the solution for shear lag effect on plates with stiffeners. The concept of effective width ratio was used to take care of the effect of shear lag. Tenchev (1996) analyzed the shear lag in orthotropic beam flanges and plates with stiffeners by using two-dimensional plane stress finite element model. The empirical formula of shear lag coefficient was obtained in term of ratios of half flange width to half length of beam, Young's modulus to shear modulus of flange, and thickness of flange to thickness of web. Longitudinal flange stiffener has been accounted for by modifying the ratio of Young's modulus to shear modulus. Similarly Eurocode 3 (2003) also gave the design of effective width factor in term of the ratios of half the width to span length, and the ratio of stiffener area to flange area.

This Chapter aims to investigate the shear lag effect on stress concentration at the mid-span of simply supported box girder with longitudinal stiffener. In the finite element analysis, three-dimensional shell element is used. The local effects due to boundary and loading conditions are carried out. Various values of geometric properties of a stiffened box girder are also considered. MARC (1994), a finite element program is used in this finite element analysis. Based on the numerical results, empirical formulas are presented to simplify the shear lag effect and to make them useful to the design engineer.

3.2 Parametric Study

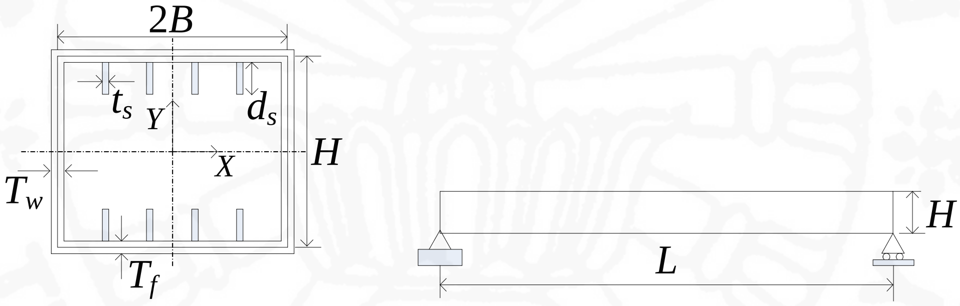
For various values that characterized the geometry of a simply supported box girder, the finite element analysis is conducted to reveal their influences on the stress concentration due to shear lag. The dependence of shear lag on the H/L, B/H and T_f/T_w ratios is recognized (Lertsima et al. 2004, Yamaguchi et al. 2008). In particular, the following values are considered: H/L = 0.025, 0.05, 0.10, 0.15, 0.20; B/H = 0.5, 1.0, 1.5, 2.0; T_f/T_w = 0.5, 1.0, 1.5, 2.0.

Shear lag effect of longitudinal stiffeners in a simply supported box girder is extended in the present study. Longitudinal stiffeners are often attached to resist the induced compressive stress and increase the buckling strength of the thin-walled box girder. To simplify the models, the stiffeners are attached to the top and bottom flange. The size and number of stiffeners are designed according to AASHTO Standard Specification (1994). The height, d_s, of one stiffener shall satisfy:

$$\frac{d_s}{t_s} \leq 0.48\sqrt{\frac{E}{F_{yc}}} \tag{3.1}$$

in which d_s = height of one stiffener; t_s = thickness of one stiffener; E = Young's modulus; and F_{yc} = specified minimum yield strength of the compression flange.

The longitudinal stiffeners in a beam are considered: A_s/A_f = 0, 0.5, 1.0 in which A_s = total area of stiffeners on each flange and A_f = area of the flange. For the actual modeling, the values of B and H/T_w are fixed equal to 600 mm and 100, respectively, while the other values are changed to attain all values of the parameters. The combination of all these values results in 240 box girders different from each other in geometry. The symbols employed in the present study for describing the structural geometry are illustrated in Figure 3.1.

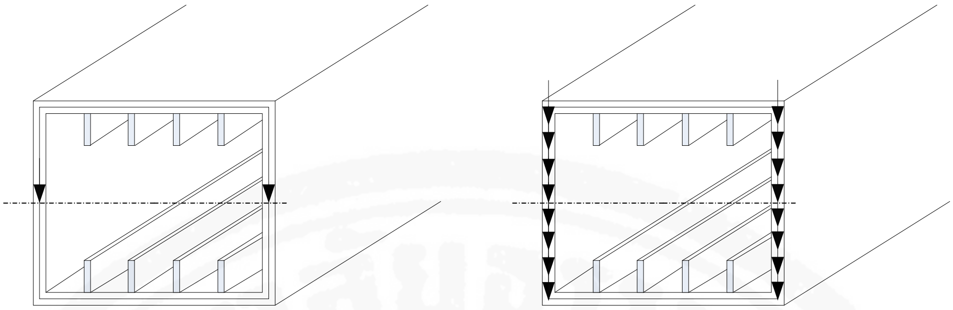


(a) Cross section (b) Side view

Figure 3.1 Structural geometry of simply supported box girder with longitudinal stiffeners

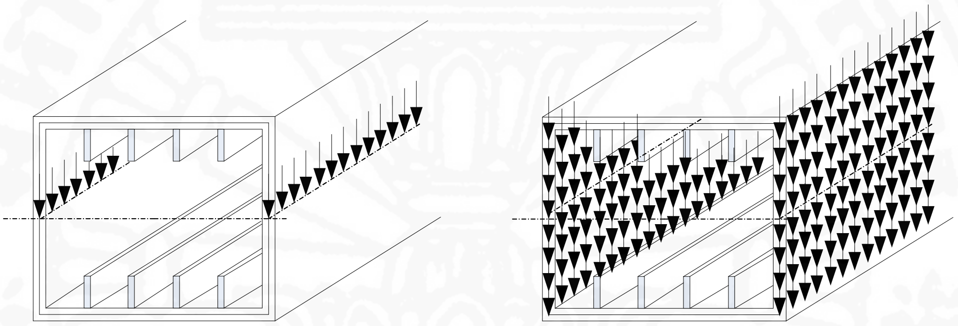
3.3 Finite Element Model

In the present study the concentrated load at the mid-span and uniformly distributed load along the beam length were applied in various ways. The loading in the plane of the web are considered. To evaluate the local effect on loading condition, two loading models are used for concentrated load: Load C-1 is a concentrated load at the middle of the web and Load C-2 is a uniformly distributed load along the height of the web, as shown in Figure 3.2. Two loading models are also used for uniformly distributed load: Load D-1 is a uniformly distributed load along the centerline of the web and Load D-2 is a uniformly distributed load not only along the beam axis but also along the web height of every cross section as shown in Figure 3.3.



(a) Load C-1 (b) Load C-2

Figure 3.2 Concentrated load



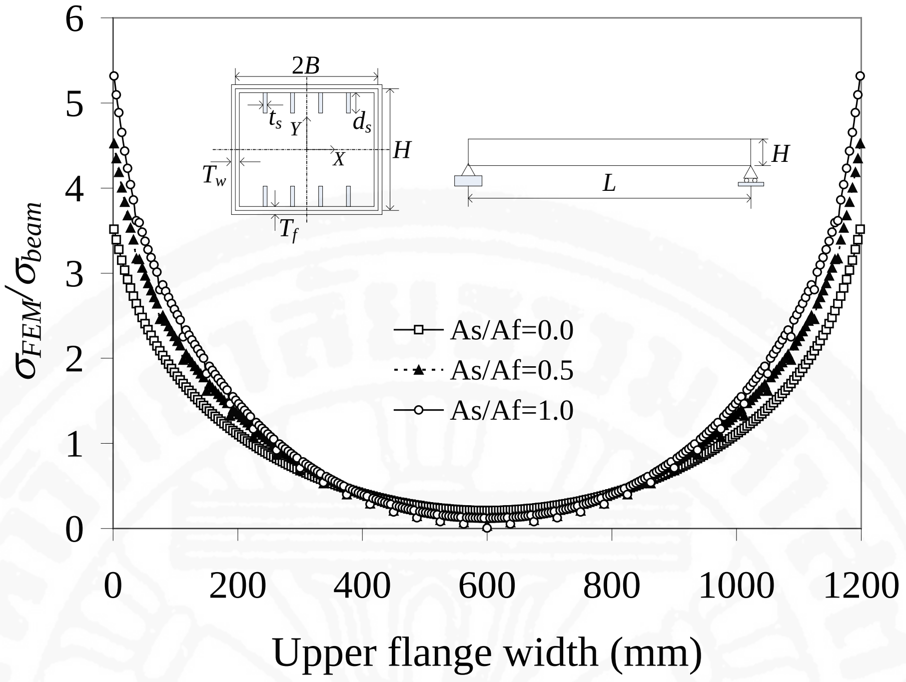
(a) Load D-1 (b) Load D-2

Figure 3.3 Distributed load

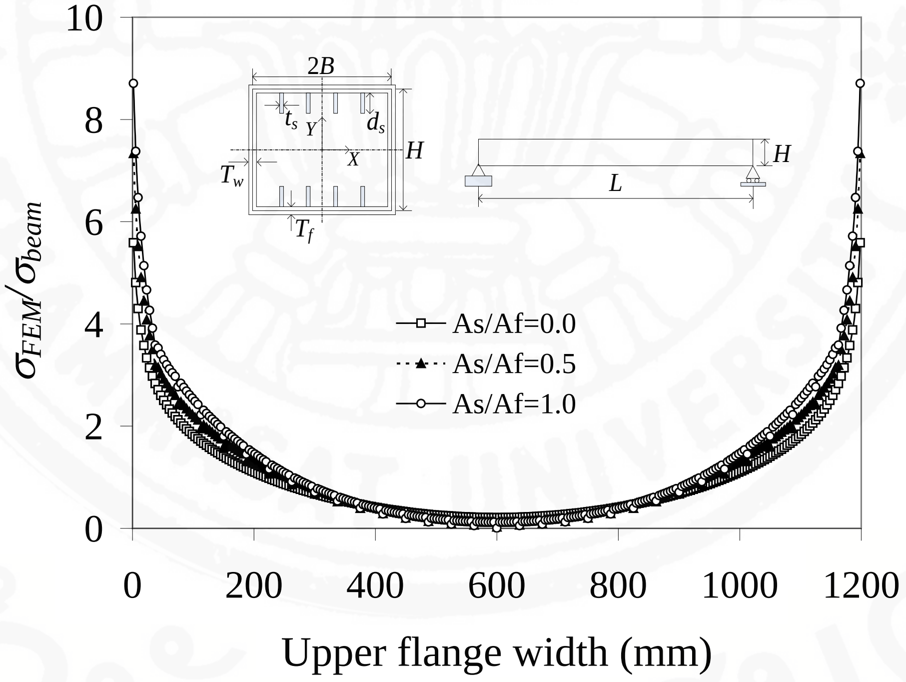
For the FEM model, the structural is modeled by using three-dimensional 4-node shell elements. Due to symmetry, a quarter of cross section is modeled. Multiple finite element meshes are used to eliminate discretization error by the multimesh extrapolation method for every girder under a specific loading condition (Cook et al. 1989, Lertsima et al. 2004, Yamaguchi et al. 2008).

3.4 Normal Stress Distribution in Upper Flange

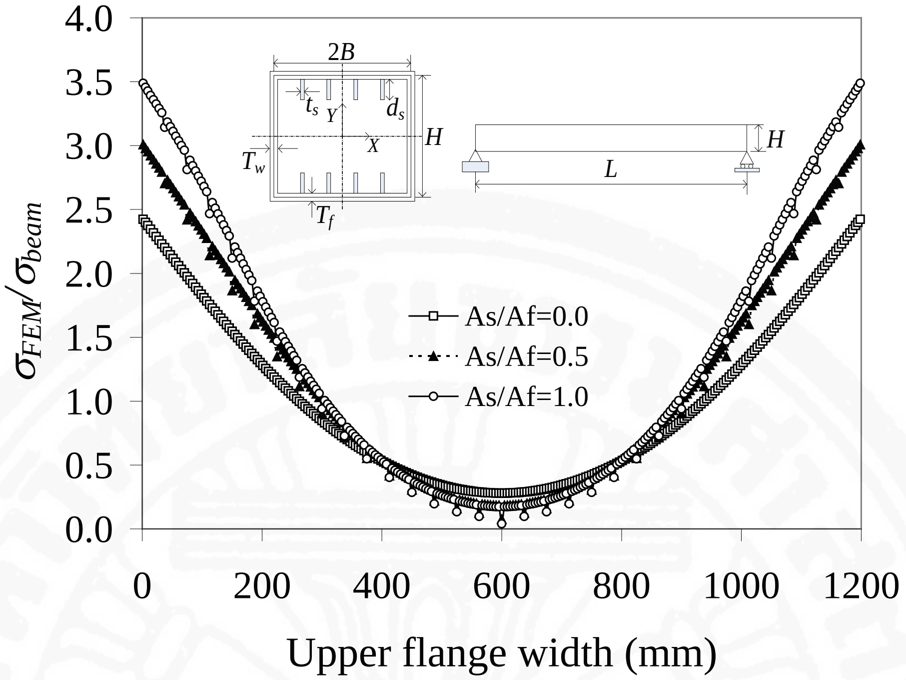
The present study put emphasis on the difference between the maximum normal stress ($\sigma_{\max}$) due to shear lag effect and the normal stress obtained by the elementary beam theory (σ_{beam}). The ratio of these stresses is defined as the stress concentration factor (K_c) for simple measure of the shear lag effect. The normal stress distribution in the upper flange at the mid span of simply supported beam with B/H = 2.0, H/L =0.2 and T_f/T_w = 2.0 is presented in Figure 3.4. K_c ($\sigma_{\max}/\sigma_{beam}$) increases with the increase of A_s/A_f.



(a) Concentrated load (C-1)



(b) Concentrated load (C-2)



(c) Distributed load (D-1)

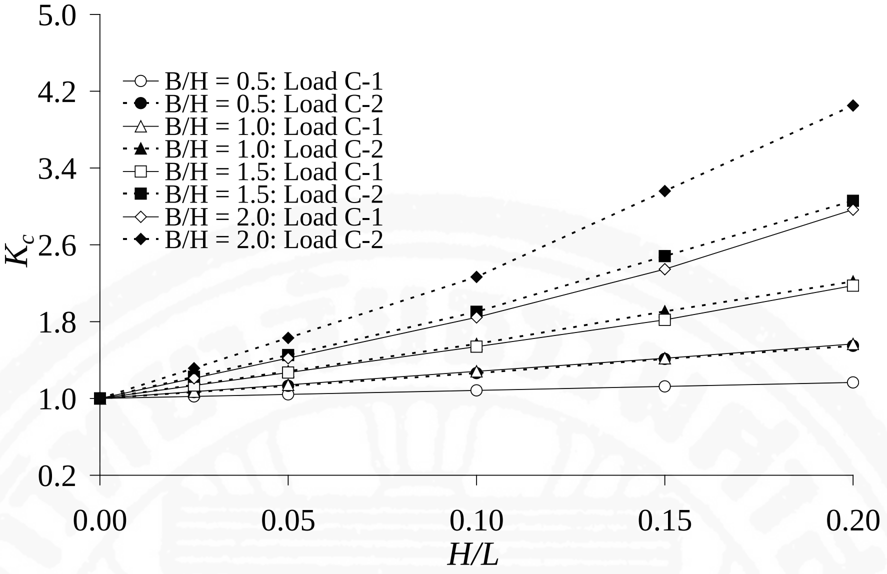
Figure 3.4 Normal stress distribution in the upper flange
(B/H = 2.0, H/L =0.2, T_f/T_w = 2.0)

The shear lag effect on simply supported box girder with longitudinal stiffeners depends upon five factors:

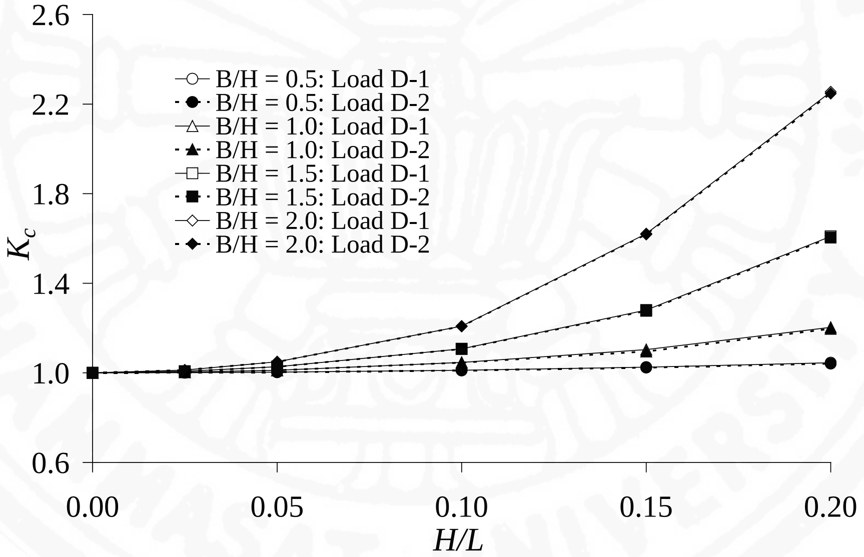
a) the type of loading
b) the half flange width/height of web ratio of the girder (B/H)
c) the height of web/span length ratio of the girder (H/L)
d) the thickness of flange/web ratio of the girder (T_f/T_w)
e) the cross sectional area of the stiffeners/area of the flange ratio of the girder (A_s/A_f)

3.5 Effect of Loading

Figure 3.5 shows K_c for cross section of T_f/T_w = 1.0 and A_s/A_f = 0 under concentrated and uniformly distributed load. The difference of K_c between Load C-1 and Load C-2 is significantly larger in case of wide flange (large vale of B/H). While the difference of shear lag effect between Load D-1 and Load D-2 is insignificant (Lertsima et al. 2004; Yamaguchi et al. 2008). Therefore Load D-2 will not be accounted in this present study.



(a) Concentrated load



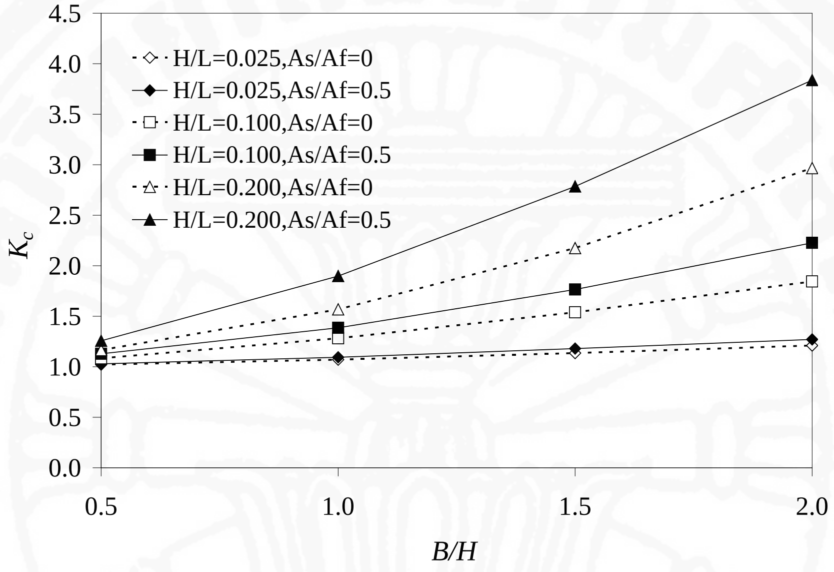
(b) Distributed load

Figure 3.5 Variation of K_c with respect to H/L ($T_f/T_w = 1.0$, $A_s/A_f = 0$)

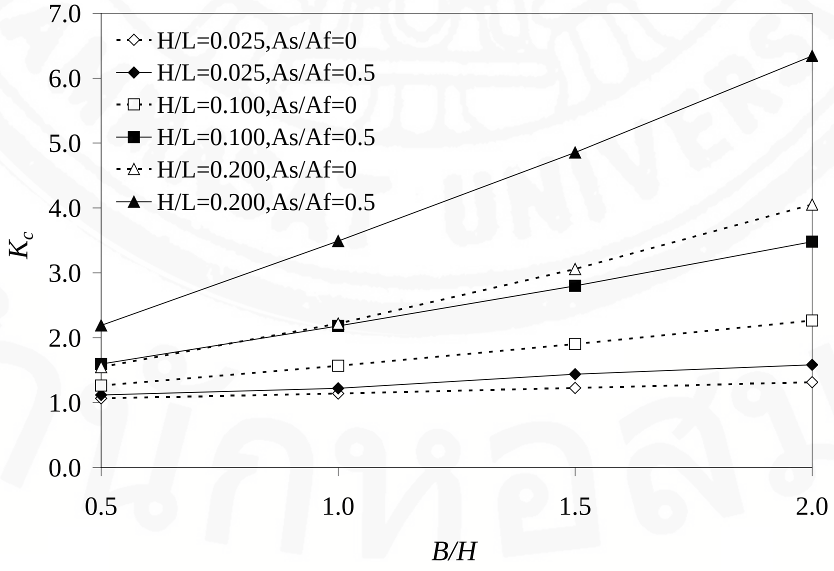
3.6 Effect of Geometric Property

3.6.1 Half Flange Width/Height of Web Ratio (*B/H*)

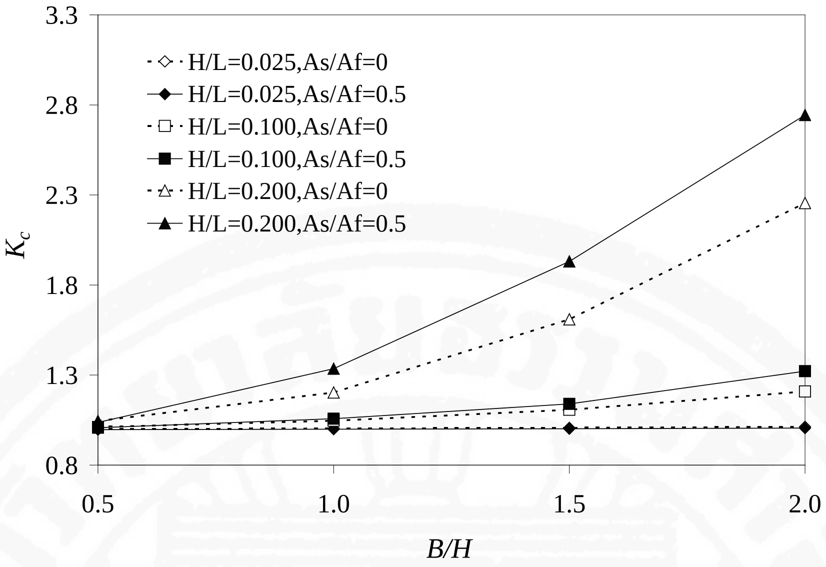
K_c becomes larger as B/H increases, as the flange get wider, follow the shear lag phenomenon. K_c tends to grow with the increase of *B/H* for large *H/L*. The influence of *B/H* on K_c is very small for *H/L* equal to 0.025 in the both case of concentrated load (C-1 and C-2) and uniformly distributed load (D-1) as shown in Figure 3.6.



(a) Load C-1



(b) Load C-2

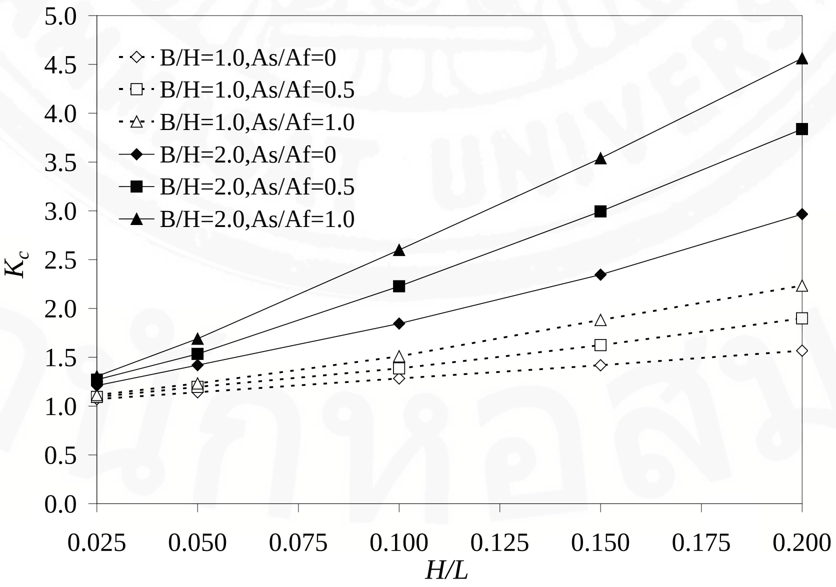


(c) Load D-1

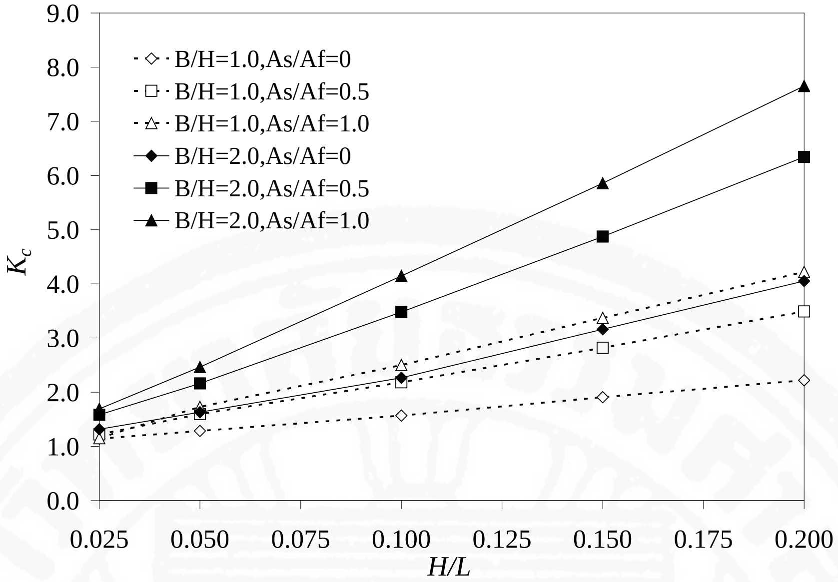
Figure 3.6 Variation of K_c with respect to *B/H* (T_f/T_w = 1.0)

3.6.2 Height of Web/Span Length Ratio (*H/L*)

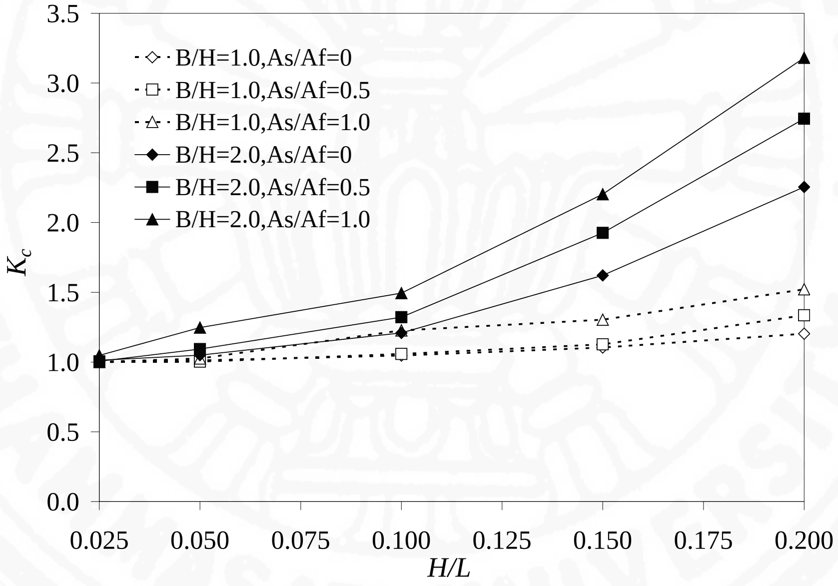
H/L has considerable influence on K_c: as *H/L* becomes larger, K_c increases in general as shown in Figure 3.7. The relationship between K_c and *H/L* is rather linear in case of the concentrated load (Load C-1 and C-2) and nonlinear in case of distributed load (Load D-1) for larger *B/H*.



(a) Load C-1



(b) Load C-2

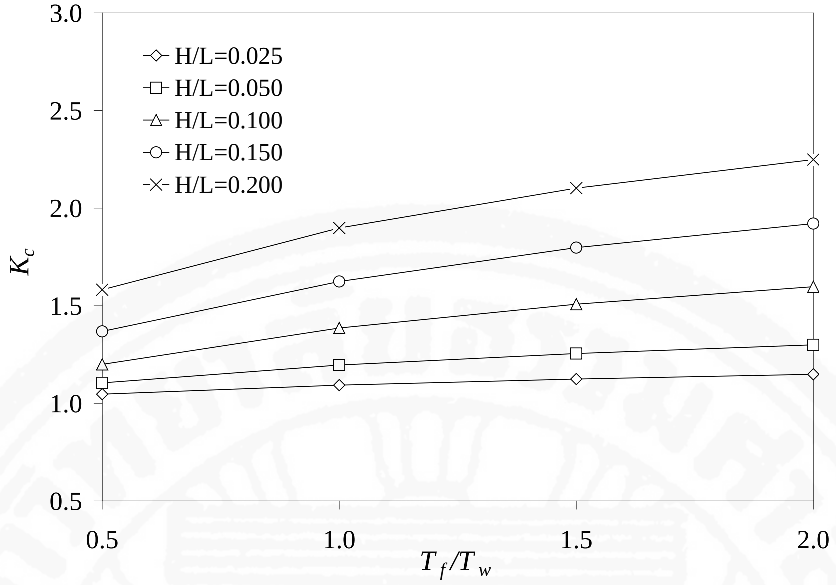


(c) Load D-1

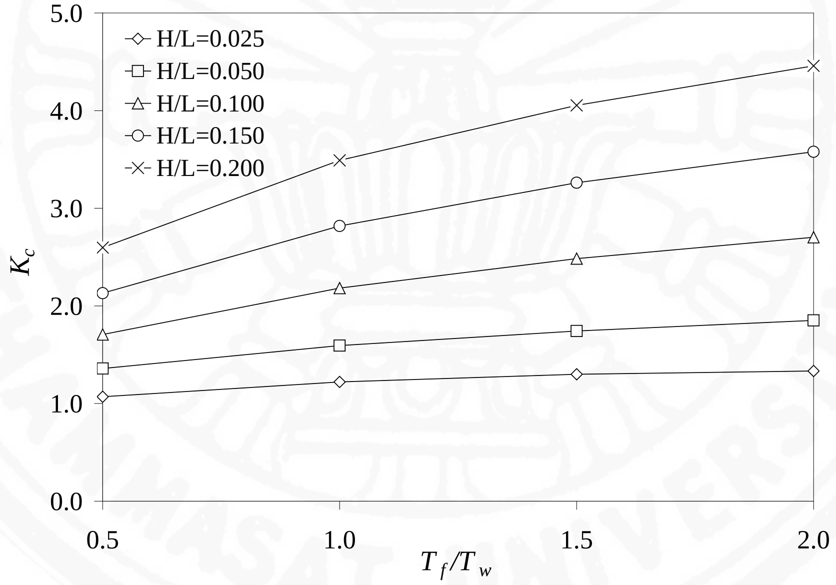
Figure 3.7 Variation of K_c with respect to H/L (T_f/T_w = 1.0)

3.6.3 Thickness of Flange/Web Ratio (T_f/T_w)

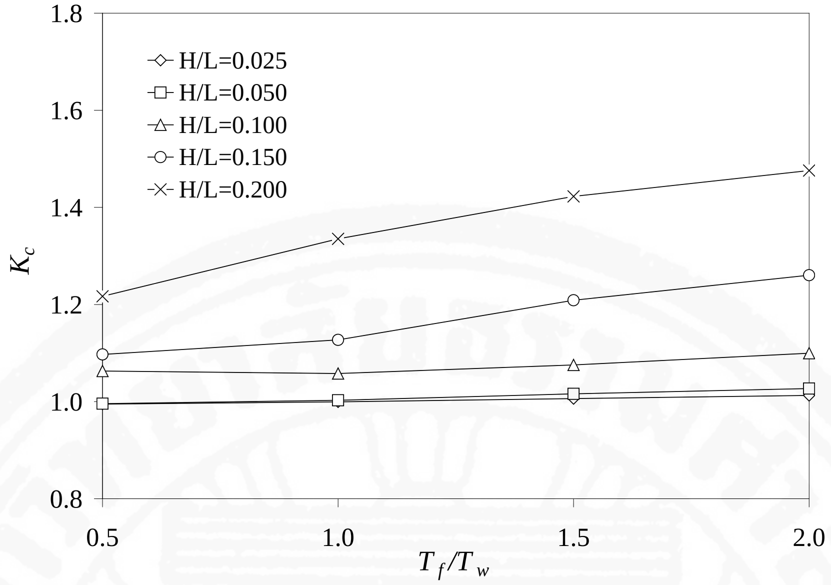
K_c tends to grow with the increase of T_f/T_w as shown in Figure 3.8. The effect of T_f/T_w is therefore also significant. The influence of T_f/T_w on K_c is very small for H/L equal to or smaller than 0.05 in the case of concentrated load and distributed load.



(a) Load C-1



(b) Load C-2

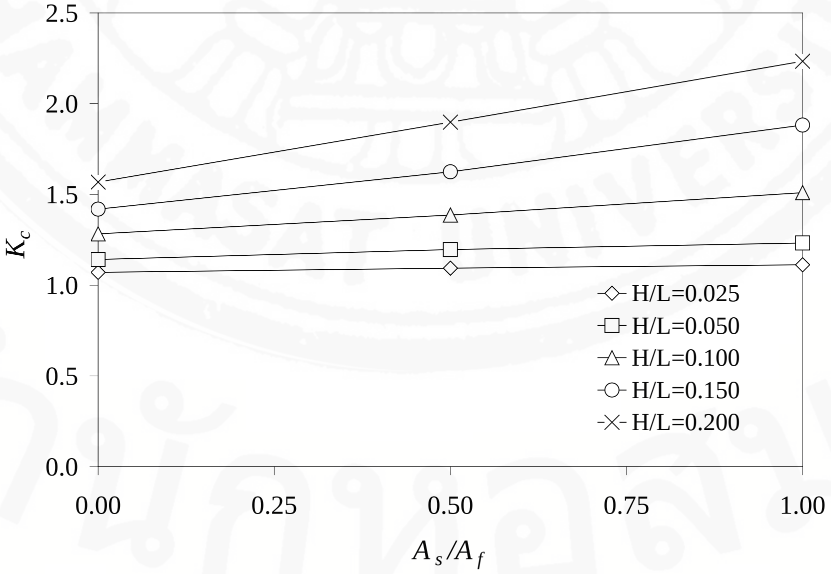


(c) Load D-1

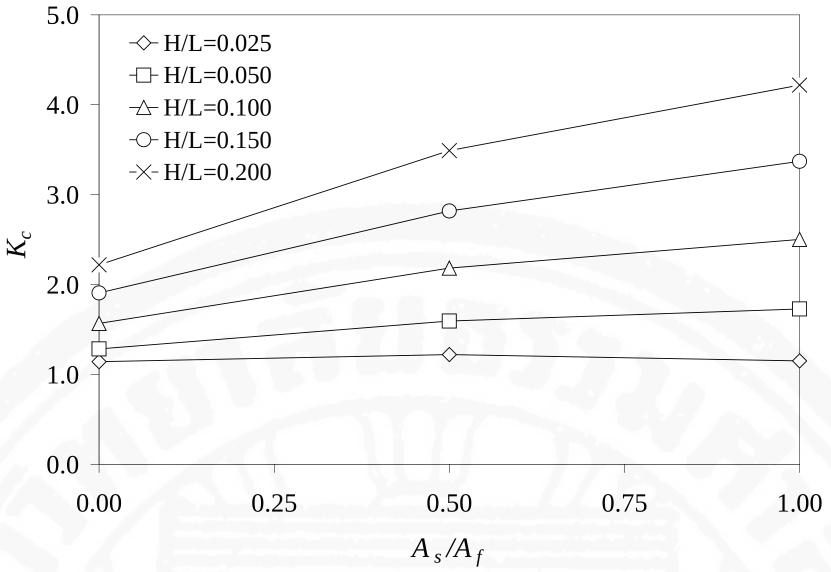
Figure 3.8 Variation of K_c with respect to T_f/T_w ($A_s/A_f = 0.5$, $B/H = 1.0$)

3.6.4 Cross Sectional Area of Stiffeners/Area of Flange Ratio (A_s/A_f)

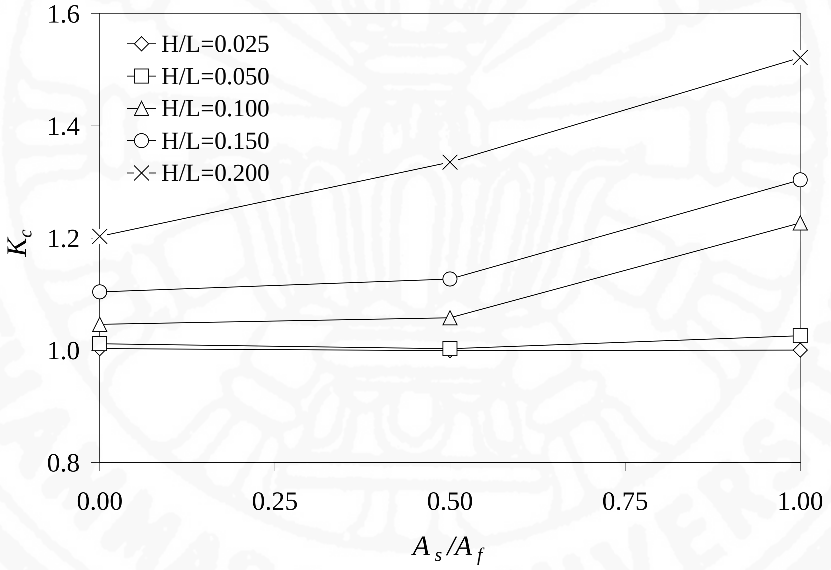
K_c tend to increase significantly with the increase of A_s/A_f as shown in Figure 3.9. The effect of the shear lag is large in case of concentrated load.



(a) Load C-1



(b) Load C-2



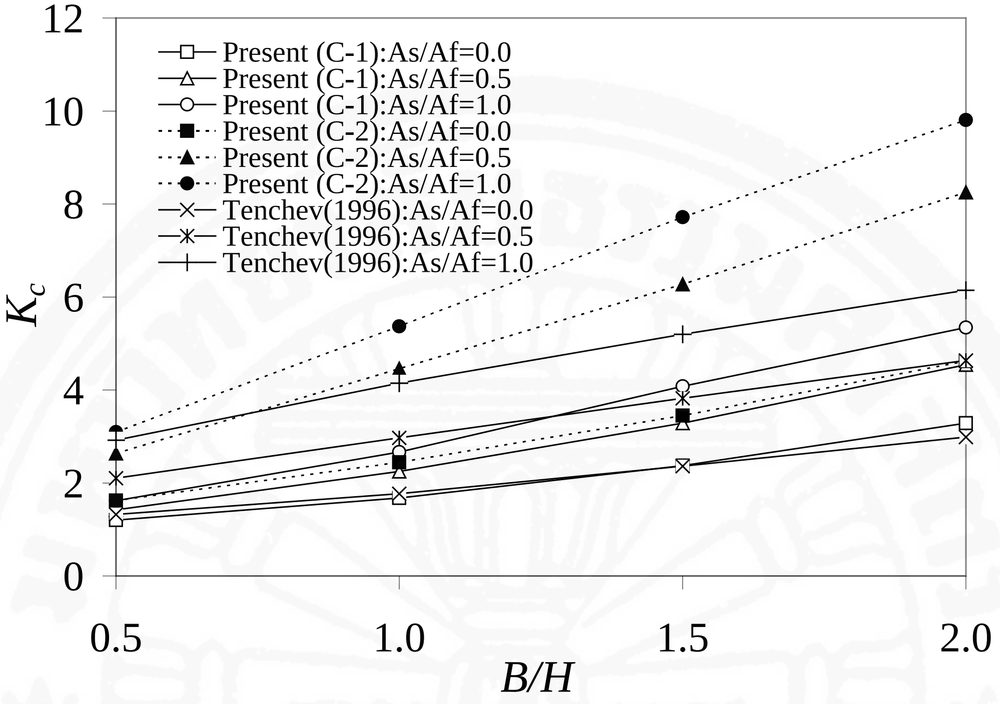
(c) Load D-1

Figure 3.9 Variation of K_c with respect to A_s/A_f (B/H = 1.0, T_f/T_w =1.0)

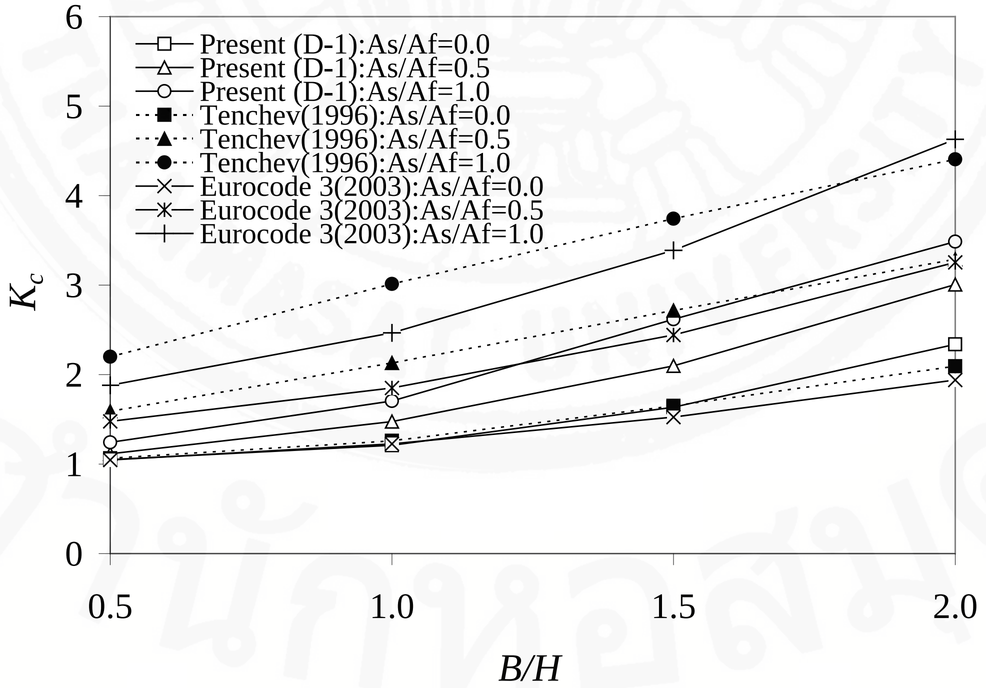
3.7 Comparison of K_c

The shear lag problem on the plate with stiffeners is studied in Tenchev (1996) and Eurocode 3 (2003). Figure 3.10 shows K_c values based on Tenchev (1996) and Eurocode 3 (2003) together with the present FEA results for a box girder with H/L =0.2 and T_f/T_w =2.0 under the concentrated load and distributed load. For the concentrated load, Load C-2 yield much larger K_c than the existing research works when the longitudinal stiffeners are attached to the box girder. On the other hand, K_c

due to present FEA is smaller than the existing researches for the uniformly distributed load (D-1).



(a) Concentrated load



(b) Distributed load

Figure 3.10 Comparison of K_c (H/L = 0.2, T_f/T_w =2.0)

3.8 Empirical Formulas

3.8.1 Proposed Formulas

Yamaguchi et al. (2008) proposed the empirical formulas for stress concentration of simply supported box girder including shear lag effect. The stress concentration factor (K_c) stands for the ratio of the maximum normal stress ($\sigma_{\max}$), which is calculated by the present FEA to the elementary beam theory stress (σ_{beam}).

To take into account the effect of longitudinal stiffeners, a multiplier factor ϕ is introduced and the empirical formulas (Yamaguchi et al., 2008) are modified. The following expression for the empirical evaluation of stress concentration factor (K_c) is proposed.

$$K_c = (\phi)\cdot(a)\cdot\left(\frac{B}{H}\right)^b\cdot\left(\frac{H}{L}\right)^c + 1 \tag{3.2}$$

where

$$\phi = \left(1+\frac{A_s}{A_f}\right)^d \tag{3.2a}$$

$$a = (e)\cdot\ln\left(\frac{T_f}{T_w}\right) + (f)\cdot\left(\frac{T_f}{T_w}\right) + g \tag{3.2b}$$

$$b = (h)\cdot\ln\left(\frac{T_f}{T_w}\right) + (i)\cdot\left(\frac{T_f}{T_w}\right) + j \tag{3.2c}$$

The unknown coefficients are determined from a data analysis program, Statistica of Statsoft, Inc. The results are presented in Table 3.1. The loading conditions C-1, C-2 and D-1 are given in Figure 3.2 and 3.3, respectively.

Table 3.1 Values of the coefficients in Eq. (3.2)

Load	c	d	e	f	g	h	i	j
C-1	1	0.9	0.832	0	2.77	-0.034	0	1.744
C-2	1	1.3	1.756	0	6.101	0.053	0	1.202
D-1	2	1.0	1.225	-0.494	6.001	-0.041	-0.006	2.371

It is note that the above formulas are applicable for $0.025 \le \frac{H}{L} \le 0.2$, $0.5 \le \frac{B}{H} \le 2.0$, $0.5 \le \frac{T_f}{T_w} \le 2.0$ and $0 \le \frac{A_s}{A_f} \le 1.0$.

3.8.2 Accuracy of the Proposed Formulas

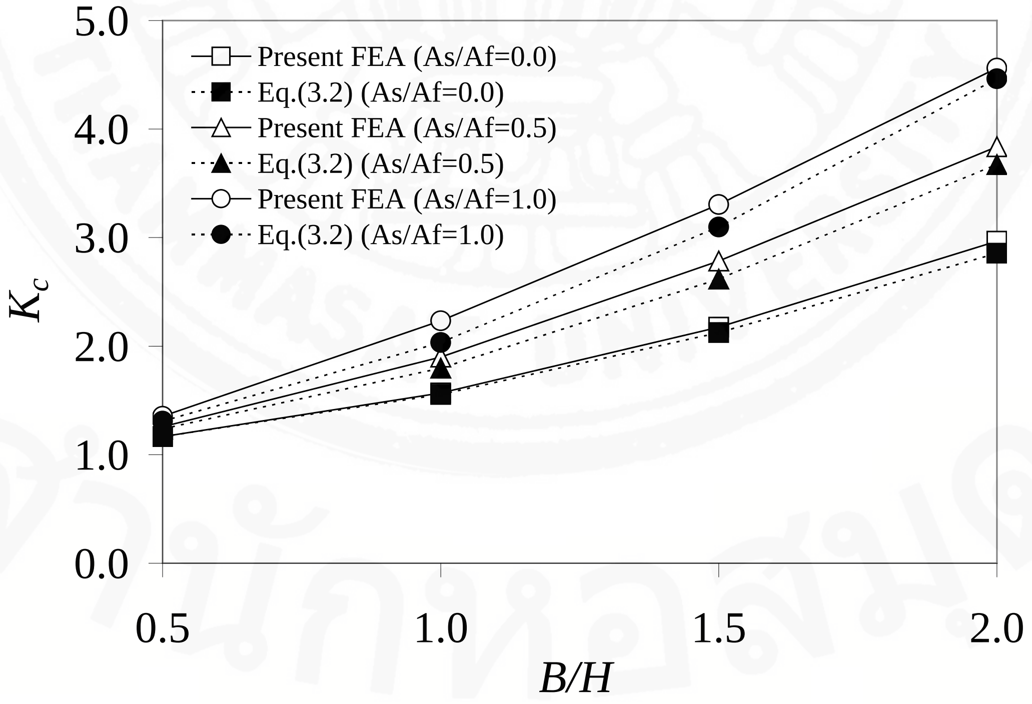
Figure 3.11 illustrates some comparisons between K_c due to the empirical formula and the present finite element analysis (FEA) with a good agreement. The overall accuracy of the proposed formula for each loading condition is calculated as the mean square error by the following equation:

$$\bar{\varepsilon} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\varepsilon_i^2} \tag{3.3}$$

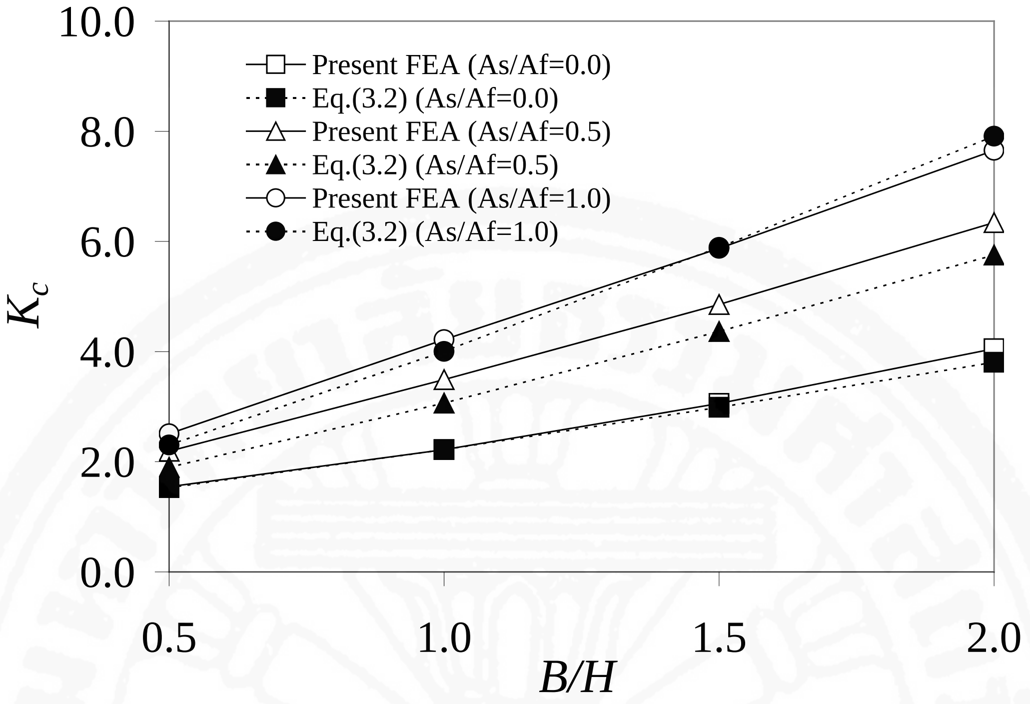
where

$$\varepsilon_i = \frac{K_{cEmp} - K_{cFEA}}{K_{cFEA}} \times 100 \quad (\%) \tag{3.4}$$

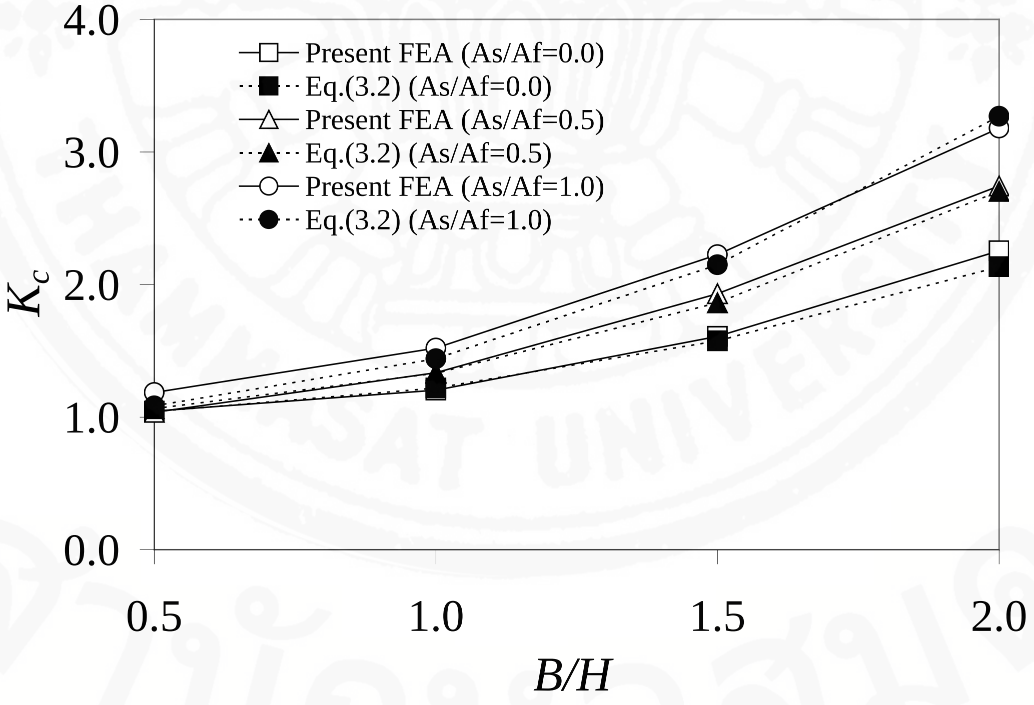
where N is the number of the present finite element results for a loading condition, and K_{cEmp} and K_{cFEA} are the K_c values obtained from the proposed formulas and the present finite element analysis, respectively. Since in the present study, the combination of the geometrical parameters has required 240 box girders to be analyzed for each loading, N in Eq. (3.3) is equal to 240. Using Eq. (3.3), the mean square error is found to be 5.73%, 8.92% and 4.34% for Load C-1, C-2 and D-1, respectively.



(a) Concentrated load (C-1)



(b) Concentrated load (C-2)



(c) Distributed load (D-1)

Figure 3.11 K_c due to proposed formulas and finite element analysis
(H/L = 0.20, T_f/T_w = 1.0)

3.9 Concluding Remarks

Three-dimensional finite element analysis of simply supported box girder with longitudinal stiffeners has been performed to reveal the shear lag problem. The stress concentration factor (K_c) is used to define the shear lag effect on stress concentration. Two loading conditions, concentrated and uniformly distributed load, are applied to various cross-sections of box girders. The numerical results show that the shear lag effect increases with the increase of H/L, B/H, T_f/T_w and A_s/A_f. Although, the longitudinal stiffeners decrease the compressive stress on the top flange, FEA show that the longitudinal stiffeners have influence on the shear lag especially in the concentrated load. Based on the numerical results, the empirical formulas are proposed for shear lag calculation in the simply supported box girder with longitudinal stiffeners suitable for practicing engineer.